Joint Optimization of User Set Selection and Transmit Power Allocation for Orthogonal Random Beamforming in Multiuser MIMO Systems

Tae-Sung Kang and Bangwon Seo

When the number of users is finite, the performance improvement of the orthogonal random beamforming (ORBF) scheme is limited in high signal-to-noise ratio regions. In this paper, to improve the performance of the ORBF scheme, the user set and transmit power allocation are jointly determined to maximize sum rate under the total transmit power constraint. First, the transmit power allocation problem is expressed as a function of a given user set. Based on this expression, the optimal user set with the maximum sum rate is determined. The suboptimal procedure is also presented to reduce the computational complexity, which separates the user set selection procedure and transmit power allocation procedure.

Keywords: Multiuser MIMO, random beamforming, power allocation, user set selection, joint optimization.

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Tae-Sung Kang (phone: +82 10 8080 6069, tourjour7@daum.net) is with the the Cloud Business Division, LG U Plus, Seoul, Rep. of Korea.

Bangwon Seo (corresponding author, seobw@etri.re.kr) is with the Advanced Communications Research Laboratory, ETRI, Daejeon, Rep. of Korea.

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I. Introduction

In downlink multiuser multiple-input multiple-output (MIMO) systems, when a base station (BS) with M transmit antennas communicates with K mobile users, each of which has a single receive antenna, sum capacity is achieved using dirty paper coding (DPC) or transmit beamforming schemes [1]-[3], and the sum capacity is linearly increased with min(M, K). However, these methods require the condition that the BS transmitter has perfect channel state information (CSI). In practice, it is difficult to satisfy this condition, particularly in frequency division duplexing systems, due to large feedback overhead caused by a large number of transmit antennas and users.

As a partial CSI feedback method, opportunistic transmission has been employed in multiuser communication systems [4]-[6]. In opportunistic beamforming (OBF), the BS randomly selects a beam for transmission and uses it to send a pilot sequence. The users send back their signal-to-noise ratio (SNR) corresponding to this beam, and the BS schedules the user with the highest SNR [4]. The performance of the OBF approaches that of the optimal beamforming for a large number of users [4].

However, since the OBF uses only one data stream at a time, it cannot obtain spatial multiplexing gain like DPC can. As a solution to this, an orthogonal random beamforming (ORBF) scheme was proposed [6]. In this scheme, a BS sends multiple orthogonal random beams simultaneously and each user reports the best beam index and its signal-to-interference plus noise ratio (SINR) to the BS. Then, the BS schedules multiple users with the highest SINRs simultaneously. The sum rate performance of the ORBF scheme approaches that of the DPC as the number of users becomes infinite.

However, in the case that the number of users is not large enough, the performance improvement of the ORBF scheme is slight in a high SNR region since there is a large multiuser interference in the received signal and it is more dominant than background noise in a high SNR region. Since the number of beams in the ORBF is equal to the number of transmit antennas, the multiuser interference increases in proportion to the number of transmit antennas. Therefore, there can be some optimal user sets to which the BS schedules beams whereby the number of users that compose a user set can be less than the number of transmit antennas.

In addition, in the ORBF scheme, the users to which beams are assigned suffer from interference. Hence, assigning more transmit power to some users may increase the interference with the other users. Improving SINRs of some users may degrade SINRs of other users. Hence, transmit power allocation should consider interference among users as well as their channel gain.

In this paper, we propose joint optimization of the user set selection and transmit power allocation to improve the performance of the ORBF scheme. In the optimal method, each user feeds back the magnitude of channel gain for each beam. Based on this information, the BS selects the best user set and optimally determines the amount of transmit power to be assigned to each beam. The optimal transmit power allocation is represented as a closed expression of a given user set using a vector-matrix form. In addition to this, we present a practical suboptimal method to reduce the huge computational complexity of the optimal method.

The rest of this paper is organized as follows. In section II, the system model and problem formulation are described. In section III, we propose joint optimization schemes for user set selection and transmit power allocation: an optimal method and a suboptimal method. Section IV shows simulation results, and section V concludes the paper.

