

# A Signal Subspace Interference Alignment Scheme with Sum Rate Maximization and Altruistic-Egoistic Bayesian Gaming

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## Abstract

In this paper, we propose a distributed signal subspace interference alignment algorithm for single beam  $K$  – user ( $K \geq 3$ ) MIMO interference channel based on sum rate maximization and game theory. A framework of game theory is provided to study relationship between interference signal subspace and altruistic-egoistic bayesian game cost function. We demonstrate that the asymptotic interference alignment under proposed scheme can be realized through a numerical algorithm using local channel state information at transmitters and receivers. Simulation results show that the proposed scheme can achieve the total degrees of freedom that is equivalent to the Cadambe-Jafar interference alignment algorithms with perfect channel state information. Furthermore, proposed scheme can effectively minimize leakage interference in desired signal subspace at each receiver and obtain a moderate average sum rate performance compared with several existing interference alignment schemes.

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**Keywords:** MIMO interference channel, Interference alignment, Signal subspace, Altruistic-Egoistic bayesian game cost function, Degrees of Freedom

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## 1. Introduction

The interference has become the major factor to impact the performance of multi-user wireless communication networks with growth of the modern systems. These effects are highlighted by both theoretical analysis [1] and experimental measurements [2][3]. The conventional interference management techniques used in practical wireless communication systems either consider interference signals as noise and orthogonalize the available channel resource. However, the former ignores the structure of interference signals and it is known to be effective only when the power of interference signals is low; while the latter leads to inefficient use of wireless channel resources. The multi-user detection schemes such as [18-20] can be used to eliminate interference and decode the desired signal in the cases where interference is strong [4], [5] or very strong [6]. However, the complexity of the multi-user detection schemes is unacceptable especially when the number of users in the interference channel exceeds three.

### A. Previous Work

In a recent study, Cadambe and Jafar [7] have shown that a total  $K / 2$  degrees of freedom (DoF) is achievable from the perspective of information theory by deploying interference alignment scheme at the transmitters and zero-forcing at the receivers for K-user ( $K \geq 2$ ) single-input single-output (SISO) frequency selective interference channels. Where the DoF is the one-order approximation of the sum rate at high SNR and it can be used to characterize the number of transmitted data streams free from interference in the channels. The key insight of the interference alignment is that the desired signal at the receivers can occupy 1/2 of the total available wireless channel resources at most in the case where suitable precoding matrices at the transmitter are designed to constrain interference subspace spanned by the interference signal.

A closed-form solution of interference alignment for transmitter precoding matrices is proposed in [7] for a 3-user interference channel. However, this solution assumes that the transmitters know global channel state information, which would become an increased overhead for system design. Moreover, the calculation of closed-form solution may become a difficult or impossible task in the cases where the number of users is greater than three. Therefore, the application of interference alignment presented in [7] is limited to only small number of users. Later on, Gomadam, Cadambe and Jafar [10] proposed a distributed interference alignment algorithms based on iterative scheme. The scheme utilizes the reciprocity of wireless networks and cycle iteration to minimize the objective function of the interference leakage to achieve asymptotical interference alignment. The algorithm requires the transmitters to obtain only the local channel state information between communication pairs (the local channel state information of transmitter and its corresponding receiver), which implies that the interference alignment can be achieved with reduced channel feedback overheads. Moreover, asymptotical interference alignment can also be achieved in the interference channel with more than 3 users in this algorithm. However, the algorithm in [10] needs to exploit channel reciprocity to alternate between the forward and reverse channels that requires tight synchronization at both ends. The processing of alternation will produce significant overheads in a dynamic communication environment where the channel varies rapidly. Another approach on the interference alignment without the requirement of reciprocal channel has proposed in [11] that alternatively optimizes the precoder matrices at transmitters and the interference subspace at receivers. Their formulation is more conducive to

mathematical and geometrical analysis. [11] uses alignment of signal subspace as the main optimization objective from the perspective of geometry, which is asymptotically optimal when the Signal to Noise Ratio(SNR) tends to infinity. However, only focusing on high SNR region and interference alignment of signal subspaces will result in linear transceivers with suboptimal performance at low to intermediate SNRs. In [15], the authors propose an approach to design the precoding vectors at each data stream in the framework of game theory that aims to provide a compromise between egoistic beamforming gain at the intended receiver; and altruistic alignment and cancellation of the interference created towards other receivers. In [16], the interference alignment scheme in frequency domain is applied to the practical heterogeneous cellular network with different cell sizes of cells and the satellite communication system including monobeam and multibeam satellites. In [17], a regularized transceiver designs is proposed to achieve a high sum-rate performance at overall SNR regime for 2-user and 3-user interference channels. Other interference alignment algorithms and interference alignment-inspired algorithms have been discussed in [21-25].

### B. Contribution

In this paper, we propose a distributed interference alignment algorithm that provides a game-theoretic interpretation for both the interference signal subspace minimization and the sum rate maximization in K-user MIMO interference channels by directly building on the egoistic and altruistic game equilibria. Furthermore, a constructive mathematical and geometrical analysis and interpretation about the egoistic and altruistic cost function for interference channel in the frame of game theory is presented. To obtain optimal system performance, each transmitter will consider its role in the interference channel from the following two aspects. On one hand, each transmitter tries to maximize its data rate by transmitting along those signaling dimensions where the desired receiver anticipates the least interference. On the other hand, each transmitter primarily tries to minimize the interference to undesired receivers. From the perspective of game theory, the former considers the impact of the interference from the egoistic aspect and the latter considers the same problem from the altruistic aspect. Therefore, we can build a game theory framework to interpret the distributed interference alignment problem. It is assumed that each transmitter only knows the local channel state information between communication pairs (local channel state information of a transmitter and its corresponding receiver) . A class of games suitable to the case of partial information based decision making, called Bayesian games, can be used in this scenario. The proposed algorithm is different from the most of the existing interference alignment schemes. The existing scheme only consider the power of the interference leakage as effective optimization objective and neglects sum rate maximization at low to intermediate SNR regions. The proposed scheme in this study is a two stage framework that is implemented by: firstly, defining the egoistic and altruistic objective functions, deriving analytically the game equilibrium and interpretation of interference alignment problem by using the mathematical and geometrical analysis; secondly adapting the obtained equilibrium solution to heuristic design of a practical beamforming technique based on the sum rate maximization scheme. The proposed approach combines the interference signal subspace minimization scheme and the sum rate maximization scheme of interference networks with a game-theoretic interpretation based on the egoistic and altruistic game equilibria. The feasibility of proposed algorithm is provided and a comparison of the DoF and sum rate performance with other interference alignment algorithms is studied using simulations. The simulation results show that the proposed scheme can effectively minimize the leakage interference in desired signal subspace at each receiver and provide a moderate average sum rate performance in comparison to several

existing interference alignment schemes.

### C. Organization and Notations

The rest of the paper is organized as follows. The system model under consideration is presented in next section. In section 3, the Bayesian game model of interference channel is presented using the mathematical and geometrical analysis and interpretation of interference alignment problem. Section 4 describes the game relationship in solving the sum rate maximization problem. In section 5, a calculation method is presented for the interference signal subspace at each receiver. Section 6 explains the convergence criterion and the concrete algorithm of proposed scheme. The simulation results and conclusion are presented in section 7 and section 8, respectively.

