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Surplus Process Perturbed by Diffusion and Subject to Two Types of Claim

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Abstract

We introduce a surplus process which follows a diffusion process with positive drift and is subject to two types of claim. We assume that type I claim occurs more frequently, however, its size is stochastically smaller than type II claim. We obtain the ruin probability that the level of the surplus becomes negative, and then, decompose the ruin probability into three parts, two ruin probabilities caused by each type of claim and the probability that the level of the surplus becomes negative naturally due to the diffusion process. Finally, we illustrate a numerical example, when the sizes of both types of claim are exponentially distributed, to compare the impacts of two types of claim on the ruin probability of the surplus along with that of the diffusion process.

Keywords: Diffusion process, integro-differential equation, ruin probability, surplus process, two types of claim.

1. Introduction

In this paper, we consider a surplus process which is both perturbed by diffusion and subject to two types of claim. The surplus in the model is initially at $u > 0$, thereafter, follows a diffusion process with drift $c > 0$ and diffusion parameter σ^2 and decreases jumpwise due to arriving claims. There are two types of claim in the model. Type I claims arrive according to a Poisson process $\{N_1(t), t \geq 0\}$ of rate $\lambda_1 > 0$ and the sizes of the claims, Y_i for $i = 1, 2, \ldots$, are i.i.d. with distribution G, meanwhile, type II claims independent of type I claims arrive according to another Poisson process $\{N_2(t), t \geq 0\}$ of rate $\lambda_2 > 0$ and the sizes of the claims, Z_j for $j = 1, 2, \ldots$, are i.i.d. with distribution H. We assume that $\lambda_1 \geq \lambda_2$ and distribution function H is stochastically larger than distribution function G. That is, type I claims occur more frequently, however, their sizes are stochastically smaller than type II claims.

Let μ_1 and μ_2 be the means of G and H, respectively. The drift parameter of the diffusion process in the model may denote the incoming premium rate, and hence, it may be represented as $c = (1 + \theta)(\lambda_1\mu_1 + \lambda_2\mu_2)$, where $\theta > 0$ is the relative security loading and $\lambda_1\mu_1 + \lambda_2\mu_2$ is the loss rate of the surplus due to the claims. The surplus process in the model can be written as

$$U(t) = u + W(t) - \sum_{i=1}^{N_1(t)} Y_i - \sum_{j=1}^{N_2(t)} Z_j,$$

where $\{W(t), t \geq 0\}$ is a Brownian motion with drift parameter $c > 0$ and diffusion parameter σ^2, that is, $W(t)$ follows normal distribution $N(ct, \sigma^2 t)$. A sample path of the surplus process in the model is

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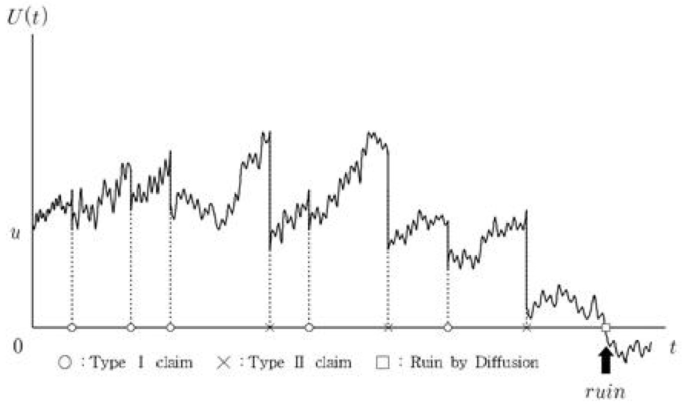

Figure 1: *Sample path of* $\{U(t), t \geq 0\}$

illustrated in Figure 1. In Figure 1, the surplus process becomes negative naturally due to the diffusion process.

