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NOTES ON A QUESTION RAISED BY E. CALABI

Yunhee Euh and Kouei Sekigawa

Abstract. We show that any orthogonal almost complex structure on a warped product Riemannian manifold of an oriented closed surface with nonnegative Gaussian curvature and a round 4-sphere is never integrable. This provides a partial answer to a question raised by E. Calabi.

1. Introduction

In our previous paper [3], we discussed the integrability of orthogonal almost complex structures on the Riemannian products of even-dimensional round spheres based on the result by Sutherland [7] and the curvature identity for Hermitian manifolds by Gray [4] and showed that such an almost complex structure is integrable if and only if it is a product of the canonical complex structures on round 2-spheres. Concomitantly, we obtained the following result ([3], Corollary 3.3).

Theorem 1.1. *Any orthogonal almost complex structure on a Riemannian product of a round 2-sphere and a round 4-sphere is never integrable.*

Theorem 1.1 gives a partial answer to the following question raised by Calabi [2].

Question 1. Does the product manifold $V^2 \times S^4$ (V^2 is any oriented closed surface) admit an integrable almost complex structure or not?

In connection with Question 1, we may note that there exists a 2-sphere bundle over a 4-sphere which admits an integrable almost complex structure. In fact, a metric twistor bundle $\mathcal{J}(S^4)$ over an oriented 4-sphere S^4 is a non-trivial 2-sphere bundle over a 4-sphere and further $\mathcal{J}(S^4)$ admits a Kähler structure $(J, \langle\,,\,\rangle)$ such that $(\mathcal{J}(S^4), J, \langle\,,\,\rangle)$ is holomorphically isometric to a 3-dimensional complex projective space $\mathbb{CP}^3$ with the Fubini-Study metric [5, 6]. In the present paper, we shall prove the following theorem.

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Theorem 1.2. *Let $V^2 \times_f S^4$ be a warped product Riemannian manifold of an oriented closed surface V^2 with nonnegative Gaussian curvature and a round 4-sphere S^4, where f is a positive-valued smooth function on V^2. Then, any orthogonal almost complex structure on $V^2 \times_f S^4$ is never integrable.*

Theorem 1.2 is a generalization of Theorem 1.1 and also gives a partial answer to Calabi's query.

2. Preliminaries

In this section, we prepare for several terminologies and basic formulas on a warped product Riemannian manifold.

Let $(B, \langle\, ,\,\rangle_B)$ and $(F, \langle\, ,\,\rangle_F)$ be Riemannian manifolds and f be a positive-valued smooth function on B. By definition, a warped product Riemannian manifold $(M, \langle\, ,\,\rangle) = (B, \langle\, ,\,\rangle_B) \times_f (F, \langle\, ,\,\rangle_F)$ (briefly, $B \times_f F$) is the product manifold $M = B \times F$ equipped with the Riemannian metric $\langle\, ,\,\rangle$ given by $\langle\, ,\,\rangle = \langle\, ,\,\rangle_B + f^2 \langle\, ,\,\rangle_F$. We denote by ∇, ∇^B and ∇^F the Levi-Civita connections of $\langle\, ,\,\rangle$, $\langle\, ,\,\rangle_B$ and $\langle\, ,\,\rangle_F$, respectively. Then, we see that the following relations hold ([1], Lemma 7.3):

$$\nabla_X Y = \nabla^B_X Y, \tag{2.1}$$

$$\nabla_U X = \frac{1}{f} X f U = \frac{1}{f} \left\langle grad^B f, X \right\rangle_B U, \tag{2.2}$$

$$\nabla_X U = \frac{1}{f} X f U = \frac{1}{f} \left\langle grad^B f, X \right\rangle_B U, \tag{2.3}$$