Boldface letters denote matrices or vectors, while upper and lower case letters denote scalars.  $(\square)^*$  refers to the conjugate transpose of  $(\square)$ . For a matrix  $\mathbf{B}$ ,  $\|\mathbf{B}\|_F$  is the Frobenius norm of  $\mathbf{B}$  and  $\text{tr}(\mathbf{B})$  is the trace of  $\mathbf{B}$ .  $E[\square]$  denotes expectation operator.  $\square^{M \times N}$  represents the set of all  $M \times N$  matrices.  $\square_j$  represents interference subspace at receiver  $j$ .  $\frac{\partial(f(\mathbf{W}))}{\partial(\mathbf{W})}$  represents the derivative for the function  $f(\mathbf{W})$ .

## 2. System Model

A multi-user MIMO interference channel with  $K$  communication pairs is shown in Fig. 1. The signal of each transmitter can be received by all the receivers; however a given transmitter only intends to have its signal decoded by the targeted receiver. It is assumed that each transmitter is equipped with  $M$  transmitting antennas and each receiving is equipped with  $N$  receiver antennas. It is also assumed that only one data stream is transmitted by each user and  $\mathbf{W}_k \in \square^{M \times 1}, k \in \{1, 2, \dots, K\}$  is the transmission precoder for transmitter  $k$  with unit norm. The received signal at receiver  $k$  is given as:

$$\mathbf{y}_k = \mathbf{H}_{kk} \mathbf{W}_k x_k + \sum_{j \neq k} \mathbf{H}_{kj} \mathbf{W}_j x_j + \mathbf{Z}_k, k \in \{1, 2, \dots, K\} \tag{1}$$

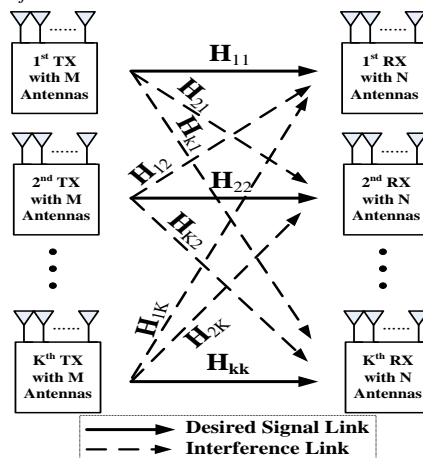


Fig. 1. A  $K$ -user MIMO interference channel where transmitters and receivers have  $M$  and  $N$  antennas respectively.

where  $\mathbf{H}_{kj} \in \mathbb{C}^{N \times M}$  is a frequency-flat fading channel between transmitter  $j$  and receiver  $k$  for  $k, j \in \{1, 2, \dots, K\}$ , where each entry is assumed as i.i.d complex Gaussian random variables with zero mean and unit variance  $\mathcal{CN}(0, 1)$ .  $\mathbf{Z}_k \in \mathbb{C}^{N \times 1}$  is a zero mean circularly symmetric additive white Gaussian noise vector (AWGN) at receiver  $k$ , and it has the covariance  $E[\mathbf{Z}_k \mathbf{Z}_k^*] = \sigma_k^2 \mathbf{I}_N$ . Here, the transmitted symbol  $x_k, k \in \{1, 2, \dots, K\}$  at the  $k$ th transmitter is generated with power budget  $P_k$ ; i.e.,  $E[x_k x_k^*] = P_k$ . In equation (1), the first term  $\mathbf{H}_{kk} \mathbf{W}_k x_k$  is the desired signal vector sent by the  $k$ th transmitter and the second term  $\sum_{j \neq k}^K \mathbf{H}_{kj} \mathbf{W}_j x_j$  represents the interference from other transmitters. Furthermore, we denote an orthonormal basis vector  $\mathbf{V}_k \in \mathbb{C}^{N \times 1}$  as receive matrix at receiver  $k$ . The instantaneous rate of communication pair  $k$  can be obtained as:

$$R_k = \log_2 \left( 1 + \frac{\mathbf{V}_k^* \mathbf{H}_{kk} \mathbf{W}_k \mathbf{W}_k^* \mathbf{H}_{kk}^* \mathbf{V}_k P_k}{\sum_{i \neq k}^K \mathbf{V}_k^* \mathbf{H}_{ki} \mathbf{W}_i \mathbf{W}_i^* \mathbf{H}_{ki}^* \mathbf{V}_k P_i + \sigma_k^2} \right) \quad (2)$$

The instantaneous sum rate of the system is:

$$\bar{R} = \sum_{k=1}^K R_k \quad (3)$$

The discussion in the following section will be based on the channel model presented in Fig. 1.

### 3. Bayesian game model of interference channel

The Bayesian game's definition of interference channel presented in [12] will be used to construct egoistic Bayesian altruistic Bayesian game cost functions. Particularly, two aspects of the problem will be considered here: 1) In absence of cooperation, each transmitter will "selfishly" choose its beamforming vector to maximize the transmission rate towards its desired receiver; 2) Each transmitter hopes that their transmitted signal causes the least interference to the desired signal at other undesired receivers. The former can be considered as the objective of egoistic Bayesian game cost function, while the latter can be mapped to the objective of altruistic Bayesian game cost function. The methods to construct these two functions are explained in the following subsections.

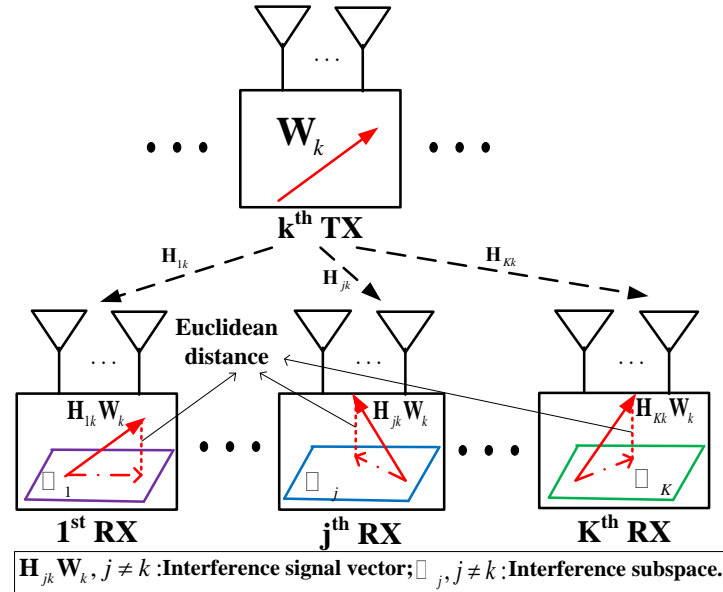
#### 3.1 Altruistic Bayesian game cost function

**Lemma 3.1:** Given  $K-1$  arbitrary  $p$ -dimensional subspaces  $\mathbb{C}^p_j$  with respective orthonormal bases  $\mathbf{Q}_j$  and a  $N \times M$  matrix  $\mathbf{H}_{jk}$ , the vector  $\mathbf{F}_k$  such that  $\mathbf{B} = \mathbf{H}_{jk} \mathbf{F}_k, \mathbf{F}_k \in \mathbb{C}^{M \times 1}$ , which minimizes the squared Euclidean distance from  $\mathbf{B}$  to the

subspaces, is equal to the eigenvector corresponding to the minimum eigenvalue of

$$\sum_{j \neq k}^K \mathbf{H}_{jk}^* (\mathbf{I} - \mathbf{Q}_j \mathbf{Q}_j^*) \mathbf{H}_{jk}$$

The proof of Lemma 1 can refer to the lemma 2 in [11] and is omitted to avoid repetition.