The ruin probability of the classical surplus process, which increases linearly and is subject to one type of claim, has been studied by many authors. The studies on the ruin probability of the classical surplus process is well summarized in Klugman *et al.* (2004). The first passage time of the surplus process to a certain level was studied by Gerber (1990) and Wang and Politis (2002). Gerber and Shiu (1997) obtained the joint distribution of the time of ruin, the surplus immediately before ruin and the deficit at ruin. Dickson and Willmot (2005) calculated the density function of the time to ruin by an inversion of its Laplace transform. Jeong and Lee (2010) suggested an optimal control policy for the surplus process and Cho *et al.* (2013) obtained the transient and stationary distributions of the surplus process.

Dufresne and Gerber (1991) generalized the classical surplus process by assuming that the surplus is perturbed by diffusion between the time points of occurrence of claim and studied the ruin probabilities of the surplus process. In the same model, Wang and Wu (2000) studied the properties of the maximum level of the surplus until the ruin and Zhang and Wang (2003) obtained the joint distribution of the time of ruin, the surplus immediately before ruin and the deficit at ruin. Tsai (2009) studied the impacts of the frequency and size of the claim on the ruin probabilities and Tsai and Lu (2010) obtained the bound of the ruin probability of the surplus process perturbed by diffusion.

Recently, the surplus process where there exist different types of claims was studied by several researchers. Chan *et al.* (2003) introduced a model with two different surplus processes and derived an integro-differential equation satisfied by two ruin probabilities. Guo *et al.* (2007) also considered the two different surplus processes and obtained the ruin probability of the surplus process represented by the sum of two different surplus processes. Li and Garrido (2005) and Lv *et al.* (2010) studied a more complicate surplus process with two types of claim where one arrives according to a Poisson process and the other arrives according to a renewal process. Li and Garrido (2005) obtained the Laplace transform of the ruin probability of the surplus process and Lv *et al.* (2010) obtained an exponential bound for the ruin probability of the surplus process.

However, until now, the surplus process subject to two types of claim and also perturbed by diffusion between claims has not been studied yet. We, in Section 2, obtain the ruin probability of our generalized surplus process by establishing an integro-differential equation for the ruin probability, solving the equation for the Laplace transform of the ruin probability and inverting the Laplace transform. In Section 3, we decompose the ruin probability into three parts, two ruin probabilities caused by each type of claim and the probability that the level of the surplus becomes negative naturally due to the diffusion process. Finally, in Section 4, we illustrate a numerical example, when the sizes of both types of claim are exponentially distributed, to compare the impacts of two types of claim on the ruin probability of the whole surplus with that of the diffusion process.

2. Ruin Probability of the Surplus

Let $\psi(u)$ be the ruin probability of the surplus process, $\{U(t), t \geq 0\}$, when the initial surplus is $u > 0$, that is,

$$\psi(u) = \Pr\{U(t) \leq 0, \text{for some } t > 0 | U(0) = u\}.$$

Observe that the following six mutually exclusive events can occur during a small interval $(0, h)$:

(i) no claims occur, in which case

$$\psi(u) = \psi(u + W(h)), \qquad \text{with probability } 1 - (\lambda_1 + \lambda_2)h + o(h).$$

(ii) a type I claim occurs with $Y \geq u + W(h')$, for $h' < h$, in which case

$$\psi(u) = 1, \qquad \text{with probability } \{\lambda_1 h + o(h)\}\{1 - \lambda_2 h + o(h)\}.$$

(iii) a type II claim occurs with $Z \geq u + W(h')$, for $h' < h$, in which case

$$\psi(u) = 1, \qquad \text{with probability } \{\lambda_2 h + o(h)\}\{1 - \lambda_1 h + o(h)\}.$$

(iv) a type I claim occurs with $Y < u + W(h')$, for $h' < h$, in which case

$$\psi(u) = \psi(u - Y + W(h)), \qquad \text{with probability } \{\lambda_1 h + o(h)\}\{1 - \lambda_2 h + o(h)\}.$$

(v) a type II claim occurs with $Z < u + W(h')$, for $h' < h$, in which case

$$\psi(u) = \psi(u - Z + W(h)), \qquad \text{with probability } \{\lambda_2 h + o(h)\}\{1 - \lambda_1 h + o(h)\}.$$

(vi) two or more claims occur, however, the probability of this event is $o(h)$.