II. System Model and Problem Formulation

Figure 1 shows a downlink multiuser MIMO system considered in this paper in which a BS with M transmit antennas communicates with K mobile users, each equipped with a single receive antenna. It is assumed that $K>M$ and the channel of each user does not vary during the scheduling interval T. As shown in Fig. 1, M users among K users are scheduled and the signals of the scheduled M users are then transmitted via a set of M random beams. Therefore, the transmitted symbol vector $\mathbf{x}=[x_1,\dots,x_M]^T$ can be written as

$$\mathbf{x}=\sum_{m=1}^{M}\sqrt{p_m}\,\mathbf{v}_m s_m, \tag{1}$$

where $\mathbf{v}_m$ is the m-th orthonormal random vector generated according to isotropic distribution [8], s_m is the transmit symbol for the m-th beam, and p_m is the transmit power for the m-th beam.

We assume the total transmit power is P, that is, $E[\|\mathbf{x}\|^2]=\sum_{m=1}^{M}p_m=P$ and $0\le p_1,\dots,p_M\le P$. The received signal, y_k, of the k-th user can be represented as

$$y_k=\sum_{m=1}^{M}\sqrt{p_m}\,\mathbf{h}_k\mathbf{v}_m s_m+w_k,\quad k=1,\dots,K, \tag{2}$$

where $\mathbf{h}_k$ is a $1\times M$ channel vector of user k, whose entries are independent and identically distributed with zero mean and unit variance, and w_k is an additive white complex Gaussian noise with zero mean and unit variance.

The BS broadcasts pilot sequences through M orthogonal random beams, and each user measures its channel vector and feeds back $|\mathbf{h}_k\mathbf{v}_m|$ for $m=1,\dots,M$ to the BS. Based on this information, the BS obtains SINRs of all users for scheduling and power allocation. The SINR value, $\gamma_{k,m}$, for the m-th

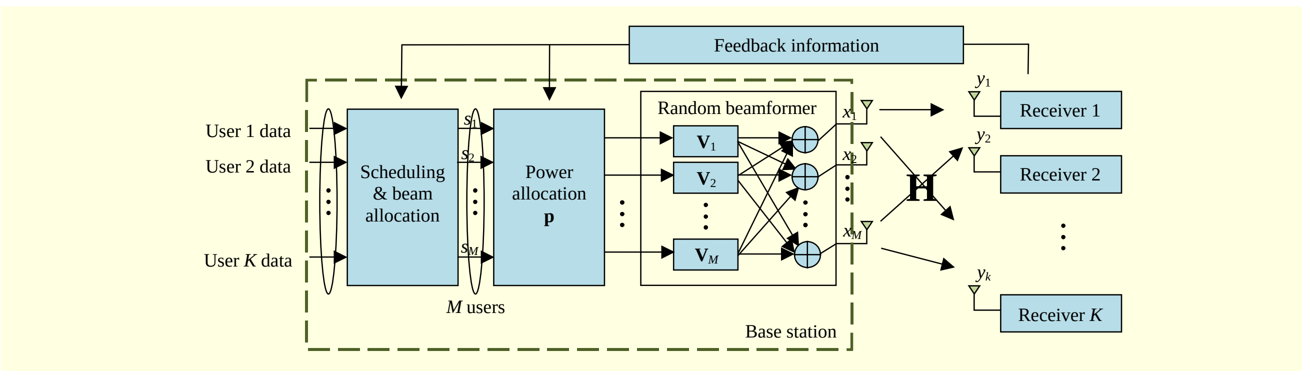

Fig. 1. MIMO downlink system model with M transmit antennas. Block diagram shows ORBF combined with power allocation, where $\mathbf{H}=[\mathbf{h}_1^T,\dots,\mathbf{h}_K^T]^T$.

received signal of the k-th user is represented as

$$\gamma_{k,m} = \frac{p_m g_{k,m}}{\sum_{i=1, i\neq m}^{M} p_i g_{k,i} + 1}, \quad m = 1, \ldots, M, \quad k = 1, \ldots, K, \tag{3}$$

where $g_{k,m} = |\mathbf{h}_k \mathbf{v}_m|^2$.

