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A SOFT TRANSFER AND SOFT ALGEBRAIC EXTENSION OF INT-SOFT SUBALGEBRAS AND IDEALS IN BCK/BCI-ALGEBRAS

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Abstract. Using the notion of soft sets, the concepts of the soft transfer, support and soft algebraic extension of an int-soft subalgebra and ideal in BCK/BCI-algebras are introduced, and related properties are investigated. Conditions for a soft set to be an int-soft subalgebra and ideal are provided. Regarding the notion of support, conditions for the soft transfer of a soft set to be an int-soft subalgebra and ideal are considered.

1. Introduction

The study of BCK-algebras was initiated by K. Iséki in 1966 as a generalization of the concept of set-theoretic difference and propositional calculus. For the general development of BCK/BCI-algebras, the ideal theory and its fuzzification play an important role. Jun (together with Kim, Meng, Song and Xin) studied fuzzy trends of several notions in BCK/BCI-algebras (see [7, 9, 10, 13]). In [11], Lee et al. discussed fuzzy translations, (normalized, maximal) fuzzy extensions and fuzzy multiplications of fuzzy subalgebras in BCK/BCI-algebras. They also investigated relations among fuzzy translations, (normalized, maximal) fuzzy extensions and fuzzy multiplications.

Various problems in many fields involve data containing uncertainties which are dealt with wide range of existing theories such as the theory of probability, (intuitionistic) fuzzy set theory, vague sets, theory of interval mathematics and rough set theory etc. All of these theories have their own difficulties which are pointed out in [14]. To overcome these difficulties, Molodtsov [14] introduced the soft set theory as a new mathematical tool for dealing with uncertainties that is free from the difficulties. Molodtsov successfully applied the soft set theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement and so on (see [14, 15, 16, 17]). Jun et al. introduced the notion

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of int-soft sets, and applied the notion of soft set theory to BCK/BCI-algebras (see [2, 3, 4, 5, 6, 8, 18]).

In this paper, using the notion of soft sets, we introduce the concepts of the soft transfer and soft algebraic extension of an int-soft subalgebra and ideal in BCK/BCI-algebras, and investigate related properties. We provide conditions for a soft set to be an int-soft subalgebra and ideal. We define the support of a soft set, and consider conditions for the soft transfer of a soft set to be an int-soft subalgebra and ideal.

2. Preliminaries

A BCK/BCI-algebra is an important class of logical algebras introduced by K. Iséki and was extensively investigated by several researchers.

An algebra $\mathcal{X} := (X; *, 0)$ of type $(2, 0)$ is called a *BCI-algebra* if it satisfies the following conditions:

(I) $(\forall x, y, z \in X)\ (((x * y) * (x * z)) * (z * y) = 0)$,
(II) $(\forall x, y \in X)\ ((x * (x * y)) * y = 0)$,
(III) $(\forall x \in X)\ (x * x = 0)$,
(IV) $(\forall x, y \in X)\ (x * y = 0, y * x = 0 \Rightarrow x = y)$.

If a BCI-algebra $\mathcal{X}$ satisfies the following identity:

(V) $(\forall x \in X)\ (0 * x = 0)$,

then $\mathcal{X}$ is called a *BCK-algebra*. Any BCK/BCI-algebra $\mathcal{X}$ satisfies the following axioms:

(a1) $(\forall x \in X)\ (x * 0 = x)$,
(a2) $(\forall x, y, z \in X)\ (x \le y \Rightarrow x * z \le y * z,\ z * y \le z * x)$,
(a3) $(\forall x, y, z \in X)\ ((x * y) * z = (x * z) * y)$,
(a4) $(\forall x, y, z \in X)\ ((x * z) * (y * z) \le x * y)$,

where $x \le y$ if and only if $x * y = 0$. A nonempty subset S of a BCK/BCI-algebra $\mathcal{X}$ is called a *subalgebra* of $\mathcal{X}$ if $x * y \in S$ for all $x, y \in S$. A subset A of a BCK/BCI-algebra $\mathcal{X}$ is called an *ideal* of $\mathcal{X}$ if it satisfies:

(b1) $0 \in A$,
(b2) $(\forall x \in X)(\forall y \in A)(x * y \in A \Rightarrow x \in A)$.

We refer the reader to the books [1, 12] for further information regarding BCK/BCI-algebras.

Molodtsov [14] defined the soft set in the following way: Let U be an initial universe set and E be a set of parameters. Let $\mathscr{P}(U)$ denotes the power set of U and $A \subset E$.