**Fig. 2.** The relationship between interference signal vectors  $\mathbf{H}_{jk} \mathbf{W}_k (j \neq k, j \in \{1, 2, \dots, K\})$  and  $K - 1$  interference subspaces  $\square_j (j \neq k, j \in \{1, 2, \dots, K\})$ .

**Remark:** In interference channel, the impact of transmitter  $k$  over  $K - 1$  undesired receivers can be embodied in the sum of the squared Euclidean distances from  $\mathbf{H}_{jk} \mathbf{W}_k (j \neq k, j \in \{1, 2, \dots, K\})$  to the  $K - 1$  interference subspaces  $\square_j (j \neq k)$  of undesired receivers. **Fig. 2** presents an illustration of relationship between interference signal vectors  $\mathbf{H}_{jk} \mathbf{W}_k (j \neq k)$  (red lines with solid arrows) from  $k^{\text{th}}$  transmitting signal source and interference subspaces  $\square_j (j \neq k)$  (planes with different colors) at  $K - 1$  interference signal destinations (from 1<sup>st</sup> Rx to  $K^{\text{th}}$  RX). Where  $\mathbf{W}_k$  is precoding vector of signal source from transmitter  $k$ ,  $\mathbf{H}_{jk} \mathbf{W}_k (j \neq k)$  is direction vector of interference signal at undesired destination, and  $\square_j (j \neq k)$  is interference subspace spanned by interference signals at undesired destination. It can be argued that the power leakage from interference subspace of  $K - 1$  undesired receivers due to the transmitter  $j$  is driven proportionally with the sum of the squared Euclidean distance. When the sum of the squared Euclidean distance is zero, the interference that transmitter  $j$  produces aligns to the interference subspace at  $K - 1$  different undesired receivers

According to the analysis provided in preceding content, it can be argued that aligning the interference produced by transmitter  $k$  to the interference signal subspace  $\square_j (j \neq k)$  at

receiver  $j$  is an effective way to reduce the interference at undesired receiver  $j$ . This is exactly an altruistic action. So, the altruistic Bayesian game cost function can be constructed by calculating the sum of the squared Euclidean distances from interference signal  $\mathbf{H}_{jk} \mathbf{W}_k (j \neq k, j \in \{1, 2, \dots, K\})$  produced by transmitter  $k$  to the  $K-1$  interference subspaces  $\square_j (j \neq k)$  of undesired receivers :

$$\phi_k(\mathbf{W}_k) = - \sum_{j \neq k}^K \|\mathbf{H}_{jk} \mathbf{W}_k - \mathbf{Q}_j \mathbf{Q}_j^* \mathbf{H}_{jk} \mathbf{W}_k\|_F^2 \quad (4)$$

To obtain the maximum benefit at transmitter  $k$ , the altruistic precoding vector  $\mathbf{W}_k^{Alt}$  can be formulated by maximizing the expression (4):

$$\mathbf{W}_k^{Alt} = \arg \max \phi_k(\mathbf{W}_k) \quad (5)$$

Then the expression (5) can be rewritten as:

$$\mathbf{W}_k^{Alt} = \arg \min_{\mathbf{Q}_j \mathbf{Q}_j^* = \mathbf{I}} \sum_{j \neq k}^K \|\mathbf{H}_{jk} \mathbf{W}_k - \mathbf{Q}_j \mathbf{Q}_j^* \mathbf{H}_{jk} \mathbf{W}_k\|_F^2 \quad (6)$$

According to Lemma 3.1, we can obtain the solution of expression (6):

$$\mathbf{W}_k^{Alt} = V^{\min} \left( \sum_{j \neq k}^K \mathbf{A}_{jk} \right) \quad (7)$$

where

$$\mathbf{A}_{jk} = \mathbf{H}_{jk}^* (\mathbf{I}_N - \mathbf{Q}_j \mathbf{Q}_j^*) \mathbf{H}_{jk} \quad (8)$$

$V^{\min}(\mathbf{A})$  represents the eigenvector corresponding to the minimum eigenvalue of  $\mathbf{A}$ . The  $\mathbf{A}_{jk}$  is considered as the altruistic Bayesian game cost function between transmitter  $k$  and receiver  $j$ .

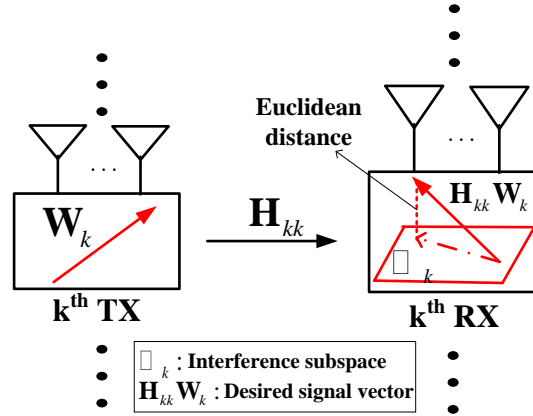
### 3.2 Egoistic Bayesian game cost function

**Lemma 3.2:** Given an arbitrary  $p$ -dimensional subspace  $\square_k$  with orthonormal base  $\mathbf{Q}_k$  and  $N \times M$  vector  $\mathbf{H}_{kk}$ , the vector  $\mathbf{F}_k$  such that  $\mathbf{B} = \mathbf{H}_{kk} \mathbf{F}_k, \mathbf{F}_k \in \square_k^{M \times 1}$ , which maximizes the squared Euclidean distance from  $\mathbf{B}$  to the subspace, is equal to the eigenvector corresponding to the maximum eigenvalues of  $\mathbf{H}_{kk}^* (\mathbf{I} - \mathbf{Q}_k \mathbf{Q}_k^*) \mathbf{H}_{kk}$ .

The proof of Lemma 3.2 is similar as lemma 3.1 therefore it is omitted to avoid repetition here.

**Remark:** To maximize the transmission rate of the desired signal from transmitter  $k$ , the optimal precoding vector at transmitter  $k$  should be designed to isolate the desired signal from the interference subspace of corresponding receiver  $k$ . Fig. 3 provides an illustration of the relationship between desired signal  $\mathbf{H}_{kk} \mathbf{W}_k$  (red line with solid arrow) from the transmitter  $k$  and the interference signal subspace  $\square_k$  (red plane) at receiver  $k$ . We choose the squared Euclidean distance from  $\mathbf{H}_{kk} \mathbf{W}_k$  to the interference subspace  $\square_k$  of receiver  $k$  as the measurement of the extent by which the desired signal is away from the interference

signal subspace at receiver  $k$ . As mentioned earlier, the interference by which the desired signal suffers from the other undesired transmitters at receiver  $k$  decreased with an increase in squared Euclidean distance. When the desired signal is independent of the interference subspace spanned by interference from unintended transmitters, zero forcing approach can be used to recover the desired signal without any interference.



**Fig. 3.** The relationship between desired signal vector  $\mathbf{H}_{kk} \mathbf{W}_k$  and interference signal subspaces  $\square_k$  at receiver  $k$ .