The sum rate is represented as

$$\sum_{k\in U, m\in S} \log\left(1 + \frac{p_m g_{k,m}}{\sum_{i=1, i\neq m}^{M} p_i g_{k,i} + 1}\right), \tag{4}$$

where $S = \{1, \ldots, M\}$ and $U \subset \{1, \ldots, K\}$ is a user index set with M elements without repetition.

III. Proposed User Selection and Transmit Power Allocation Scheme

1. Optimal Joint Scheme for User Set Selection and Transmit Power Allocation

A joint optimization problem for optimal user set selection and transmit power allocation can be formulated to achieve the maximum sum rate under the total transmit power constraint:

$$\max_{U, p_1, \cdots, p_M} \sum_{k\in U, m\in S} \log\left(1 + \frac{p_m g_{k,m}}{\sum_{i=1, i\neq m}^{M} p_i g_{k,i} + 1}\right),$$

$$\text{subject to} \sum_{m=1}^{M} p_m = P, \quad 0 \le p_1, \ldots, p_M \le P. \tag{5}$$

Therefore, the optimal user set U^* and the optimal transmit power $\mathbf{p}^* = [p_1^*, \ldots, p_M^*]$ are jointly found by solving the optimization problem (5).

In (5), there are $\binom{K}{M}$ combinations for choosing M users among K users and there are $M!$ permutations for beam assignment for each selected user set. Hence, the transmit power allocation is performed for $\binom{K}{M}$ user sets and $M!$ beam permutations; then, the best user set and the best beam assignment are selected.

We define U_n, $1 \le n \le \binom{K}{M}$, as the n-th ordered user set of U; $S_{n,j}$, $1 \le n \le M$, as the j-th ordered beam assignment for the user set U_n; and $p_{n,m}$, $1 \le m \le M$, as the assigned power to the m-th user in the user set U_n. Then, the optimization problem (5) can be rewritten as

$$\max_{\substack{U_n, \\ 1\le n\le \binom{K}{M}}} \max_{\substack{S_{n,j}, \\ 1\le j\le M!}} \max_{p_{n,1},\cdots,p_{n,M}} \sum_{k_m\in U_n, m\in S_{n,j}} \log\left(1 + \frac{p_{n,m} g_{k_m,m}}{\sum_{i\in S_{n,j}, i\neq m} p_{n,i} g_{k_m,i} + 1}\right),$$

$$\text{subject to} \sum_{m=1}^{M} p_{n,m} = P, \; 0 \le p_{n,1}, \cdots, p_{n,M} \le P. \tag{6}$$

The optimal transmit power allocation is first done for all user sets U_n, $1 \le n \le \binom{K}{M}$, and all beam assignments $S_{n,j}$, $1 \le n \le M!$. For a given U_n and $S_{n,j}$, the Lagrange multiplier method can be used to express $P_{n,m}$ in terms of U_n. We define a Lagrangian as

$$L_n = \sum_{k_m\in U_n, m\in S_{n,j}} \log\left(1 + \frac{p_{n,m} g_{k_m,m}}{\sum_{i\in S_{n,j}, i\neq m} p_{n,i} g_{k_m,i} + 1}\right) - \lambda_n\left(\sum_{m=1}^{M} p_{n,m} - P\right), \tag{7}$$

where λ_n is a Lagrange multiplier. The solution can be obtained by solving $\partial L_n / \partial p_{n,m} = 0$. Then, we obtain the following:

$$\frac{\partial L_n}{\partial p_{n,m}} = \frac{g_{k_m,m}}{p_{n,m} g_{k_m,m} + \sum_{i\in S_{n,j}, i\neq m} p_{n,i} g_{k_m,i} + 1} - \lambda_n = 0, \tag{8}$$

$$\Rightarrow p_{n,m} = \frac{1}{\lambda_n} - \frac{1}{g_{k_m,m}} - \sum_{i\in S_{n,j}, i\neq m} p_{n,i} \frac{g_{k_m,i}}{g_{k_m,m}}, \quad m = 1, \ldots, M. \tag{9}$$