From these relations, we can have

$$\begin{aligned}
\psi(u) = &\{1 - (\lambda_1 + \lambda_2)h + o(h)\}E[\psi(u + W(h))] \\
&+ \{\lambda_1 h + o(h)\}\{1 - \lambda_2 h + o(h)\}E[\Pr(Y \geq u + W(h))] \\
&+ \{\lambda_2 h + o(h)\}\{1 - \lambda_1 h + o(h)\}E[\Pr(Z \geq u + W(h))] \\
&+ \{\lambda_1 h + o(h)\}\{1 - \lambda_2 h + o(h)\}E\left[\int_0^{u+W(h')} \psi(u - y + W(h))dG(y)\right] \\
&+ \{\lambda_2 h + o(h)\}\{1 - \lambda_1 h + o(h)\}E\left[\int_0^{u+W(h')} \psi(u - z + W(h))dH(z)\right] \\
&+ o(h).
\end{aligned} \tag{2.1}$$

Applying Taylor series expansion to $E[\psi(u + W(h))]$ gives

$$E[\psi(u+W(h))] = E\left[\psi(u) + \frac{\psi'(u)}{1!}W(h) + \frac{\psi''(u)}{2!}W(h)^2 + o(h)\right]$$
$$= \psi(u) + ch\psi'(u) + \frac{\sigma^2}{2}h\psi''(u) + o(h).$$

Here, $E[W(h)] = ch$ and $E[W(h)^2] = \mathrm{Var}[W(h)] + [E(W(h))]^2 = \sigma^2 h + o(h)$. Inserting the expansion into (2.1), dividing each side of (2.1) by h and letting $h \to 0$, we have the following integro-differential equation for $\psi(u)$:

$$-c\psi'(u) = -(\lambda_1 + \lambda_2)\psi(u) + \frac{\sigma^2}{2}\psi''(u) + \lambda_1\bar{G}(u) + \lambda_2\bar{H}(u) + \lambda_1\int_0^u \psi(u-y)dG(y) + \lambda_2\int_0^u \psi(u-z)dH(z), \tag{2.2}$$

where $\bar{G}(u) = 1 - G(u)$ and $\bar{H}(u) = 1 - H(u)$.

Let $\psi^*(s) = \int_0^\infty e^{-su}\psi(u)du$, for $s > 0$, be the Laplace transform of $\psi(u)$. Taking the Laplace transforms on both sides of (2.2) and solving the equation for $\psi^*(s)$, we have

$$\psi^*(s) = \frac{-c\psi(0) + \frac{\lambda_1}{s}\{1 - g^*(s)\} + \frac{\lambda_2}{s}\{1 - h^*(s)\} - \frac{\sigma^2}{2}\{s\psi(0) + \psi'(0)\}}{-cs + \lambda_1 + \lambda_2 - \lambda_1 g^*(s) - \lambda_2 h^*(s) - \frac{\sigma^2}{2}s^2}, \tag{2.3}$$

where $g^*(s) = \int_0^\infty e^{-su}dG(u)$ and $h^*(s) = \int_0^\infty e^{-su}dH(u)$ are the Laplace-Stieltjes transform of G and H, respectively. Here, notice that $\psi(0) = 1$ due to the property of the Brownian motion which oscillates infinitely many times even in a very small interval. To find $\psi'(0)$, we put $s \to 0$ in (2.3), then the denominator of $\psi^*(s)$ goes to zero. For $\psi^*(0)$ to exist, the numerator of $\psi^*(s)$ should also go to zero when $s \to 0$. This results in

$$\psi'(0) = \frac{2c}{\sigma^2}\left(\frac{\lambda_1\mu_1 + \lambda_2\mu_2}{c} - 1\right).$$

Inserting $\psi'(0)$ into (2.3), we can see that $\psi^*(s)$ is written as

$$\psi^*(s) = \frac{-(\lambda_1\mu_1 + \lambda_2\mu_2) + \frac{\lambda_1}{s}\{1 - g^*(s)\} + \frac{\lambda_2}{s}\{1 - h^*(s)\} - \frac{\sigma^2}{2}s}{-cs + \lambda_1\{1 - g^*(s)\} + \lambda_2\{1 - h^*(s)\} - \frac{\sigma^2}{2}s^2}. \tag{2.4}$$