Now, the egoistic Bayesian game cost function can be constructed by calculating the squared Euclidean distance from desired signal  $\mathbf{H}_{kk} \mathbf{W}_k$  over transmitter  $k$  to the interference subspaces  $\square_k$  of desired receiver  $k$ :

$$\phi_k(\mathbf{W}_k) = \|\mathbf{H}_{kk} \mathbf{W}_k - \mathbf{Q}_k \mathbf{Q}_k^* \mathbf{H}_{kk} \mathbf{W}_k\|_F^2 \tag{9}$$

To obtain the maximum benefit at transmitter  $k$ , the egoistic precoding vector  $\mathbf{W}_k^{\text{Ego}}$  can be formulated by maximizing the expression (9):

$$\mathbf{W}_k^{\text{Ego}} = \arg \max \phi_k(\mathbf{W}_k) \tag{10}$$

Then the expression (10) can be rewritten as:

$$\mathbf{W}_k^{\text{Ego}} = \arg \max \|\mathbf{H}_{kk} \mathbf{W}_k - \mathbf{Q}_k \mathbf{Q}_k^* \mathbf{H}_{kk} \mathbf{W}_k\|_F^2 \tag{11}$$

According to Lemma 3.2, we can obtain the solution of expression (11):

$$\mathbf{W}_k^{\text{Ego}} = V^{\max}(\mathbf{E}_k) \tag{12}$$

where

$$\mathbf{E}_k = \mathbf{H}_{kk}^* (\mathbf{I} - \mathbf{Q}_k \mathbf{Q}_k^*) \mathbf{H}_{kk} \tag{13}$$

$V^{\max}(\mathbf{A})$  represents the eigenvector corresponding to the maximum eigenvalue of  $\mathbf{A}$ .  $\mathbf{E}_k$  denotes the egoistic Bayesian game cost function between transmitter  $k$  and receiver  $k$ .



#### 4. Game Relationship in Sum Rate

In this section, the precoding vector that maximizes sum rate for the multi-user MIMO interference channel will be solved using the approach of Lagrange multipliers. The relationship will be explored between the sum rate and Bayesian game cost functions mentioned in the preceding section. The description of sum rate for  $K$  – user MIMO interference channel in section 2 is given as:

$$\bar{R} = \sum_{k=1}^K R_k \quad (14)$$

where  $R_k = \log_2 \left( 1 + \frac{\mathbf{V}_k^* \mathbf{H}_{kk} \mathbf{W}_{kk} \mathbf{W}_k^* \mathbf{H}_{kk}^* \mathbf{V}_k P_k}{\sum_{i \neq k}^K \mathbf{V}_k^* \mathbf{H}_{ki} \mathbf{W}_i \mathbf{W}_i^* \mathbf{H}_{ki}^* \mathbf{V}_k P_i + \sigma_k^2} \right)$  and the definitions of parameters

in this expression are explained in the system model in section 2. The constraint conditions can be employed, expression (14) and  $\mathbf{W}_k^* \mathbf{W}_k = 1$ , to construct a Lagrange function for solving sum rate maximization problem:

$$\text{larg}(\mathbf{W}_k, \mu) = \bar{R} - \mu_{\max} (\mathbf{W}_k^* \mathbf{W}_k - 1) \quad (15)$$

where  $\mu_{\max}$  is the Lagrangian. The necessary condition for maximizing sum rate  $\bar{R}$  in the expression (14) is:

$$\frac{\partial \text{larg}(\mathbf{W}_k, \mu)}{\partial \mathbf{W}_k^*} = 0 \quad (16)$$

The above expressions can be derived to:

$$\frac{\partial R_k}{\partial \mathbf{W}_k^*} + \sum_{k=1}^K \frac{\partial R_j}{\partial \mathbf{W}_k^*} - \mu_{\max} \mathbf{W}_k = 0 \quad (17)$$

Substituting (2) into (17):

$$\begin{aligned} & \frac{\mathbf{H}_{kk}^* \mathbf{V}_k \mathbf{V}_k^* \mathbf{H}_{kk} P_k \mathbf{W}_k \left( \sum_{i \neq k}^K \mathbf{V}_k^* \mathbf{H}_{ki} \mathbf{W}_i \mathbf{W}_i^* \mathbf{H}_{ki}^* \mathbf{V}_k P_i + \sigma_k^2 \right)}{\sum_{i=1}^K \mathbf{V}_k^* \mathbf{H}_{ki} \mathbf{W}_i \mathbf{W}_i^* \mathbf{H}_{ki}^* \mathbf{V}_k P_i + \sigma_k^2} \times \frac{1}{\sum_{i \neq k}^K \mathbf{V}_k^* \mathbf{H}_{ki} \mathbf{W}_i \mathbf{W}_i^* \mathbf{H}_{ki}^* \mathbf{V}_k P_i + \sigma_k^2} \\ & \sum_{j \neq k}^K \frac{\sum_{i \neq j}^K \mathbf{V}_j^* \mathbf{H}_{ji} \mathbf{W}_i \mathbf{W}_i^* \mathbf{H}_{ji}^* \mathbf{V}_j P_i + \sigma_j^2}{\sum_{i=1}^K \mathbf{V}_j^* \mathbf{H}_{ji} \mathbf{W}_i \mathbf{W}_i^* \mathbf{H}_{ji}^* \mathbf{V}_j P_i + \sigma_j^2} \times \frac{(\mathbf{H}_{jk}^* \mathbf{V}_j \mathbf{V}_j^* \mathbf{H}_{jk} P_k \mathbf{W}_k)(\mathbf{V}_j^* \mathbf{H}_{jj} \mathbf{W}_j \mathbf{W}_j^* \mathbf{H}_{jj}^* \mathbf{V}_j P_j)}{\left( \sum_{i \neq j}^K \mathbf{V}_j^* \mathbf{H}_{ji} \mathbf{W}_i \mathbf{W}_i^* \mathbf{H}_{ji}^* \mathbf{V}_j P_i + \sigma_j^2 \right)^2} \\ & = \mu_{\max} \mathbf{W}_k \end{aligned} \quad (18)$$

Simplifying the expression (18):

$$\begin{aligned}
 & \frac{\mathbf{H}_{kk}^* \mathbf{V}_k \mathbf{V}_k^* \mathbf{H}_{kk} P_k \mathbf{W}_k}{\sum_{i=1}^K \mathbf{V}_k^* \mathbf{H}_{ki} \mathbf{W}_i \mathbf{W}_i^* \mathbf{H}_{ki}^* \mathbf{V}_k P_i + \sigma_k^2} - \frac{1}{\sum_{j \neq k} \sum_{i=1}^K \mathbf{V}_j^* \mathbf{H}_{ji} \mathbf{W}_i \mathbf{W}_i^* \mathbf{H}_{ji}^* \mathbf{V}_j P_i + \sigma_j^2} \times \\
 & \frac{(\mathbf{H}_{jk}^* \mathbf{V}_j \mathbf{V}_j^* \mathbf{H}_{jk} P_k \mathbf{W}_k)(\mathbf{V}_j^* \mathbf{H}_{jj} \mathbf{W}_j \mathbf{W}_j^* \mathbf{H}_{jj}^* \mathbf{V}_j P_j)}{\left( \sum_{i \neq j}^K \mathbf{V}_j^* \mathbf{H}_{ji} \mathbf{W}_i \mathbf{W}_i^* \mathbf{H}_{ji}^* \mathbf{V}_j P_i + \sigma_j^2 \right)} \\
 & = \mu_{\max} \mathbf{W}_k \tag{19}
 \end{aligned}$$