Equation (9) can be rewritten as a vector-matrix form given by

$$(\mathbf{G}_n + \mathbf{I}_M)\mathbf{p}_n = \left(\frac{1}{\lambda_n}\mathbf{1}_M - \mathbf{f}_n\right), \tag{10}$$

where $\mathbf{1}_M = [1, \ldots, 1]^T$ is an $M\times 1$ vector, $\mathbf{I}_M$ is an $M\times M$ identity matrix, $\mathbf{p}_n = [p_{n,1}, \ldots, p_{n,M}]^T$, $\mathbf{f}_n = [1/g_{k_1,1}, \cdots, 1/g_{k_M,M}]^T$, $k_m \in U_n$, $m=1,\ldots, M$, and the matrix $\mathbf{G}_n$ is given by

$$\mathbf{G}_n = \begin{bmatrix} 0 & g_{k_1,2}/g_{k_1,1} & \cdots & g_{k_1,M}/g_{k_1,1} \\ g_{k_2,1}/g_{k_2,2} & 0 & \cdots & g_{k_2,M}/g_{k_2,2} \\ \vdots & \vdots & \ddots & \vdots \\ g_{k_M,1}/g_{k_M,M} & g_{k_M,2}/g_{k_M,M} & \cdots & 0 \end{bmatrix}. \tag{11}$$

In (10), λ_n can be chosen to satisfy $\sum_{m=1}^{M} p_{n,m} = P$, $p_{n,m} \ge 0$, $m=1,\ldots, M$. In the interference matrix $\mathbf{G}_n$, the (m, i)th element represents the relative ratio of the m-th beam power to the *i*-th beam power.

Note that the vector $\mathbf{p}_n$ is an implicit function of U_n. By putting $P_{n,m}$, m=1,…, M, of (10) into (6), the optimal user set with maximum sum rate is obtained as

$$\{n^*, j^*\} = \arg \max_{\substack{U_n, \\ 1\le n \le \binom{K}{M}}} \max_{\substack{S_{n,j}, \\ 1\le j \le M!}} \sum_{k_m \in U_n, m\in S_{n,j}} \log\left(1+\frac{p_{n,m} g_{k_m,m}}{\sum_{i\in S_{n,j}, i\neq m} p_{n,i} g_{k_m,i} + 1}\right). \quad (12)$$

Then, the optimal user set $U^* = U_{n^*}$, and the optimal transmit power $\mathbf{p}^* = \mathbf{p}_{n^*}$.

Note that since the transmit power allocation (10) requires the knowledge of the channel gain of all the beams for all users, it is difficult to express a simple form like a water-filling solution in orthogonal channels [7]. As a special case, when matrix $\mathbf{G}_n$ is a zero matrix, $\mathbf{p}_n^* = [\mathbf{f}_n]^+$, that is, the proposed power allocation method becomes the water-filling solution [7].

2. Suboptimal Joint Scheme for User Set Selection and Transmit Power Allocation

In the optimal joint scheme, the transmit power allocation is performed for all possible $\binom{K}{M}$ user sets and $M!$ beam selections. Then, the computational complexity is very high for a large K or a large M. Hence, we propose a suboptimal iterative user selection and transmit power allocation scheme to reduce the computational complexity.

In each iteration of the suboptimal scheme, the optimization procedure is divided into two steps. In the first step, a user set and a beam set assignment are selected by maximizing the sum rate based on the given power allocation. Then, the second step will determine the transmit power allocation for both the user set and a beam set assignment selected in the first step. Since the transmit power allocation is obtained in the second step for a given user set and beam set assignment, the procedure will be repeated using the updated power allocation. In this way, the two steps will be repeated until no further improvement can be made in the sum rate. Note that two or three iterations are enough to achieve sufficient convergence.

The following algorithm represents the proposed suboptimal iterative method.

Step 1. Initialize: t=1, $p_m^{(0)} = \frac{P}{M}$, m=1,…, M.

Step 2. Iteration: Repeat the following until $\|\mathbf{p}^{(t)} - \mathbf{p}^{(t-1)}\| < \varepsilon$.

Table 1. Computational complexity comparison.