A pair $(\tilde{f}, A)$ is called a *soft set* over U (see [14]) if $\tilde{f}$ is a mapping given by

$$\tilde{f} : A \to \mathscr{P}(U).$$

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A$, $\tilde{f}(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set (F, A). Clearly, a soft set is not a set. For illustration, Molodtsov considered several examples in [14].

3. A soft transfer of int-soft subalgebras and ideals

In what follows, let $\mathcal{X}$ denote a BCK/BCI-algebra unless otherwise specified, and we take $\mathcal{X}$ as a set of parameters.

Definition 3.1 ([5])**.** A soft set $(\tilde{f}, \mathcal{X})$ over U is called an *int-soft subalgebra* of $\mathcal{X}$ over U if it satisfies the following condition:

$$(\forall x, y \in X)\left(\tilde{f}(x) \cap \tilde{f}(y) \subseteq \tilde{f}(x * y)\right). \tag{3.1}$$

Definition 3.2 ([5])**.** A soft set $(\tilde{f}, \mathcal{X})$ over U is called an *int-soft ideal* of $\mathcal{X}$ over U if it satisfies the following condition:

$$(\forall x \in X)\left(\tilde{f}(x) \subseteq \tilde{f}(0)\right), \tag{3.2}$$

$$(\forall x, y \in X)\left(\tilde{f}(x * y) \cap \tilde{f}(y) \subseteq \tilde{f}(x)\right). \tag{3.3}$$

For any soft set $(\tilde{f}, \mathcal{X})$ over U, we consider the set

$$\Sigma_{\tilde{f}} := U \setminus \bigcup_{x \in X} \tilde{f}(x). \tag{3.4}$$

It is clear that $\Sigma_{\tilde{f}}$ and $\tilde{f}(x)$ are disjoint for all $x \in X$.

Definition 3.3. For a soft set $(\tilde{f}, \mathcal{X})$ over U and $\varepsilon \subseteq \Sigma_{\tilde{f}}$, a soft set $(\tilde{f}^t_\varepsilon, \mathcal{X})$ is called the *soft transfer* of $(\tilde{f}, \mathcal{X})$ with respect to ε (briefly, ε-soft transfer of $(\tilde{f}, \mathcal{X})$) where

$$\tilde{f}^t_\varepsilon : X \to \mathscr{P}(U),\ x \mapsto \tilde{f}(x) \cup \varepsilon.$$

Proposition 3.4. *Let $(\tilde{f}, \mathcal{X})$ be a soft set over U and $\varepsilon \subseteq \Sigma_{\tilde{f}}$. If $(\tilde{f}, \mathcal{X})$ is an int-soft subalgebra of $\mathcal{X}$ over U, then the ε-soft transfer of $(\tilde{f}, \mathcal{X})$ satisfies*:

$$(\forall x \in L)\left(\tilde{f}^t_\varepsilon(0) \supseteq \tilde{f}^t_\varepsilon(x)\right). \tag{3.5}$$

If $(\tilde{f}, \mathcal{X})$ is an int-soft ideal of $\mathcal{X}$ over U, then the ε-soft transfer of $(\tilde{f}, \mathcal{X})$ satisfies:

$$(\forall x, y \in X)\left(x \leq y \ \Rightarrow\ \tilde{f}^t_\varepsilon(x) \supseteq \tilde{f}^t_\varepsilon(x)\right), \tag{3.6}$$

$$(\forall x, y, z \in X)\left(x * y \leq z \ \Rightarrow\ \tilde{f}^t_\varepsilon(x) \supseteq \tilde{f}^t_\varepsilon(y) \cap \tilde{f}^t_\varepsilon(z)\right). \tag{3.7}$$

Proof. Let $(\tilde{f}, \mathcal{X})$ be an int-soft subalgebra of $\mathcal{X}$ over U. For any $x \in X$, we have $\tilde{f}^t_\varepsilon(0) = \tilde{f}(0) \cup \varepsilon = \tilde{f}(x * x) \cup \varepsilon \supseteq (\tilde{f}(x) \cap \tilde{f}(x)) \cup \varepsilon = \tilde{f}(x) \cup \varepsilon = \tilde{f}^t_\varepsilon(x)$. Assume that $(\tilde{f}, \mathcal{X})$ is an int-soft ideal of $\mathcal{X}$ over U. Let $x, y \in X$ be such that $x \leq y$. Then $x * y = 0$, and so $\tilde{f}^t_\varepsilon(y) = \tilde{f}(y) \cup \varepsilon = (\tilde{f}(y) \cap \tilde{f}(0)) \cup \varepsilon =$