According to the relationship between orthonormal base  $\mathbf{Q}_k$  for received interference signal subspace  $\square_k$  and linear receiver  $\mathbf{V}_k$  at receiver  $k$ , we can obtain:

$$\mathbf{V}_k \mathbf{V}_k^* = \mathbf{I}_N - \mathbf{Q}_k \mathbf{Q}_k^* \tag{20}$$

Multiplying by  $\frac{1}{P_k} \left( \sum_{i=1}^K \mathbf{V}_k^* \mathbf{H}_{ki} \mathbf{W}_i \mathbf{W}_i^* \mathbf{H}_{ki}^* \mathbf{V}_k P_i + \sigma_k^2 \right)$  on both sides of expression (19), the following expression can be constructed:

$$\begin{aligned}
 & \mathbf{H}_{kk}^* (\mathbf{I}_N - \mathbf{Q}_k \mathbf{Q}_k^*) \mathbf{H}_{kk} \mathbf{W}_k - \sum_{j \neq k} \frac{\sum_{i=1}^K \mathbf{V}_k^* \mathbf{H}_{ki} \mathbf{W}_i \mathbf{W}_i^* \mathbf{H}_{ki}^* \mathbf{V}_k P_i + \sigma_k^2}{\sum_{i=1}^K \mathbf{V}_j^* \mathbf{H}_{ji} \mathbf{W}_i \mathbf{W}_i^* \mathbf{H}_{ji}^* \mathbf{V}_j P_i + \sigma_j^2} \\
 & \times \frac{(\mathbf{V}_j^* \mathbf{H}_{jj} \mathbf{W}_j \mathbf{W}_j^* \mathbf{H}_{jj}^* \mathbf{V}_j P_j)}{\sum_{i \neq j}^K \mathbf{V}_j^* \mathbf{H}_{ji} \mathbf{W}_i \mathbf{W}_i^* \mathbf{H}_{ji}^* \mathbf{V}_j P_i + \sigma_j^2} \times \mathbf{H}_{jk}^* (\mathbf{I}_N - \mathbf{Q}_j \mathbf{Q}_j^*) \mathbf{H}_{jk} \mathbf{W}_k \\
 & = \mu_{\max} \mathbf{W}_k \tag{21}
 \end{aligned}$$

Substituting (8) and (13) into (21)

$$\left( \mathbf{E}_k + \sum_{j \neq k} \lambda_{jk}^{opt} \mathbf{A}_{jk} \right) \mathbf{W}_k = \mu_{\max}' \mathbf{W}_k \tag{22}$$

According to expression (21):

$$\lambda_{jk}^{opt} = \frac{\sum_{i=1}^K \mathbf{V}_k^* \mathbf{H}_{ki} \mathbf{W}_i \mathbf{W}_i^* \mathbf{H}_{ki}^* \mathbf{V}_k P_i + \sigma_k^2}{\sum_{i=1}^K \mathbf{V}_j^* \mathbf{H}_{ji} \mathbf{W}_i \mathbf{W}_i^* \mathbf{H}_{ji}^* \mathbf{V}_j P_i + \sigma_j^2} \times \frac{(\mathbf{V}_j^* \mathbf{H}_{jj} \mathbf{W}_j \mathbf{W}_j^* \mathbf{H}_{jj}^* \mathbf{V}_j P_j)}{\sum_{i \neq j}^K \mathbf{V}_j^* \mathbf{H}_{ji} \mathbf{W}_i \mathbf{W}_i^* \mathbf{H}_{ji}^* \mathbf{V}_j P_i + \sigma_j^2} \tag{23}$$

$$\mu_{\max}' = \frac{\left( \sum_{i=1}^K \mathbf{V}_k^* \mathbf{H}_{ki} \mathbf{W}_i \mathbf{W}_i^* \mathbf{H}_{ki}^* \mathbf{V}_k P_i + \sigma_k^2 \right)}{P_k} \mu_{\max} \tag{24}$$

In expression (22),  $\mathbf{E}_k$  and  $\mathbf{A}_{jk}$  correspond to the egoistic and the altruistic Bayesian

game cost functions. Therefore, the principle of altruism and egoism in sum rate maximization is synthesized through a linear combination. According to the matrix theory, the precoding vector  $\mathbf{W}_k$  for transmitter  $k$  is just the dominant eigenvector of the linear combination

$\mathbf{E}_k + \sum_{j \neq k}^K \lambda_{jk}^{opt} \mathbf{A}_{jk}$ , where  $\lambda_{jk}^{opt}$  plays a role of balancing factor between the egoistic and the

altruistic Bayesian game cost functions. The parameter  $\lambda_{jk}^{opt}$  in expression (22) requires the global channel state information, which implies excessive training and a significant amount of feedback overhead in practical system. To eliminate this dependency on global state information, the parameter  $\lambda_{jk}^{opt}$  can be obtained through a suboptimal egoism-altruism balancing method mentioned in [12]. In this paper, the focus is on the feasibility of solution method for the parameter  $\lambda_{jk}^{opt}$  in practical wireless communication systems. Although a suboptimal egoism-altruism balancing method is demonstrated in [12], the parameter  $\lambda_{jk}^{opt}$  is still obtained through a complicated expression that requires a significant amount of channel state information and calculations. Therefore, an empirical estimation scheme for the estimation of parameter  $\lambda_{jk}^{opt}$  is presented in this study to improve our algorithm's application value. Section 7 explains the impact of the parameter  $\lambda_{jk}^{opt}$  on the algorithm across the different performance factors including the average sum-rate, the average number of iterations for our algorithm, the ratio of leakage interference in desired signal space.

## 5. Calculation of Interference Signal Subspace at Receivers

In the discussion in preceding section, it is assumed that the orthonormal base  $\mathbf{Q}_k, k \in (1, 2, \dots, K)$  for interference signal subspace  $\square_k$  at receiver  $k$  is known by transmitter  $k$ , thus an appropriate precoding vector  $\mathbf{W}_k, k \in (1, 2, \dots, K)$  can be designed to achieve interference alignment at each transmitter. In this section, the interference signal subspace of each receiver will be solved for fixed precoding vector  $\mathbf{W}_k, k \in (1, 2, \dots, K)$ , assuring that the interference signal from undesired transmitters falls into the interference signal subspace as much as possible.

Let us assume that each transmitter knows its local Channel State Information (CSI) and local CSI of the corresponding receiver. The local CSI at each transmitter can be obtained by estimating the pilot signal sent from each receiver. Each transmitter can obtain the local CSI of the corresponding receiver through the feedback link from the corresponding receiver.

**Lemma 5.1:** Given  $K-1$  arbitrary vectors  $\mathbf{H}_{jk} \mathbf{W}_k \in \square^{N \times 1} (k \neq j)$ , the  $p$ -dimensional subspace  $\square_j$  with minimum overall Euclidean distance to all vectors  $\mathbf{H}_{jk} \mathbf{W}_k (k \neq j)$  has orthonormal basis  $\mathbf{Q}_j$ , where the columns of  $\mathbf{Q}_j$  are the eigenvectors associated with the  $p$

largest eigenvalues of  $\sum_{k \neq j}^K \mathbf{H}_{jk} \mathbf{W}_k \mathbf{W}_k^* \mathbf{H}_{jk}^*$ .