$$\nabla_U V = \nabla^F_U V - f \langle U, V \rangle_F \, grad^B f \tag{2.4}$$

for X, $Y \in \mathfrak{X}(B)$ and U, $V \in \mathfrak{X}(F)$, where $\mathfrak{X}(B)$ and $\mathfrak{X}(F)$ denote the Lie algebras of all smooth vector fields on B and F, respectively. We denote the curvature tensors of $(M, \langle\, ,\,\rangle)$, $(B, \langle\, ,\,\rangle_B)$ and $(F, \langle\, ,\,\rangle_F)$ by R, R^B and R^F defined by

$$R(\bar{X}, \bar{Y})\bar{Z} = [\nabla_{\bar{X}}, \nabla_{\bar{Y}}]\bar{Z} - \nabla_{[\bar{X}, \bar{Y}]}\bar{Z}, \tag{2.5}$$

$$R^B(X, Y)Z = [\nabla^B_X, \nabla^B_Y]Z - \nabla^B_{[X,Y]}Z, \tag{2.6}$$

$$R^F(U, V)W = [\nabla^F_U, \nabla^F_V]W - \nabla^F_{[U,V]}W \tag{2.7}$$

for X, Y, $Z \in \mathfrak{X}(B)$, U, V, $W \in \mathfrak{X}(F)$ and $\bar{X}$, $\bar{Y}$, $\bar{Z} \in \mathfrak{X}(M)$. Then, from (2.1)$\sim$(2.7), we have

$$R(X, Y)Z = R^B(X, Y)Z, \tag{2.8}$$

$$R(X, Y)U = 0, \tag{2.9}$$

$$R(X, U)Y = \frac{1}{f} Hess^B f(X, Y)U, \tag{2.10}$$

$$R(U,V)X = 0, \tag{2.11}$$

$$\begin{aligned} &R(U,V)W \\ &= R^F(U,V)W - |\,grad^B f|_B^2 \big(\langle V,W\rangle_F U - \langle U,W\rangle_F V \big), \\ &= R^F(U,V)W - \frac{1}{f^2}|\,grad^B f|_B^2 \big(\langle V,W\rangle U - \langle U,W\rangle V \big) \end{aligned} \tag{2.12}$$

for X, Y, $Z \in \mathfrak{X}(B)$ and U, V, $W \in \mathfrak{X}(F)$ ([1], Lemma 7.4). From (2.8)$\sim$(2.12), we have further

$$R(X,Y,Z,Z') = R^B(X,Y,Z,Z'), \tag{2.13}$$

$$R(X,Y,Z,U) = 0, \tag{2.14}$$

$$R(X,Y,U,V) = 0, \tag{2.15}$$

$$R(X,U,Y,V) = \frac{1}{f}(Hess^B f)(X,Y)\langle U,V\rangle\,, \tag{2.16}$$

$$R(U,V,W,X) = 0, \tag{2.17}$$

$$\begin{aligned} &R(U,V,W,W') \\ &= \big\langle R^F(U,V)W,W' \big\rangle \\ &\quad - \frac{1}{f^2}|grad^B f|_B^2 \left(\langle V,W\rangle \langle U,W'\rangle - \langle U,W\rangle \langle V,W'\rangle \right) \end{aligned} \tag{2.18}$$

for X, Y, Z, $Z' \in \mathfrak{X}(B)$ and U, V, W, $W' \in \mathfrak{X}(F)$.

3. Proof of Theorem 1.2

In this section, we shall show Theorem 1.2 by making use of the fundamental formulas prepared in § 2. In the sequel, we assume that $(B, \langle\,,\,\rangle_B) = (V^2, \langle\,,\,\rangle_{V^2})$ and $(F, \langle\,,\,\rangle_F) = (S^4(\beta), \langle\,,\,\rangle_{S^4(\beta)})$, where $(V^2, \langle\,,\,\rangle)$ is an oriented closed surface with nonnegative Gaussian curvature α and $(S^4(\beta), \langle\,,\,\rangle_{S^4(\beta)})$ is a round 4-sphere of constant sectional curvature β and further $(M, \langle\,,\,\rangle) = (V^2, \langle\,,\,\rangle_{V^2}) \times_f (S^4(\beta), \langle\,,\,\rangle_{S^4(\beta)})$, where f is a positive-valued smooth function on V^2. First, we recall the result due to Gray [4] which plays an essential role in the proof of Theorem 1.2.