$(\tilde{f}(y) \cap \tilde{f}(x * y)) \cup \varepsilon \subseteq \tilde{f}(x) \cup \varepsilon = \tilde{f}^t_\varepsilon(x)$. Now let $x, y, z \in X$ be such that $x * y \leq z$. Then $\tilde{f}^t_\varepsilon(x * y) \supseteq \tilde{f}^t_\varepsilon(z)$ by (3.6). It follows from (3.3) that

$$\begin{aligned} \tilde{f}^t_\varepsilon(x) &= \tilde{f}(x) \cup \varepsilon \\ &\supseteq (\tilde{f}(x * y) \cap \tilde{f}(y)) \cup \varepsilon \\ &= (\tilde{f}(x * y) \cup \varepsilon) \cap (\tilde{f}(y) \cup \varepsilon) \\ &= \tilde{f}^t_\varepsilon(x * y) \cap \tilde{f}^t_\varepsilon(y) \\ &\supseteq \tilde{f}^t_\varepsilon(y) \cap \tilde{f}^t_\varepsilon(z). \end{aligned}$$

This completes the proof. □

Theorem 3.5. *If $(\tilde{f}, \mathcal{X})$ is an int-soft subalgebra of $\mathcal{X}$ over U, then the ε-soft transfer of $(\tilde{f}, \mathcal{X})$ is an int-soft subalgebra of $\mathcal{X}$ over U for all $\varepsilon \subseteq \Sigma_{\tilde{f}}$.*

Proof. Using (3.1), we have

$$\begin{aligned} \tilde{f}^t_\varepsilon(x * y) &= \tilde{f}(x * y) \cup \varepsilon \\ &\supseteq (\tilde{f}(x) \cap \tilde{f}(y)) \cup \varepsilon \\ &= (\tilde{f}(x) \cup \varepsilon) \cap (\tilde{f}(y) \cup \varepsilon) \\ &= \tilde{f}^t_\varepsilon(x) \cap \tilde{f}^t_\varepsilon(y) \end{aligned}$$

for all $x, y \in X$ and $\varepsilon \subseteq \Sigma_{\tilde{f}}$. Therefore $(\tilde{f}^t_\varepsilon, \mathcal{X})$ is an int-soft subalgebra of $\mathcal{X}$ over U. □

Theorem 3.6. *If $(\tilde{f}, \mathcal{X})$ is an int-soft ideal of $\mathcal{X}$ over U, then the ε-soft transfer of $(\tilde{f}, \mathcal{X})$ is an int-soft ideal of $\mathcal{X}$ over U for all $\varepsilon \subseteq \Sigma_{\tilde{f}}$.*

Proof. Using (3.2) and (3.3), we have

$$\tilde{f}^t_\varepsilon(x) = \tilde{f}(x) \cup \varepsilon \subseteq \tilde{f}(0) \cup \varepsilon = \tilde{f}^t_\varepsilon(0)$$

and

$$\begin{aligned} \tilde{f}^t_\varepsilon(x) &= \tilde{f}(x) \cup \varepsilon \\ &\supseteq (\tilde{f}(x * y) \cap \tilde{f}(y)) \cup \varepsilon \\ &= (\tilde{f}(x * y) \cup \varepsilon) \cap (\tilde{f}(y) \cup \varepsilon) \\ &= \tilde{f}^t_\varepsilon(x * y) \cap \tilde{f}^t_\varepsilon(y) \end{aligned}$$

for all $x, y \in X$ and $\varepsilon \subseteq \Sigma_{\tilde{f}}$. Therefore $(\tilde{f}^t_\varepsilon, \mathcal{X})$ is an int-soft ideal of $\mathcal{X}$ over U. □

We consider the converse of Theorems 3.5 and 3.6.

Theorem 3.7. *Let $(\tilde{f}, \mathcal{X})$ be a soft set over U. If there exists a subset ε of $\Sigma_{\tilde{f}}$ such that the ε-soft transfer of $(\tilde{f}, \mathcal{X})$ is an int-soft subalgebra of $\mathcal{X}$ over U, then $(\tilde{f}, \mathcal{X})$ is an int-soft subalgebra of $\mathcal{X}$ over U.*