**Proof:** The proof of **Lemma 5.1** is relegated to the Appendix

**Remark:** At receiver  $j$ , the interference suffering from  $K-1$  unintended transmitters can be

embodied in the sum of the squared Euclidean distance from  $K - 1$  matrices  $\mathbf{H}_{jk} \mathbf{W}_k (k \neq j)$  to the interference signal subspace  $\square_j, j \neq k$ . Fig. 4 provides an illustration of relationship between interference signal vectors  $\mathbf{H}_{jk} \mathbf{W}_k (k \neq j)$  (colorized lines with solid arrows) from  $K - 1$  undesired transmitting signal sources (from 1<sup>st</sup> TX to  $K^{\text{th}}$  TX) and interference subspaces  $\square_j (j \neq k)$  (planes with different colors) at receiver  $j$ . It is argued that the smaller the sum of the squared Euclidean distance is, the less power of the interference that  $K - 1$  undesired transmitters produce at receiver  $j$  leaks out from interference subspace of the  $j^{\text{th}}$  receiver. When the sum of the squared Euclidean distance is equal to zero, the interference from  $K - 1$  unintended transmitters aligns to interference subspaces  $\square_j$  at receiver  $j$ . Fig. 3 illustrates the relationship between  $K - 1$  interference vectors  $\mathbf{H}_{jk} \mathbf{W}_k (k \neq j)$  and interference signal subspace at receiver  $j$ . According to the Lemma 5.1, the optimal orthonormal base  $\mathbf{Q}_k, k \in (1, 2, \dots, K)$  for interference signal subspace  $\square_k$  at receiver  $k$  can be obtained by expression (30) in the Appendix.

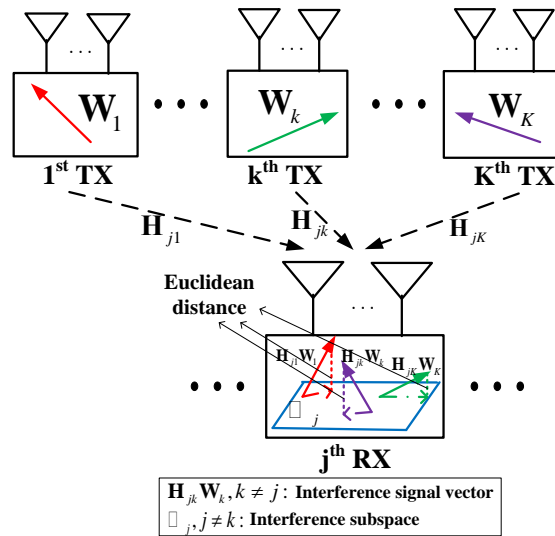


Fig. 4. The relationship between  $K - 1$  interference signal vectors  $\mathbf{H}_{jk} \mathbf{W}_k (k \neq j)$  and interference signal subspaces  $\square_j$  at receiver  $j, j \neq k$ .

### 6. Convergence criterion and algorithm

At receiver  $j$ , the total leakage-interference caused by  $K - 1$  undesired transmitters ( $k \neq j$ ) is given by:

$$IL_j = \mathbf{V}_j^* \mathbf{O}_j \mathbf{V}_j \tag{31}$$

where

$$\mathbf{O}_j = \sum_{k=1, k \neq j}^K P_k \mathbf{H}_{jk} \mathbf{W}_k \mathbf{W}_k^* \mathbf{H}_{jk}^* \tag{32}$$

and  $\mathbf{V}_j$  is linear receiver at receiver  $j$ .

The metric representing average interference ratio in desired signal space is defined as:

$$MeanR_{IL} = \frac{1}{K} \sum_{j=1}^K \frac{\mathbf{O}_j}{IL_j} \quad (33)$$

The value of  $MeanR_{IL}$  is reduced by each iteration of the algorithm. Since  $MeanR_{IL}$  is bounded below a threshold value, this implies that the algorithm must converge. According to the convergence criterion, the proposed algorithm can be described as follows:

**Step 1:** start with  $K$  arbitrary  $M \times 1$  precoding vectors  $\mathbf{W}_k (k=1,2,\dots,K)$  at transmitters and guarantee  $\mathbf{W}_k \mathbf{W}_k^* = \mathbf{I} (k=1,2,\dots,K)$ .

**Begin iteration.**

**Step 2:** calculate the interference signal subspace according to (30) and obtain the linear receiver according to (20) for each receiver.

**Step 3:** calculate the altruistic Bayesian game cost function  $A_{jk} (j \neq k, j=1,2,\dots,K)$  between the transmitter  $k$  and each undesired receiver  $j$  according to (8) and the egoistic Bayesian game cost function  $E_k$  between the transmitter  $k$  and the receiver  $k$  according to (13).

**Step 4:** calculate the precoding vectors  $\mathbf{W}_k (k=1,2,\dots,K)$  at transmitters according to (22), the rearrangement of the equation is given as:

$$\mathbf{W}_k = V^{min} \left( E_k + \sum_{j \neq k}^K \lambda_{jk}^{opt} A_{jk} \right) \quad (34)$$

where  $\lambda_{jk}^{opt}$  is determined by statistical channel information.

**Step 5:** go to step 2.

**Step 6:** continue until parameter  $MeanR_{IL}$  converges.

## 7. Simulation result

In this section, the numerical evaluation performance of the proposed algorithm is presented considering two major factors: 1) average sum-rate; 2) average ratio of leakage interference in desired signal space. The algorithm realization for 3-user interference channel has been considered with 2 antennas at each node. The transmission power budgets are set to 1 for all the transmitters and noise power  $\sigma_k^2$  is set equally for each transmitter, where  $\sigma_k^2 = 1 / (10^{SNR/10})$ . An uncorrelated fading channel model is used with channel coefficients generated from the complex Gaussian distribution  $\mathbf{CN}(0,1)$ . Each simulation result is averaged over 100 random channel realizations and the permitted maximum number of iterations for the proposed algorithm is 2000. In these simulations, an average of the instantaneous sum-rate (we call it average sum-rate in simulations) defined in formula (3) is chosen and average ratio of leakage interference to desired signal space defined in formula (33) is selected as the performance metrics.

According to Section 4, the parameter  $\lambda_{jk}^{opt}$  in (23) has been defined as a balancing factor between the egoistic Bayesian game cost function and the altruistic Bayesian game cost

function. The parameter  $\lambda_{jk}^{opt}$  used in this study is given as:

$$\lambda_{jk}^{opt} = \lambda * (10^{SNR/10}), (j, k) \in (1, 2, \dots, K) \tag{35}$$

The performance of the proposed algorithm is evaluated with different values of parameter  $\lambda$  in (35) to obtain the optimal balancing factor  $\lambda_{jk}^{opt}$ .