Lemma 3.1. *Let $M = (M, J, \langle\,,\,\rangle)$ be a Hermitian manifold. Then, we have*

$$\begin{aligned} &R(\bar X,\bar Y,\bar Z,\bar W) + R(J\bar X,J\bar Y,J\bar Z,J\bar W) - R(J\bar X,J\bar Y,\bar Z,\bar W) \\ &- R(J\bar X,\bar Y,J\bar Z,\bar W) - R(J\bar X,\bar Y,\bar Z,J\bar W) - R(\bar X,J\bar Y,J\bar Z,\bar W) \\ &- R(\bar X,J\bar Y,\bar Z,J\bar W) - R(\bar X,\bar Y,J\bar Z,J\bar W) = 0 \end{aligned}$$

for $\bar X$, $\bar Y$, $\bar Z$, $\bar W \in \mathfrak{X}(M)$.

Now, it is known that $M = V^2 \times S^4(\beta)$ admits an almost complex structure [2, 7]. Let J be an orthogonal almost complex structure on $(M, \langle\, ,\, \rangle)$. We may identify $T_{(p_1,p_2)}(V^2 \times S^4(\beta))$ with $T_{p_1}V^2 \oplus T_{p_2}S^4(\beta)$ for each point $p = (p_1, p_2) \in V^2 \times S^4(\beta)$ in the natural way. Let $\{e_i\}_{1\leq i\leq 6}$ be a local orthonormal frame field on $(M, \langle\, ,\, \rangle)$ such that $\{e_1, e_2\}$ and $\{e_3, e_4, e_5, e_6\}$ are tangent to V^2 and $S^4(\beta)$, respectively. We here set

$$Je_a = \sum_b J_{ab}e_b + \sum_v J_{av}e_v, \quad Je_u = \sum_b J_{ub}e_b + \sum_v J_{uv}e_v \tag{3.1}$$

for $1 \leq a, b, \ldots \leq 2$ and $3 \leq u, v, \ldots \leq 6$. Then, we may easily check that the following equalities hold:

$$J_{ij} = -J_{ji}, \quad \sum_{k=1}^{6} J_{ik}J_{jk} = \delta_{ij} \tag{3.2}$$

for $1 \leq i, j \leq 6$. Then, from (2.8)~(2.12), taking account of (3.1) and (3.2), we have

$$R(e_1, e_2, e_1, e_2) = -\alpha, \tag{3.3}$$

(3.4)