We will invert $\psi^*(s)$ to find $\psi(u)$. To do that, define $G_e(u) = (1/\mu_1)\int_0^u \bar{G}(x)dx$ and $H_e(u) = (1/\mu_2)\int_0^u \bar{H}(x)dx$ as the equilibrium distributions of G and H, then it can be shown that the Laplace-Stieltjes transforms of G_e and H_e are given by

$$g_e^*(s) = \int_0^\infty e^{-su}dG_e(u) = \frac{1}{\mu_1 s}\{1 - g^*(s)\},$$
$$h_e^*(s) = \int_0^\infty e^{-su}dH_e(u) = \frac{1}{\mu_2 s}\{1 - h^*(s)\}.$$

Hence, $\psi^*(s)$ can be, now, written as

$$\psi^*(s) = \frac{1}{s} - \frac{\left(1 - \frac{\lambda_1\mu_1 + \lambda_2\mu_2}{c}\right)\frac{1}{s}}{1 + \frac{\sigma^2}{2c}s - \frac{\lambda_1\mu_1}{c}g_e^*(s) - \frac{\lambda_2\mu_2}{c}h_e^*(s)}. \tag{2.5}$$

After some algebras, we can show that $\psi^*(s)$ can be represented as the following infinite series:

$$\psi^*(s) = \sum_{k=1}^{\infty}\left(1 - \frac{\lambda_1\mu_1 + \lambda_2\mu_2}{c}\right)\left(\frac{\lambda_1\mu_1 + \lambda_2\mu_2}{c}\right)^{k-1}$$
$$\times\left[\frac{1}{s} - \left(\frac{2c/\sigma^2}{2c/\sigma^2 + s}\right)^k \frac{1}{s}\left(\frac{\lambda_1\mu_1}{\lambda_1\mu_1 + \lambda_2\mu_2} g_e^*(s) + \frac{\lambda_2\mu_2}{\lambda_1\mu_1 + \lambda_2\mu_2} h_e^*(s)\right)^{k-1}\right].$$

Inverting $\psi^*(s)$, finally, gives

$$\psi(u) = \sum_{k=1}^{\infty}\left(1 - \frac{\lambda_1\mu_1 + \lambda_2\mu_2}{c}\right)\left(\frac{\lambda_1\mu_1 + \lambda_2\mu_2}{c}\right)^{k-1}\left[1 - D^{(k)} \cdot M^{(k-1)}(u)\right], \tag{2.6}$$

where $D(u) = 1-\exp(-(2c/\sigma^2)u)$ and $M(u) = \{\lambda_1\mu_1/(\lambda_1\mu_1 + \lambda_2\mu_2)\}G_e(u)+\{\lambda_2\mu_2/(\lambda_1\mu_1 + \lambda_2\mu_2)\}H_e(u)$. Moreover, $\cdot$ and (k) denote the operators of the Stieltjes convolution and the k-fold recursive Stieltjes convolution, respectively.

3. Decomposition of the Ruin Probability

Let $\psi_1(u)$ and $\psi_2(u)$ be the ruin probabilities that the level of the surplus becomes negative jumpwise by type I claim and type II claim, respectively. Let $\psi_d(u)$ be the probability that the level of the surplus becomes negative continuously due to the diffusion process. In this section, we obtain $\psi_1(u)$, $\psi_2(u)$ and $\psi_d(u)$ and show that

$$\psi(u) = \psi_1(u) + \psi_2(u) + \psi_d(u), \quad \text{for } u \geq 0.$$

We, first, obtain $\psi_1(u)$. Observe again that the following six mutually exclusive events can occur during a small interval (o, h) and $\psi_1(u)$ satisfies the following relations:

(i) no claims occur, then, $\psi_1(u) = \psi_1(u + W(h))$.

(ii) a type I claim occurs with $Y \geq u + W(h')$, for $h' < h$, then, $\psi_1(u) = 1$.