In Fig. 5 and Fig. 6, different markers are used to present each of the cases of  $\lambda = 0.01, 0.1, 1, 10, 100$ . Fig. 5 shows a plot of the average sum-rate values versus the SNRs for the 3-user interference channel with 2 antennas at each node in response to different values of parameters  $\lambda$ . Fig. 6 illustrates the variation of average ratio of leakage interference in desired signal space under different values of parameter  $\lambda$  as the values of SNR increase in the same scenario as the Fig. 5. It can be observed in Fig. 5 that the proposed algorithm provides almost the best performance on the average sum-rate when  $\lambda = 10$ . From the Fig. 6, Ittt can be noticed that the least average of leakage interference in desired signal space is obtained in the case of  $\lambda = 100$ ; and the performance gap between  $\lambda = 100$  and  $\lambda = 10$  is very small. Combining the results from these two simulations, the value  $10 * (10^{SNR/10})$  is selected for  $\lambda_{jk}^{opt}$  as the optimal balancing factor for 3-user interference channel with 2 antennas at each node.

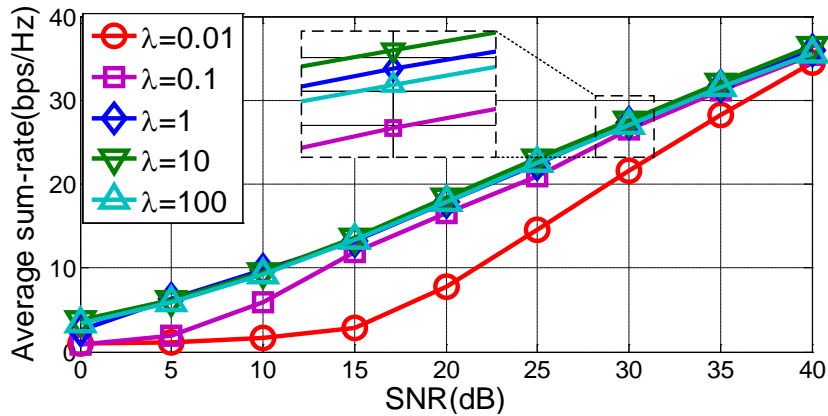


Fig. 5. Average sum-rate versus SNR with different values of the parameter  $\lambda = 0.01, 0.1, 1, 10, 100$  for the 3-user interference channel with 2 antennas at each node

In the coming simulations, the performance of the proposed algorithm using the selected balancing factor is compared with the following algorithms on beamformer design for multiuser interference channel:

- Reciprocal IA-CJ08[10]
- Alternating-Minimization[11]
- MMSE[13]
- Max SINR IA-CJ08[10]
- WMMSE[14]

In Fig. 7, the performance on the average sum-rate for 3-user interference is measured with 2 antennas at each node, where the results from all of the above mentioned algorithms are averaged over 100 random channel realizations while all other configurations are same as in the simulation in Fig. 5. Further, it must also be noted that the number of spatial dimensions

(i.e., antennas) is equal to two for all the algorithms. The result shows that the proposed algorithm produced almost similar sum-rate performance in comparison to Reciprocal IA-CJ08 and Max SINR IA-CJ08 at high SNR, which implies that the proposed scheme can achieve same total DoF as achieved by these interference alignment schemes with perfect CSI. The sum-rate advantage of the algorithms such as Reciprocal IA-CJ08, Max SINR IA-CJ08 and the proposed algorithm over others such as MMSE and WMMSE at high SNR is due to the interference alignment in the vector space. However, the Reciprocal IA-CJ08 and Max SINR IA-CJ08 require the reciprocity of channel and perfect CSI which indicate that both algorithms have limited application scenario in realistic system. Although the proposed algorithm does not produce best performance on the average sum-rate, it takes advantage of reduced external constrains over other two algorithms, hence it becomes a better alternative in practical applications. From the graph in Fig. 7, it can be seen that MMSE and WMMSE have better performance in low SNR that indicates a suboptimal "selfish" approach where a transmitter ignores the interference it causes and aims simply to maximize desired signal rate. These algorithms have their own advantages in low SNR's scenarios.

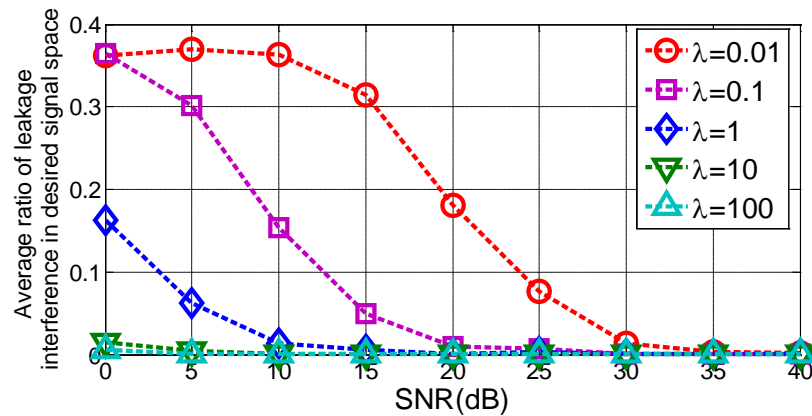


Fig. 6. Average ratio of leakage interference in desired signal space versus SNR with different values of the parameter  $\lambda = 0.01, 0.1, 1, 10, 100$  for the 3-user interference channel with 2 antennas at each node

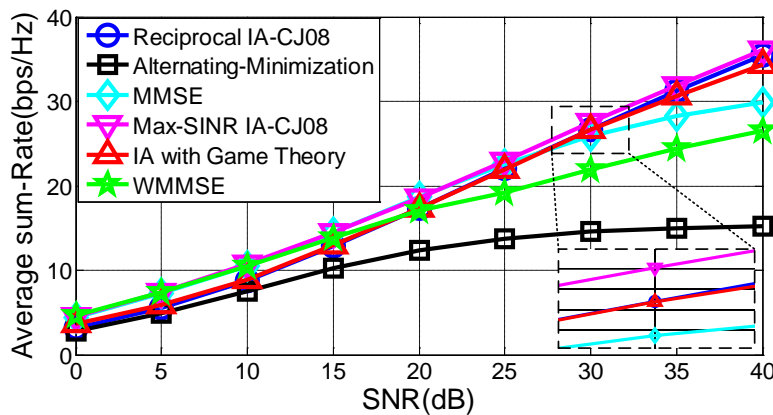
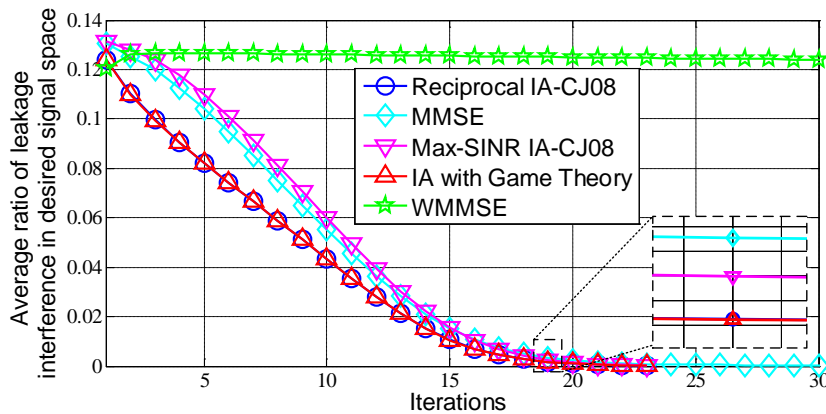