$$\begin{aligned}
&R(Je_1, Je_2, Je_1, Je_2)\\
&= R\Big(\sum_a J_{1a}e_a + \sum_u J_{1u}e_u, \sum_b J_{2b}e_b + \sum_v J_{2v}e_v,\\
&\quad \sum_c J_{1c}e_c + \sum_w J_{1w}e_w, \sum_d J_{2d}e_d + \sum_z J_{2z}e_z\Big)\\
&= \sum_{a,b,c,d} J_{1a}J_{2b}J_{1c}J_{2d}R(e_a, e_b, e_c, e_d) + \sum_{a,b,c,z} J_{1a}J_{2b}J_{1c}J_{2z}R(e_a, e_b, e_c, e_z)\\
&\quad + \sum_{a,b,w,d} J_{1a}J_{2b}J_{1w}J_{2d}R(e_a, e_b, e_w, e_d) + \sum_{a,b,w,z} J_{1a}J_{2b}J_{1w}J_{2z}R(e_a, e_b, e_w, e_z)\\
&\quad + \sum_{a,v,c,d} J_{1a}J_{2v}J_{1c}J_{2d}R(e_a, e_v, e_c, e_d) + \sum_{a,v,c,z} J_{1a}J_{2v}J_{1c}J_{2z}R(e_a, e_v, e_c, e_z)\\
&\quad + \sum_{a,v,w,d} J_{1a}J_{2v}J_{1w}J_{2d}R(e_a, e_v, e_w, e_d) + \sum_{a,v,w,z} J_{1a}J_{2v}J_{1w}J_{2z}R(e_a, e_v, e_w, e_z)\\
&\quad + \sum_{u,b,c,d} J_{1u}J_{2b}J_{1c}J_{2d}R(e_u, e_b, e_c, e_d) + \sum_{u,b,c,z} J_{1u}J_{2b}J_{1c}J_{2z}R(e_u, e_b, e_c, e_z)\\
&\quad + \sum_{u,b,w,d} J_{1u}J_{2b}J_{1w}J_{2d}R(e_u, e_b, e_w, e_d) + \sum_{u,b,w,z} J_{1u}J_{2b}J_{1w}J_{2z}R(e_u, e_b, e_w, e_z)\\
&\quad + \sum_{u,v,c,d} J_{1u}J_{2v}J_{1c}J_{2d}R(e_u, e_v, e_c, e_d) + \sum_{u,v,c,z} J_{1u}J_{2v}J_{1c}J_{2z}R(e_u, e_v, e_c, e_z)
\end{aligned}$$

$$
\begin{aligned}
&+ \sum_{u,v,w,d} J_{1u}J_{2v}J_{1w}J_{2d}R(e_u,e_v,e_w,e_d)\\
&+ \sum_{u,v,w,z} J_{1u}J_{2v}J_{1w}J_{2z}R(e_u,e_v,e_w,e_z)\\
= &\sum_{a,b,c,d} J_{1a}J_{2b}J_{1c}J_{2d}R(e_a,e_b,e_c,e_d) + \sum_{a,v,c,z} J_{1a}J_{2v}J_{1c}J_{2z}R(e_a,e_v,e_c,e_z)\\
&+ \sum_{a,v,w,d} J_{1a}J_{2v}J_{1w}J_{2d}R(e_a,e_v,e_w,e_d) + \sum_{u,b,c,z} J_{1u}J_{2b}J_{1c}J_{2z}R(e_u,e_b,e_c,e_z)\\
&+ \sum_{u,b,w,d} J_{1u}J_{2b}J_{1w}J_{2d}R(e_u,e_b,e_w,e_d) + \sum_{u,v,w,z} J_{1u}J_{2v}J_{1w}J_{2z}R(e_u,e_v,e_w,e_z)\\
= &-\alpha J_{12}^4 + \frac{1}{f}\sum_{a,v,c,z} J_{1a}J_{2v}J_{1c}J_{2z}Hess^{V^2}f(e_a,e_c)\delta_{vz}\\
&- \frac{1}{f}\sum_{a,v,w,d} J_{1a}J_{2v}J_{1w}J_{2d}Hess^{V^2}f(e_a,e_d)\delta_{vw}\\
&- \frac{1}{f}\sum_{u,b,c,z} J_{1u}J_{2b}J_{1c}J_{2z}Hess^{V^2}f(e_b,e_c)\delta_{uz}\\
&+ \frac{1}{f}\sum_{u,b,w,d} J_{1u}J_{2b}J_{1w}J_{2d}Hess^{V^2}f(e_d,e_b)\delta_{uw}\\
&+ \sum_{u,v,w,z} J_{1u}J_{2v}J_{1w}J_{2z}\big(\big\langle R^F(e_u,e_v)e_w,e_z\big\rangle\\
&- \frac{1}{f^2}|\operatorname{grad}^B f|_B^2\,\langle\delta_{vw}e_u-\delta_{uw}e_v,e_z\rangle\big)\\
= &-\alpha J_{12}^4 + \frac{1}{f}J_{12}^2(1-J_{12}^2)Hess^{V^2}f(e_2,e_2) + \frac{1}{f}J_{12}^2(1-J_{12}^2)Hess^{V^2}f(e_1,e_1)\\
&+ \frac{1}{f^2}\big(\beta-|\operatorname{grad}^B f|_B^2\big)\sum_{u,v,w,z} J_{1u}J_{2v}J_{1w}J_{2z}(\delta_{vw}\delta_{uz}-\delta_{uw}\delta_{vz})\\
= &-\alpha J_{12}^4 + \frac{1}{f}J_{12}^2(1-J_{12}^2)\Delta^{V^2}f - \frac{1}{f^2}\big(\beta-|grad^{V^2}f|_B^2\big)(1-J_{12}^2)^2,
\end{aligned}
$$