(iii) a type II claim occurs with $Z \geq u + W(h')$, for $h' < h$, then, $\psi_1(u) = 0$.

(iv) a type I claim occurs with $Y < u + W(h')$, then, $\psi_1(u) = \psi_1(u - Y + W(h))$.

(v) a type II claim occurs with $Z < u + W(h')$, then, $\psi_1(u) = \psi_1(u - Z + W(h))$.

(vi) two or more claims occur, however, the probability of this event is $o(h)$.

From these relations, we can have

$$\begin{aligned}\psi_1(u) = {} & \{1 - (\lambda_1 + \lambda_2)h + o(h)\}E[\psi_1(u + W(h))] \\ & + \{\lambda_1 h + o(h)\}\{1 - \lambda_2 h + o(h)\}E[\Pr(Y \geq u + W(h))] \\ & + \{\lambda_1 h + o(h)\}\{1 - \lambda_2 h + o(h)\}E\left[\int_0^{u+W(h')} \psi_1(u - y + W(h))dG(y)\right] \\ & + \{\lambda_2 h + o(h)\}\{1 - \lambda_1 h + o(h)\}E\left[\int_0^{u+W(h')} \psi_1(u - z + W(h))dH(z)\right] \\ & + o(h).\end{aligned} \tag{3.1}$$

Applying the same approach adopted in the previous section, we can derive the following integro-differential equation for $\psi_1(u)$:

$$-c\psi_1'(u) = -(\lambda_1 + \lambda_2)\psi_1(u) + \frac{\sigma^2}{2}\psi_1''(u) + \lambda_1\bar{G}(u) \qquad (3.2)$$
$$+ \lambda_1 \int_0^u \psi_1(u-y)dG(y) + \lambda_2 \int_0^u \psi_1(u-z)dH(z).$$

Let $\psi_1^*(s) = \int_0^\infty e^{-su}\psi_1(u)du$, for $s > 0$, be the Laplace transform of $\psi_1(u)$. Taking the Laplace transforms on both sides of (3.2) and solving the equation for $\psi_1^*(s)$, we have

$$\psi_1^*(s) = \frac{-c\psi_1(0) + \frac{\lambda_1}{s}\{1 - g^*(s)\} - \frac{\sigma^2}{2}\{s\psi_1(0) + \psi_1'(0)\}}{-cs + \lambda_1 + \lambda_2 - \lambda_1 g^*(s) - \lambda_2 h^*(s) - \frac{\sigma^2}{2}s^2}. \qquad (3.3)$$

Notice that $\psi_1(0) = 0$. Moreover, if we put $s \to 0$ in (3.3), then the denominator of $\psi_1^*(s)$ goes to zero. For $\psi_1^*(0)$ to exist, the numerator of $\psi_1^*(s)$ should also go to zero when $s \to 0$. This results in

$$\psi_1'(0) = \frac{2\lambda_1\mu_1}{\sigma^2}.$$

Inserting $\psi_1'(0)$ into (3.3) and inverting the Laplace transform $\psi_1^*(s)$ by a similar method used to obtain $\psi(u)$ in Section 2, we have

$$\psi_1(u) = \sum_{k=1}^{\infty}\left(\frac{\lambda_1\mu_1 + \lambda_2\mu_2}{c}\right)^k \left(\frac{\lambda_1\mu_1}{\lambda_1\mu_1 + \lambda_2\mu_2}\right) D^{(k)} \cdot M^{(k-1)} \cdot \bar{G}_e(u). \qquad (3.4)$$

By symmetry, $\psi_2(u)$, the probability of the ruin caused by type II claim, is given by

$$\psi_2(u) = \sum_{k=1}^{\infty}\left(\frac{\lambda_1\mu_1 + \lambda_2\mu_2}{c}\right)^k \left(\frac{\lambda_2\mu_2}{\lambda_1\mu_1 + \lambda_2\mu_2}\right) D^{(k)} \cdot M^{(k-1)} \cdot \bar{H}_e(u). \qquad (3.5)$$

To obtain $\psi_d(u)$, the probability that the level of the surplus becomes negative continuously due to the diffusion process, observe again that the following six mutually exclusive events can occur during a small interval $(0, h)$ and $\psi_d(u)$ satisfies the following relations:

(i) no claims occur, then, $\psi_d(u) = \psi_d(u + W(h))$.