Proof. Assume that the ε-soft transfer $(\tilde{f}^t_\varepsilon, \mathcal{X})$ of $(\tilde{f}, \mathcal{X})$ is an int-soft subalgebra of $\mathcal{X}$ over U for some $\varepsilon \subseteq \Sigma_{\tilde{f}}$. Then

$$\begin{aligned}\tilde{f}(x * y) \cup \varepsilon &= \tilde{f}^t_\varepsilon(x * y) \supseteq \tilde{f}^t_\varepsilon(x) \cap \tilde{f}^t_\varepsilon(y)\\ &= (\tilde{f}(x) \cup \varepsilon) \cap (\tilde{f}(y) \cup \varepsilon)\\ &= (\tilde{f}(x) \cap \tilde{f}(y)) \cup \varepsilon\end{aligned}$$

for all $x, y \in X$. Now, if $z \in \tilde{f}(x) \cap \tilde{f}(y)$, then

$$z \in (\tilde{f}(x) \cap \tilde{f}(y)) \cup \varepsilon \subseteq \tilde{f}(x * y) \cup \varepsilon$$

and $z \notin \varepsilon$ since $\tilde{f}(x)$ and ε are disjoint for all $x \in X$. Hence $z \in \tilde{f}(x * y)$, and so $\tilde{f}(x) \cap \tilde{f}(y) \subseteq \tilde{f}(x * y)$. Therefore $(\tilde{f}, \mathcal{X})$ is an int-soft subalgebra of $\mathcal{X}$ over U. □

Theorem 3.8. *Let $(\tilde{f}, \mathcal{X})$ be a soft set over U. If there exists a subset ε of $\Sigma_{\tilde{f}}$ such that the ε-soft transfer of $(\tilde{f}, \mathcal{X})$ is an int-soft ideal of $\mathcal{X}$ over U, then $(\tilde{f}, \mathcal{X})$ is an int-soft ideal of $\mathcal{X}$ over U.*

Proof. Assume that the ε-soft transfer $(\tilde{f}^t_\varepsilon, \mathcal{X})$ of $(\tilde{f}, \mathcal{X})$ is an int-soft ideal of $\mathcal{X}$ over U for some $\varepsilon \subseteq \Sigma_{\tilde{f}}$. Then

$$\tilde{f}(x) \cup \varepsilon = \tilde{f}^t_\varepsilon(x) \subseteq \tilde{f}^t_\varepsilon(0) = \tilde{f}(0) \cup \varepsilon$$

and

$$\begin{aligned}\tilde{f}(x) \cup \varepsilon &= \tilde{f}^t_\varepsilon(x) \supseteq \tilde{f}^t_\varepsilon(x * y) \cap \tilde{f}^t_\varepsilon(y)\\ &= (\tilde{f}(x * y) \cup \varepsilon) \cap (\tilde{f}(y) \cup \varepsilon)\\ &= (\tilde{f}(x * y) \cap \tilde{f}(y)) \cup \varepsilon\end{aligned}$$

for all $x, y \in X$. Since $\tilde{f}(x)$ and ε are disjoint for all $x \in X$, it follows that $\tilde{f}(0) \supseteq \tilde{f}(x)$ and $\tilde{f}(x) \supseteq \tilde{f}(x * y) \cap \tilde{f}(y)$ for all $x, y \in X$. Therefore $(\tilde{f}, \mathcal{X})$ is an int-soft ideal of $\mathcal{X}$ over U. □

For any soft set $(\tilde{f}, \mathcal{X})$ over U, consider a set

$$\mathcal{X}^\varepsilon_\delta := \{x \in X \mid \delta \setminus \varepsilon \subseteq \tilde{f}(x)\},$$

where $\varepsilon \subseteq \Sigma_{\tilde{f}}$ and $\delta \in \mathscr{P}(U)$ with $\varepsilon \subseteq \delta$. We say that $\mathcal{X}^\varepsilon_\delta$ is the (δ, ε)-support of $(\tilde{f}, \mathcal{X})$. Note that

$$(\forall x \in X)\left(\delta \setminus \varepsilon \subseteq \tilde{f}(x) \;\Leftrightarrow\; \delta \subseteq \tilde{f}(x) \cup \varepsilon = \tilde{f}^t_\varepsilon(x)\right).$$

Hence $\mathcal{X}^\varepsilon_\delta := \{x \in X \mid \delta \subseteq \tilde{f}^t_\varepsilon(x)\}$.

Theorem 3.9. *For any $\varepsilon \subseteq \Sigma_{\tilde{f}}$, if a soft set $(\tilde{f}, \mathcal{X})$ over U is an int-soft subalgebra (resp. ideal) of $\mathcal{X}$ over U, then the (δ, ε)-support of $(\tilde{f}, \mathcal{X})$ is a subalgebra (resp. ideal) of $\mathcal{X}$ for all $\delta \in \mathscr{P}(U)$ with $\varepsilon \subseteq \delta$.*