Scheme	Complexity
OBF [4]	$O(K)$
ORBF [6]	$O(MK)$
Proposed optimal method	$O(K^M M^3)$
Proposed suboptimal method	$\max\{O(K^M), O(M^3)\}$

Step 2-1. Select a user set $U_n^{(t)}$ and a beam set assignment $S_{n,j}^{(t)}$, maximizing the sum rate for a given power allocation as follows:

$$\{n^{(t)}, j^{(t)}\} = \arg \max_{\substack{U_n, \\ 1\le n \le \binom{K}{M}}} \max_{\substack{S_{n,j}, \\ 1\le j \le M!}} \sum_{k_m \in U_n, m\in S_{n,j}} \log\left(1+\frac{p_m^{(t-1)} g_{k_m,m}}{\sum_{i\in S_{n,j}, i\neq m} p_i^{(t-1)} g_{k_m,i} + 1}\right). \quad (13)$$

Set $U^{(t)} = U_{n^{(t)}}$ and $S^{(t)} = S_{n^{(t)}, j^{(t)}}$.

Step 2-2. Using the vector-matrix form power allocation solution (10), determine the amount of the transmit power $\mathbf{p}^{(t)}$ for the user set $U^{(t)}$ and the beam assignment $S^{(t)}$ obtained in Step 2-1.

Step 2-3. If $\|\mathbf{p}^{(t)} - \mathbf{p}^{(t-1)}\| < \varepsilon$, stop the iteration. Otherwise, set t=t+1 and go to Step 2-1.

In Table 1, we compare the computational complexity for OBF [4], ORBF [6], the proposed optimal scheme, and the proposed suboptimal scheme. From the table, it can be observed that the proposed suboptimal scheme reduces the computational complexity of the optimal scheme.

IV. Simulation Results

In this section, we show simulation results to verify the performance of the proposed joint optimization scheme for user set selection and transmit power allocation in a downlink multiuser MIMO system. We assume that there are K mobile users located at the same distance from a BS and each user is equipped with a single receive antenna and a BS with M transmit antennas. The channel of each user is assumed to be independent and identically distributed with Rayleigh distribution, that is, each entry of $\mathbf{h}_k$ is independently $CN(0, 1)$ distributed. Simulation is performed more than 10,000 runs. The number of transmit antennas is M=4, which is fixed for all simulation runs. Background noise variance is assumed to be 1 for simplicity. Therefore, the average SNR is P.

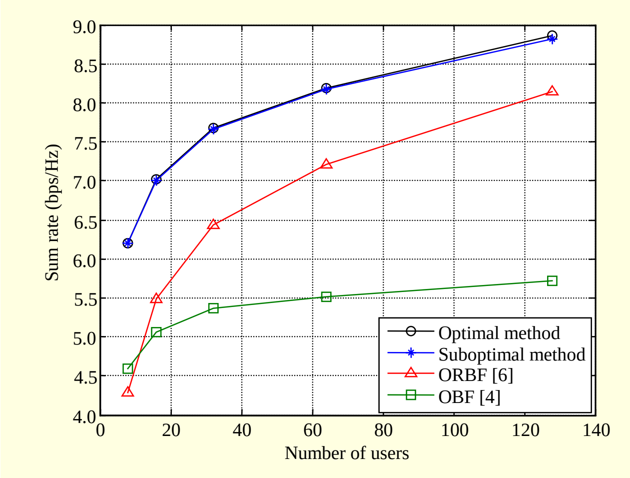

Fig. 2. Performance comparison: sum rate (bps/Hz) vs. number of users (K) for M=4, SNR=10 dB.

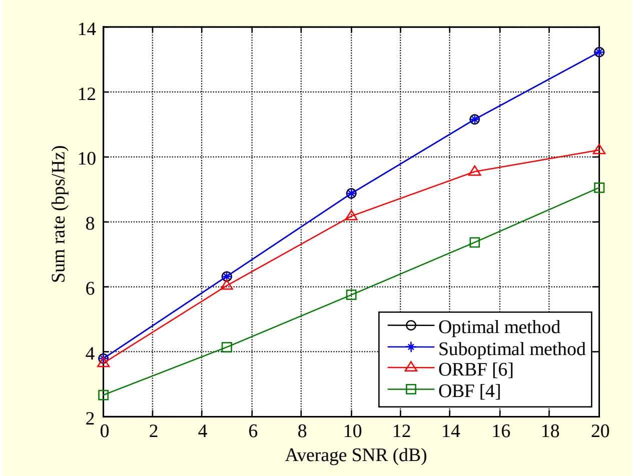

Fig. 4. Performance comparison: sum rate (bps/Hz) vs. average SNR (dB) for M=4, K=128.