(ii) a type I claim occurs with $Y \geq u + W(h')$, for $h' < h$, then, $\psi_d(u) = 0$.

(iii) a type II claim occurs with $Z \geq u + W(h')$, for $h' < h$, then, $\psi_d(u) = 0$.

(iv) a type I claim occurs with $Y < u + W(h')$, then, $\psi_d(u) = \psi_d(u - Y + W(h))$.

(v) a type II claim occurs with $Z < u + W(h')$, then, $\psi_d(u) = \psi_d(u - Z + W(h))$.

(vi) two or more claims occur, however, the probability of this event is $o(h)$.

Table 1: Ruin probabilities when $\sigma^2 = 0.1$

σ^2	λ_1	μ_1	λ_2	μ_2	$\psi(u)$	$\psi_1(u)$	$\psi_2(u)$	$\psi_d(u)$
0.1	1	1	0.50	2	0.2155	0.0637(29.54)	0.1491(69.18)	0.0028(1.29)
			0.20	5	0.3954	0.0513(12.96)	0.3418(86.43)	0.0024(0.61)
			0.05	20	0.5798	0.0200(3.46)	0.5588(96.38)	0.0010(0.17)
			0.01	100	0.6482	0.0053(0.82)	0.6427(99.14)	0.0002(0.04)
	1.5	0.67	1	1	0.0775	0.0285(36.83)	0.0471(60.84)	0.0018(2.33)
	2	0.50			0.0615	0.0179(29.17)	0.0420(68.29)	0.0016(2.55)
	5	0.20			0.0377	0.0048(12.62)	0.0318(84.41)	0.0011(2.97)
	10	0.10			0.0315	0.0020(6.43)	0.0285(90.46)	0.0010(3.11)

From these relations, we can derive the following integro-differential equation for $\psi_d(u)$, by applying the same approach used to obtain that of $\psi(u)$ in Section 2:

$$\begin{aligned} -c\psi_d'(u) &= -(\lambda_1+\lambda_2)\psi_d(u) + \frac{\sigma^2}{2}\psi_d''(u) \\ &\quad + \lambda_1\int_0^u \psi_d(u-y)dG(y) + \lambda_2\int_0^u \psi_d(u-z)dH(z). \end{aligned} \tag{3.6}$$

Let $\psi_d^*(s) = \int_0^\infty e^{-su}\psi_d(u)du$, for $s > 0$, be the Laplace transform of $\psi_d(u)$. Taking the Laplace transforms on both sides of (3.6), solving the equation for $\psi_d^*(s)$ with boundary conditions $\psi_d(0) = 1$ and $\psi_d'(0) = -2c/\sigma^2$ and inverting the Laplace transform $\psi_d^*(s)$ by a similar method used to obtain $\psi(u)$ in Section 2, we have

$$\psi_d(u) = \sum_{k=1}^{\infty}\left(\frac{\lambda_1\mu_1+\lambda_2\mu_2}{c}\right)^k\left(\frac{\sigma^2/2}{\lambda_1\mu_1+\lambda_2\mu_2}\right)D^{(k-1)}\cdot M^{(k-1)}\cdot d(u), \tag{3.7}$$

where $d(u) = (d/du)D(u) = (2c/\sigma^2)\exp(-(2c/\sigma^2)u)$.

Finally, by making use of Laplace transforms of $\psi(u), \psi_1(u), \psi_2(u)$ and $\psi_d(u)$, we can show that $\psi(u) = \psi_1(u) + \psi_2(u) + \psi_d(u)$, for all $u \geq 0$.