Proof. Assume that $(\tilde{f}, \mathcal{X})$ is an int-soft subalgebra (resp. ideal) of $\mathcal{X}$ over U. Let $x, y \in X$. If $x, y \in \mathcal{X}_\delta^\varepsilon$, then $\delta \setminus \varepsilon \subseteq \tilde{f}(x)$ and $\delta \setminus \varepsilon \subseteq \tilde{f}(y)$. Hence

$$\tilde{f}(x * y) \supseteq \tilde{f}(x) \cap \tilde{f}(y) \supseteq \delta \setminus \varepsilon$$

and $\tilde{f}(0) \supseteq \tilde{f}(x) \supseteq \delta \setminus \varepsilon$ by (3.1) and (3.2), and so $x * y \in \mathcal{X}_\delta^\varepsilon$ and $0 \in \mathcal{X}_\delta^\varepsilon$. Suppose that $x * y \in \mathcal{X}_\delta^\varepsilon$ and $y \in \mathcal{X}_\delta^\varepsilon$. Then $\delta \setminus \varepsilon \subseteq \tilde{f}(x * y)$ and $\delta \setminus \varepsilon \subseteq \tilde{f}(y)$. Using (3.3), we have

$$\tilde{f}(x) \supseteq \tilde{f}(x * y) \cap \tilde{f}(y) \supseteq \delta \setminus \varepsilon$$

and thus $x \in \mathcal{X}_\delta^\varepsilon$. Therefore the (δ, ε)-support of $(\tilde{f}, \mathcal{X})$ is a subalgebra (resp. ideal) of $\mathcal{X}$. □

Using Theorems 3.7, 3.8 and 3.9, we obtain the following corollary.

Corollary 3.10. *For a soft set $(\tilde{f}, \mathcal{X})$ over U, if there exists a subset ε of $\Sigma_{\tilde{f}}$ such that the ε-soft transfer of $(\tilde{f}, \mathcal{X})$ is a subalgebra (resp. ideal) of $\mathcal{X}$ over U, then the (δ, ε)-support of $(\tilde{f}, \mathcal{X})$ is a subalgebra (resp. ideal) of $\mathcal{X}$ for all $\delta \in \mathscr{P}(U)$ with $\varepsilon \subseteq \delta$.*

Given $\varepsilon \subseteq \Sigma_{\tilde{f}}$ and any $\delta \in \mathscr{P}(U)$ with $\varepsilon \subseteq \delta$, let $(\tilde{f}, \mathcal{X})$ be a soft set over U such that the (δ, ε)-support of $(\tilde{f}, \mathcal{X})$ is a subalgebra of $\mathcal{X}$. Let $x, y \in X$ be such that $\tilde{f}_\varepsilon^t(x) = \delta_x$ and $\tilde{f}_\varepsilon^t(y) = \delta_y$. If we take $\delta = \delta_x \cap \delta_y$, then $\tilde{f}_\varepsilon^t(x) = \delta_x \supseteq \delta$ and $\tilde{f}_\varepsilon^t(y) = \delta_y \supseteq \delta$. Thus $x, y \in \mathcal{X}_\delta^\varepsilon$, and so $x * y \in \mathcal{X}_\delta^\varepsilon$. It follows that

$$\tilde{f}_\varepsilon^t(x * y) \supseteq \delta = \delta_x \cap \delta_y = \tilde{f}_\varepsilon^t(x) \cap \tilde{f}_\varepsilon^t(y).$$

Therefore we have the following theorem.

Theorem 3.11. *Given $\varepsilon \subseteq \Sigma_{\tilde{f}}$ and any $\delta \in \mathscr{P}(U)$ with $\varepsilon \subseteq \delta$, if the (δ, ε)-support of a soft set $(\tilde{f}, \mathcal{X})$ over U is a subalgebra of $\mathcal{X}$, then the ε-soft transfer of $(\tilde{f}, \mathcal{X})$ is an int-soft subalgebra of $\mathcal{X}$ over U.*

Corollary 3.12. *Given $\varepsilon \subseteq \Sigma_{\tilde{f}}$ and any $\delta \in \mathscr{P}(U)$ with $\varepsilon \subseteq \delta$, if the (δ, ε)-support of a soft set $(\tilde{f}, \mathcal{X})$ over U is a subalgebra of $\mathcal{X}$, then $(\tilde{f}, \mathcal{X})$ is an int-soft subalgebra of $\mathcal{X}$ over U.*

Theorem 3.13. *Let $\mathcal{X}$ be a BCK-algebra. Given $\varepsilon \subseteq \Sigma_{\tilde{f}}$ and any $\delta \in \mathscr{P}(U)$ with $\varepsilon \subseteq \delta$, if the (δ, ε)-support of a soft set $(\tilde{f}, \mathcal{X})$ over U is an ideal of $\mathcal{X}$, then the ε-soft transfer of $(\tilde{f}, \mathcal{X})$ is an int-soft ideal of $\mathcal{X}$ over U.*