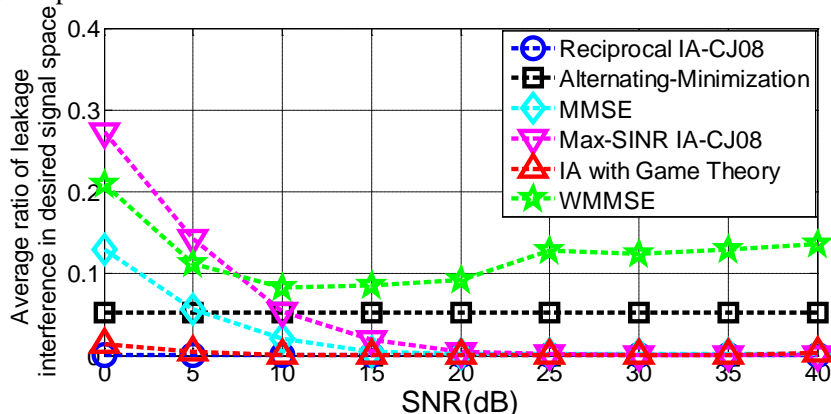
Fig. 7. Comparison of average sum-rate versus SNR for the 3-user interference channel with 2 antennas at each node

**Fig. 8** illustrates the convergence behaviors of the proposed algorithm, Reciprocal IA-CJ08 algorithm [10], MMSE algorithm [13], Max SINR IA-CJ08 algorithm [10] and WMMSE algorithm [14] for the case where SNR=30 (dB). The graph shows that the proposed algorithm and Reciprocal IA-CJ08 algorithm have better performance and converge in about 23 steps, where fast convergence means low algorithm overload.



**Fig. 8.** Comparison of average ratio of leakage interference in desired signal space versus Iterations for the 3-user interference channel with 2 antennas at each node

**Fig. 9** presents the performance of different algorithms on average ratio of leakage interference in desired signal space. It can be observed that the Reciprocal IA-CJ08 algorithm has significantly better performance compared to other algorithms. This is due to the fact that the Reciprocal IA-CJ08 aims to maximize the achievable DoF by minimizing the leakage interference in desired signal space. Other algorithms that address the maximization of the SINR or Sum-Rate have shown worse performance on leakage interference. By analyzing the structures of these algorithms such as MMSE and WMMSE, it is noticed that these algorithms always egoistically maximize the rate for data streams of desired signal by increasing the power of transmitted and thses does not altruistically suppress the interference to other undesired receivers. From the plot in **Fig. 7**, it can be seen that the performance of the proposed algorithm is very close to the optimal Reciprocal IA-CJ08 algorithm almost at all the SNRs. The simulation results indicate that the proposed algorithm try to maximize the sum-rate of the 3-user interference channel while keeping the leakage interference in the desired signal space to minimum.



**Fig. 9.** Comparison of average ratio of leakage interference in desired signal space versus SNR for the 3-user interference channel with 2 antennas at each node



## 8. Conclusion

In this paper, a numerical algorithm to achieve spatial interference alignment for  $K$  – user interference channel has been presented that is based on the framework of game theory. The proposed algorithm accounts for the impact of interference management scheme on both DoF and sum rate performance for  $K$  – user interference channel. The achievable DoF of the interference channel and the average sum rate of systems have been optimized to provide a better alternate to existing interference alignment schemes. While the signal subspace analytical approaches can only solve the minimization problem of the leakage interference power from the perspective of DoF, and the conventional optimization approaches are always used to deal with the sum rate maximization problem of system. The main contribution of this work is the design of a numerical interference alignment algorithm aiming to maximize sum rate with signal subspace analytical method and game theory. Furthermore, in the proposed algorithm only requires each transmitter only require the information about the channel between the corresponding communication pairs to obtain the better performance, which implies that this algorithm can be used in a distributed manner in realistic systems. In the future work, we will consider how we could utilize the other interference management approaches to obtain a medium performance if each transmitter only knows the local channel state information.

## Appendix

The problem is formulated by minimizing the sum of the squared Euclidean distances between all vectors  $\mathbf{H}_{jk} \mathbf{W}_k$  ( $k \neq j$ ) and their orthogonal projections onto  $\square_j$ :

$$\mathbf{Q}_j^{opt} = \arg \min_{\mathbf{W}_k^* \mathbf{W}_k = \mathbf{I}} \sum_{k \neq j}^K \|\mathbf{H}_{jk} \mathbf{W}_k - \mathbf{Q}_j \mathbf{Q}_j^* \mathbf{H}_{jk} \mathbf{W}_k\|_F^2 \quad (25)$$

where  $\mathbf{Q}_j$  is an orthonormal basis of  $\square_j$ . According to matrix theory, square of Frobenius norm has the following property:

$$\|\mathbf{B}\|_F^2 = \left( \sum_{i=1}^n \sum_{j=1}^m |b_{ij}|^2 \right) = \text{Tr}(\mathbf{B}^* \mathbf{B}) \quad (26)$$

Expression (25) can be extended as:

$$\mathbf{Q}_j^{opt} = \arg \min_{\mathbf{W}_k^* \mathbf{W}_k = \mathbf{I}} \text{Tr} \left( \sum_{k \neq j}^K (\mathbf{H}_{jk} \mathbf{W}_k)^* \mathbf{H}_{jk} \mathbf{W}_k - (\mathbf{H}_{jk} \mathbf{W}_k)^* \mathbf{Q}_j \mathbf{Q}_j^* \mathbf{H}_{jk} \mathbf{W}_k \right) \quad (27)$$

In expression (27), the first term of  $\text{Tr}(\bullet)$  is constant and only the second term of  $\text{Tr}(\bullet)$  is related to  $\mathbf{Q}_j^{opt}$ , it can lead to the following expression:

$$\mathbf{Q}_j^{opt} = \arg \max_{\mathbf{W}_k^* \mathbf{W}_k = \mathbf{I}} \text{Tr} \left( \sum_{k \neq j}^K (\mathbf{H}_{jk} \mathbf{W}_k)^* \mathbf{Q}_j \mathbf{Q}_j^* \mathbf{H}_{jk} \mathbf{W}_k \right) \quad (28)$$

Due to  $\text{Tr}(\mathbf{AB}) = \text{Tr}(\mathbf{BA})$  in linear algebra, expression (28) can be rewritten as follows:

$$\mathbf{Q}_j^{opt} = \arg \max_{\mathbf{W}_k^* \mathbf{W}_k = \mathbf{I}} \text{Tr} \left( \mathbf{Q}_j^* \left( \sum_{k \neq j}^K \mathbf{H}_{jk} \mathbf{W}_k (\mathbf{H}_{jk} \mathbf{W}_k)^* \right) \mathbf{Q}_j \right) \quad (29)$$

According to the matrix theory, the solution of  $\mathbf{Q}_j^{opt}$  for expression (29) is equal to the  $p$

dominant eigenvectors of  $\sum_{k \neq j}^K (\mathbf{H}_{jk} \mathbf{W}_k)^* \mathbf{H}_{jk} \mathbf{W}_k$ , so the columns of  $\mathbf{Q}_j^{opt}$  can be given by:

$$\mathbf{Q}_j^{opt} = V_p^{\max} \left( \sum_{k \neq j}^K \mathbf{H}_{jk} \mathbf{W}_k (\mathbf{H}_{jk} \mathbf{W}_k)^* \right) \quad (30)$$

where  $V_p^{\max}(\mathbf{A})$  represents the sets of eigenvectors corresponding to the  $p$  maximum eigenvalues of  $\mathbf{A}$ .

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