4. Numerical Example

In this section, we calculate ruin probabilities $\psi(u), \psi_1(u), \psi_2(u)$ and $\psi_d(u)$ and compare them numerically, when the claim sizes of both types follow independently exponential distributions, that is, when

$$G(y) = 1 - e^{-\frac{y}{\mu_1}}, \quad \text{for } y > 0 \quad \text{and} \quad H(z) = 1 - e^{-\frac{z}{\mu_2}}, \quad \text{for } z > 0.$$

It is assumed that $\lambda_1\mu_1 = \lambda_2\mu_2$, that is, the expected total amount brought by type I claims per unit time is same as that amount brought by type II claims. Here, without loss of generality, we assume that $\lambda_1\mu_1 = \lambda_2\mu_2 = 1$ with $\lambda_1 \geq \lambda_2$ and $\mu_1 \leq \mu_2$. It is also assumed that $u = 10$ and $c = 2.5$.

In the following two tables, by varying the values of $\lambda_1, \mu_1, \lambda_2, \mu_2$ and σ^2, we compare the impacts of two types of claim on the ruin probability of the surplus along with that of the diffusion process. The numbers in the parentheses denote the proportions (percentages) of $\psi_1(u), \psi_2(u)$ and $\psi_d(u)$ in $\psi(u)$.

In both tables, we can see that $\psi_2(u) \geq \psi_1(u)$, that is, type II claims have more impact on the ruin probability of the surplus than type I claims. We can also see that $\psi_2(u)$ increases rapidly as μ_2 increases even though λ_2 decreases, which results in also the rapid increase of $\psi(u)$. This shows that

Table 2: Ruin probabilities when $\sigma^2 = 1$

σ^2	λ_1	μ_1	λ_2	μ_2	$\psi(u)$	$\psi_1(u)$	$\psi_2(u)$	$\psi_d(u)$
1	1	1	0.50	2	0.2517	0.0672(26.71)	0.1548(61.52)	0.0296(11.77)
			0.20	5	0.4168	0.0520(12.47)	0.3405(81.70)	0.0243(5.83)
			0.05	20	0.5864	0.0205(3.50)	0.5561(94.84)	0.0097(1.66)
			0.01	100	0.6499	0.0059(0.91)	0.6415(98.71)	0.0025(0.39)
	1.5	0.67	1	1	0.1172	0.0358(30.54)	0.0581(49.59)	0.0233(19.87)
	2	0.50			0.0996	0.0239(24.01)	0.0543(54.56)	0.0213(21.43)
	5	0.20			0.0705	0.0073(10.33)	0.0459(65.11)	0.0173(24.56)
	10	0.10			0.0619	0.0033(5.27)	0.0428(69.06)	0.0159(25.67)

the size of the claim has more impact on the ruin probability of the surplus than the rate of occurrence of the claim, which is a well-known fact in the risk theory.

It can be also seen that $\psi_1(u)$ as well as $\psi_2(u)$ decrease as μ_1 decreases even though λ_1 increases, however, $\psi_2(u)$ decreases relatively slowly compared to $\psi_1(u)$ and so the proportion of $\psi_2(u)$ in $\psi(u)$ increases. This shows again that type II claims which have larger losses contribute more to the ruin probability of the surplus than type I claims that occur more frequently.

Finally, we can see that $\psi(u)$ in Table 2 with $\sigma^2 = 1$ is larger than $\psi(u)$ in Table 1 with $\sigma^2 = 0.1$, no matter what the values of $\lambda_1, \mu_1, \lambda_2$ and μ_2 are, which, of course, results from the increase of $\psi_d(u)$, the probability that the level of the surplus becomes negative continuously due to the diffusion process.

5. Concluding Remarks

In this paper, we studied the ruin probabilities of a surplus process that follows a diffusion process with positive drift and is subject to two types of claim. Type I claim occurs more frequently, however, its size is stochastically smaller than type II claim. We obtained the ruin probability of the whole surplus process, two ruin probabilities caused by each type of claim and the ruin probability caused by diffusion process. We compared these ruin probabilities numerically when the sizes of both types of claim are exponentially distributed. However, when the sizes of both types of claim are generally distributed, the formulas of ruin probabilities obtained in the paper are too complicate to be applied in practice. Hence, it will be very practical to develope in the future simple approximation formulas for these theoretically complex ruin probabilities.

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