Proof. Suppose that the (δ, ε)-support $\mathcal{X}_\delta^\varepsilon$ of a soft set $(\tilde{f}, \mathcal{X})$ over U is an ideal of $\mathcal{X}$. Then $\mathcal{X}_\delta^\varepsilon$ is a subalgebra of $\mathcal{X}$ because every ideal is a subalgebra in a BCK-algebra $\mathcal{X}$. It follows from Theorem 3.11 that the ε-soft transfer of $(\tilde{f}, \mathcal{X})$ is an int-soft subalgebra of $\mathcal{X}$ over U and from Theorem 3.7 that $(\tilde{f}, \mathcal{X})$ is an int-soft subalgebra of $\mathcal{X}$ over U. Using (3.5) in Proposition 3.4, we have $\tilde{f}_\varepsilon^t(0) \supseteq \tilde{f}_\varepsilon^t(x)$ for all $x \in X$. Let $x, y \in X$ be such that $\tilde{f}_\varepsilon^t(x * y) = \delta_{x*y}$ and $\tilde{f}_\varepsilon^t(y) = \delta_y$. If we take $\delta := \delta_{x*y} \cap \delta_y$, then $\tilde{f}_\varepsilon^t(x * y) = \delta_{x*y} \supseteq \delta$ and

$\tilde{f}^t_\varepsilon(y) = \delta_y \supseteq \delta$, that is, $x * y \in \mathcal{X}^\varepsilon_\delta$ and $y \in \mathcal{X}^\varepsilon_\delta$. Since $\mathcal{X}^\varepsilon_\delta$ is an ideal of $\mathcal{X}$, we have $x \in \mathcal{X}^\varepsilon_\delta$ by (b2). Thus $\tilde{f}^t_\varepsilon(x) \supseteq \delta = \delta_{x*y} \cap \delta_y = \tilde{f}^t_\varepsilon(x*y) \cap \tilde{f}^t_\varepsilon(y)$. Therefore the ε-soft transfer of $(\tilde{f}, \mathcal{X})$ is an int-soft ideal of $\mathcal{X}$ over U. □

Definition 3.14. Let $(\tilde{f}, \mathcal{X})$ and $(\tilde{g}, \mathcal{X})$ be soft sets over U. We say that $(\tilde{g}, \mathcal{X})$ is a *soft algebraic extension* of $(\tilde{f}, \mathcal{X})$ if it satisfies:

(1) $(\forall x \in X)\left(\tilde{f}(x) \subseteq \tilde{g}(x)\right)$,

(2) $(\tilde{g}, \mathcal{X})$ is an int-soft subalgebra of $\mathcal{X}$ over U whenever $(\tilde{f}, \mathcal{X})$ is an int-soft subalgebra of $\mathcal{X}$ over U.

Example 3.15. If $(\tilde{f}, \mathcal{X})$ is an int-soft subalgebra of $\mathcal{X}$ over U, then the ε-soft transfer of $(\tilde{f}, \mathcal{X})$ is a soft algebraic extension of $(\tilde{f}, \mathcal{X})$ for all $\varepsilon \subseteq \Sigma_{\tilde{f}}$.

The following example shows that there exist soft sets $(\tilde{f}, \mathcal{X})$ and $(\tilde{g}, \mathcal{X})$ over U such that

(1) $(\tilde{f}, \mathcal{X})$ is an int-soft subalgebra of $\mathcal{X}$ over U.

(2) $(\tilde{g}, \mathcal{X})$ is a soft algebraic extension of $(\tilde{f}, \mathcal{X})$.

(3) $(\tilde{g}, \mathcal{X})$ is not the ε-soft transfer of $(\tilde{f}, \mathcal{X})$ for any $\varepsilon \subseteq \Sigma_{\tilde{g}}$.

Example 3.16. Let $X = \{0, 1, 2, 3, 4\}$ be a set with the following Cayley table:

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	2	0
3	3	1	3	0	1
4	4	4	4	4	0

Then $\mathcal{X} := (X, *, 0)$ is a BCK-algebra (see [12]). Let $(\tilde{f}, \mathcal{X})$ be a soft set over $U = \{1, 2, 3, 4, 5, 6\}$ in which $\tilde{f}$ is defined by

$$\tilde{f} : X \to \mathscr{P}(U),\ x \mapsto \begin{cases} \{1,2,3,5\} & \text{if } x = 0, \\ \{1,3\} & \text{if } x = 1, \\ \{1,5\} & \text{if } x = 2, \\ \{3,5\} & \text{if } x = 3, \\ \{1\} & \text{if } x = 4. \end{cases}$$

Then $(\tilde{f}, \mathcal{X})$ is an int-soft subalgebra of $\mathcal{X}$ over U.