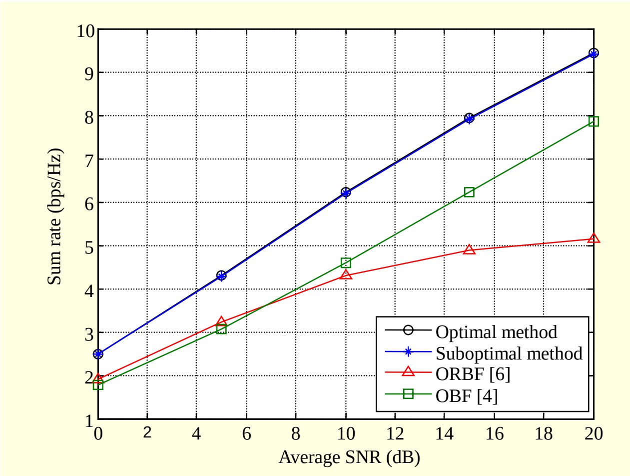

Fig. 3. Performance comparison: sum rate (bps/Hz) vs. average SNR (dB) for M=4, K=8.

Figure 2 shows the sum rate versus the number of users when the SNR is 10 dB. Regarding sum rate performance, we compare the proposed optimal and suboptimal schemes with the conventional ORBF scheme [6] and OBF scheme [4]. From the figure, it can be observed that the proposed optimal and suboptimal schemes significantly outperform the conventional ORBF and OBF schemes. The figure also shows that the performance of the proposed suboptimal scheme is very close to that of the proposed optimal scheme.

Figure 3 shows the sum rate versus the average SNR for a small number of users, that is, K=8. From the figure, it can be observed that both the proposed optimal scheme and the proposed suboptimal scheme have a sum rate that is proportional to the average SNR. However, the sum rate of the conventional ORBF scheme is saturated in high SNR because the residual interference in this scheme is dominant in the received signal. The performance difference between the proposed schemes and the conventional ORBF scheme increases in proportion to the average SNR value. The sum rate of the ORBF scheme is lower than that of the OBF scheme in a high SNR region, due to the fact that it is difficult to find the user set with good spatial separation in the ORBF scheme if the number of users is small, and poor spatial separation causes high inter-user interference.

Figure 4 shows the sum rate versus the average SNR in the case that the number of users is large, that is, K=128. In the figure, we can observe that the sum rate of the ORBF scheme is higher than that of the OBF scheme. The sum rate is higher because it is easy for the ORBF scheme to find the user set with good spatial separation if the number of users is large, and the ORBF scheme can provide spatial multiplexing gain.

V. Conclusion

In this paper, joint user set selection and transmit power allocation schemes were proposed to improve the performance of the conventional ORBF scheme: an optimal scheme and a suboptimal scheme. In the proposed optimal scheme, the optimal user set and transmit power allocation were jointly found to maximize the sum rate under the total transmit power constraint. To reduce the computation complexity of the optimal scheme, we proposed the practical suboptimal iterative scheme. Simulation results showed that the proposed schemes have better sum rate performance than that of the conventional ORBF and OBF schemes.

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Tae-Sung Kang received his BS, MS, and PhD in electrical engineering and computer science (EECS) from the Korea Advanced Institute of Science and Technology (KAIST), Daejeon, Rep. of Korea, in 2000, 2002, and 2009, respectively. Since 2009, he has been with LG U+ in Seoul, Rep. of Korea. His research interests include resource allocation in multiuser MIMO systems, OFDM systems, and wireless LAN.

Bangwon Seo received his BS, MS, and PhD in electrical engineering from the Korea Advanced Institute of Science and Technology (KAIST), Daejeon, Rep. of Korea, in 1997, 1999, and 2010, respectively. Since 2004, he has been with ETRI, Daejeon, Rep. of Korea. His research interests include receiver design, MC-CDMA systems, OFDM systems, MIMO systems, CoMP, device-to-device (D2D) communications, and cognitive radio networks.