(1) Let $(\tilde{g}, \mathcal{X})$ be a soft set over $U = \{1, 2, 3, 4, 5, 6\}$ in which $\tilde{g}$ is defined by

$$\tilde{g} : X \to \mathscr{P}(U),\ x \mapsto \begin{cases} \{1,2,3,5\} & \text{if } x = 0, \\ \{1,3,5\} & \text{if } x = 1, \\ \{1,2,5\} & \text{if } x = 2, \\ \{2,3,5\} & \text{if } x = 3, \\ \{1,2\} & \text{if } x = 4. \end{cases}$$

Then $\tilde{f}(x) \subseteq \tilde{g}(x)$ for all $x \in X$, and $(\tilde{g}, \mathcal{X})$ is an int-soft subalgebra of $\mathcal{X}$ over U. Therefore $(\tilde{g}, \mathcal{X})$ is a soft algebraic extension of $(\tilde{f}, \mathcal{X})$. Note that $\Sigma_{\tilde{f}} = \{4, 6\}$ and there is no $\varepsilon \subseteq \Sigma_{\tilde{g}}$ such that $(\tilde{g}, \mathcal{X})$ is the ε-soft transfer of $(\tilde{f}, \mathcal{X})$.

(2) Let $(\tilde{h}, \mathcal{X})$ be a soft set over $U = \{1, 2, 3, 4, 5, 6\}$ in which $\tilde{g}$ is defined by

$$\tilde{h} : X \to \mathscr{P}(U),\ x \mapsto \begin{cases} \{1,2,3,5,6\} & \text{if } x = 0, \\ \{1,3,6\} & \text{if } x = 1, \\ \{1,5,6\} & \text{if } x = 2, \\ \{3,5,6\} & \text{if } x = 3, \\ \{1,6\} & \text{if } x = 4. \end{cases}$$

Then $(\tilde{h}, \mathcal{X}) = (\tilde{f}^t_\varepsilon, \mathcal{X})$ where $\varepsilon = \{6\} \subseteq \Sigma_{\tilde{f}}$, that is, $(\tilde{h}, \mathcal{X})$ is the ε-soft transfer of $(\tilde{f}, \mathcal{X})$. Thus it is a soft algebraic extension of $(\tilde{f}, \mathcal{X})$.

Let $(\tilde{f}, \mathcal{X})$ be an int-soft subalgebra of $\mathcal{X}$ over U. For every $\varepsilon \subseteq \Sigma_{\tilde{f}}$, the ε-soft transfer of $(\tilde{f}, \mathcal{X})$ is an int-soft subalgebra of $\mathcal{X}$ over U. If $(\tilde{g}, \mathcal{X})$ is a soft algebraic extension of $(\tilde{f}^t_\varepsilon, \mathcal{X})$, then there exists $\delta \subseteq \Sigma_{\tilde{f}}$ such that $\varepsilon \subseteq \delta$ and $\tilde{f}^t_\delta(x) \subseteq \tilde{g}(x)$ for all $x \in X$. Also the ε_1-soft transfer and the ε_2-soft transfer of $(\tilde{f}, \mathcal{X})$ are int-soft subalgebras of $\mathcal{X}$ over U for any subsets ε_1 and ε_2 of $\Sigma_{\tilde{f}}$ by Theorem 3.5. If $\varepsilon_1 \subseteq \varepsilon_2$, then $\tilde{f}^t_{\varepsilon_1}(x) = \tilde{f}(x) \cup \varepsilon_1 \subseteq \tilde{f}(x) \cup \varepsilon_2 = \tilde{f}^t_{\varepsilon_2}(x)$ for all $x \in X$. Hence we have the following theorem.

Theorem 3.17. *Let $(\tilde{f}, \mathcal{X})$ be an int-soft subalgebra of $\mathcal{X}$ over U and $\varepsilon \subseteq \Sigma_{\tilde{f}}$. For every soft algebraic extension $(\tilde{g}, \mathcal{X})$ of the ε-soft transfer of $(\tilde{f}, \mathcal{X})$, there exists $\delta \subseteq \Sigma_{\tilde{f}}$ such that $\varepsilon \subseteq \delta$ and $(\tilde{g}, \mathcal{X})$ is a soft algebraic extension of the δ-soft transfer of $(\tilde{f}, \mathcal{X})$. Also for any subsets ε_1 and ε_2 of $\Sigma_{\tilde{f}}$ with $\varepsilon_1 \subseteq \varepsilon_2$, the ε_2-soft transfer of $(\tilde{f}, \mathcal{X})$ is a soft algebraic extension of the ε_1-soft transfer of $(\tilde{f}, \mathcal{X})$.*

The following example illustrates the first part of Theorem 3.17.

Example 3.18. Let $X = \{0, 1, 2, 3, 4\}$ be a set with the following Cayley table:

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	0	1	1
2	2	2	0	2	2
3	3	3	3	0	3
4	4	4	4	4	0

Then $\mathcal{X} := (X, *, 0)$ is a BCK-algebra (see [12]). Let $(\tilde{f}, \mathcal{X})$ be a soft set over $U = \{1, 2, 3, 4, 5, 6, 7\}$ in which $\tilde{f}$ is defined by

$$\tilde{f} : X \to \mathscr{P}(U),\ x \mapsto \begin{cases} \{1,3,5,7\} & \text{if } x = 0, \\ \{1,5,7\} & \text{if } x = 1, \\ \{1,3,5\} & \text{if } x = 2, \\ \{1,3\} & \text{if } x = 3, \\ \{1\} & \text{if } x = 4. \end{cases}$$

Then $(\tilde{f}, \mathcal{X})$ is an int-soft subalgebra of $\mathcal{X}$ over U and $\Sigma_{\tilde{f}} = \{2, 4, 6\}$. If we take $\varepsilon = \{6\}$, then the ε-soft transfer $(\tilde{f}^t_\varepsilon, \mathcal{X})$ of $(\tilde{f}, \mathcal{X})$ is given by

$$\tilde{f}^t_\varepsilon : X \to \mathscr{P}(U),\ x \mapsto \begin{cases} \{1,3,5,6,7\} & \text{if } x = 0, \\ \{1,5,6,7\} & \text{if } x = 1, \\ \{1,3,5,6\} & \text{if } x = 2, \\ \{1,3,6\} & \text{if } x = 3, \\ \{1,6\} & \text{if } x = 4, \end{cases}$$

and it is an int-soft subalgebra of $\mathcal{X}$ over U by Theorem 3.5. Let $(\tilde{g}, \mathcal{X})$ be a soft set over $U = \{1, 2, 3, 4, 5, 6, 7\}$ in which $\tilde{g}$ is defined by

$$\tilde{g} : X \to \mathscr{P}(U),\ x \mapsto \begin{cases} \{1,2,3,4,5,6,7\} & \text{if } x = 0, \\ \{1,2,4,5,6,7\} & \text{if } x = 1, \\ \{1,2,3,5,6\} & \text{if } x = 2, \\ \{1,2,3,6\} & \text{if } x = 3, \\ \{1,2,6\} & \text{if } x = 4. \end{cases}$$

Then $(\tilde{g}, \mathcal{X})$ is an int-soft subalgebra of $\mathcal{X}$ over U and $\tilde{f}^t_\varepsilon(x) \subseteq \tilde{g}(x)$ for all $x \in X$. Thus $(\tilde{g}, \mathcal{X})$ is a soft algebraic extension of the ε-soft transfer $(\tilde{f}^t_\varepsilon, \mathcal{X})$ of $(\tilde{f}, \mathcal{X})$. But $(\tilde{g}, \mathcal{X})$ is not the ε-soft transfer of $(\tilde{f}, \mathcal{X})$ for all $\varepsilon \subseteq \Sigma_{\tilde{f}}$. If we take $\delta = \{2, 6\}$, then $\varepsilon \subseteq \delta$ and the δ-soft transfer $(\tilde{f}^t_\delta, \mathcal{X})$ of $(\tilde{f}, \mathcal{X})$ is given as follows:

$$\tilde{f}^t_\delta : X \to \mathscr{P}(U),\ x \mapsto \begin{cases} \{1,2,3,5,6,7\} & \text{if } x = 0, \\ \{1,2,5,6,7\} & \text{if } x = 1, \\ \{1,2,3,5,6\} & \text{if } x = 2, \\ \{1,2,3,6\} & \text{if } x = 3, \\ \{1,2,6\} & \text{if } x = 4. \end{cases}$$

Then $\tilde{g}(x) \supseteq \tilde{f}^t_\delta(x)$ for all $x \in X$ and $(\tilde{f}^t_\delta, \mathcal{X})$ is an int-soft subalgebra of $\mathcal{X}$ over U. Therefore $(\tilde{g}, \mathcal{X})$ is a soft algebraic extension of the δ-soft transfer of $(\tilde{f}, \mathcal{X})$.

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