(3.5)
$$
\begin{aligned}
&R(Je_1,Je_2,e_1,e_2)\\
&= R\Big(\sum_a J_{1a}e_a+\sum_u J_{1u}e_u,\sum_b J_{2b}e_b+\sum_v J_{2v}e_v,e_1,e_2\Big)\\
&= \sum_{a,b} J_{1a}J_{2b}R(e_a,e_b,e_1,e_2) + \sum_{a,v} J_{1a}J_{2v}R(e_a,e_v,e_1,e_2)\\
&\quad + \sum_{b,u} J_{1u}J_{2b}R(e_u,e_b,e_1,e_2) + \sum_{u,v} J_{1u}J_{2v}R(e_u,e_v,e_1,e_2)
\end{aligned}
$$

$$= \sum_{a,b} J_{1a}J_{2b}(\delta_{b1}\delta_{a2} - \delta_{a1}\delta_{b2})$$
$$= -\alpha J_{12}^2,$$

(3.6)
$$\begin{aligned}
&R(Je_1, e_2, Je_1, e_2)\\
&= R\big(\sum_a J_{1a}e_a + \sum_u J_{1u}e_u, e_2, \sum_b J_{1b}e_b + \sum_v J_{1v}e_v, e_2\big)\\
&= \sum_{a,b} J_{1a}J_{1b}R(e_a, e_2, e_b, e_2) + \sum_{a,v} J_{1a}J_{1v}R(e_a, e_2, e_v, e_2)\\
&\quad + \sum_{b,u} J_{1u}J_{1b}R(e_u, e_2, e_b, e_2) + \sum_{u,v} J_{1u}J_{1v}R(e_u, e_2, e_v, e_2)\\
&= \sum_{a,b} J_{1a}J_{1b}(\delta_{b1}\delta_{a1} - \delta_{a1}\delta_{b1}) + \sum_{u,v} J_{1u}J_{1v}R(e_u, e_2, e_v, e_2)\\
&= \sum_{u,v} J_{1u}J_{1v}R(e_2, e_u, e_2, e_v)\\
&= \frac{1}{f}\sum_{u,v} J_{1u}J_{1v}Hess^{V^2}f(e_2, e_2)\,\langle e_u, e_v\rangle\\
&= \frac{1}{f}(1 - J_{12}^2)Hess^{V^2}f(e_2, e_2),
\end{aligned}$$

(3.7)
$$\begin{aligned}
&R(Je_1, e_2, e_1, Je_2)\\
&= R\big(\sum_a J_{1a}e_a + \sum_u J_{1u}e_u, e_2, e_1, \sum_b J_{2b}e_b + \sum_v J_{2v}e_v\big)\\
&= \sum_{a,b} J_{1a}J_{2b}R(e_a, e_2, e_1, e_b) + \sum_{a,v} J_{1a}J_{2v}R(e_a, e_2, e_1, e_v)\\
&\quad + \sum_{b,u} J_{1u}J_{2b}R(e_u, e_2, e_1, e_b) + \sum_{u,v} J_{1u}J_{2v}R(e_u, e_2, e_1, e_v)\\
&= \sum_{u,v} J_{1u}J_{2v}R(e_u, e_2, e_1, e_v)\\
&= -\frac{1}{f}\sum_{u,v} J_{1u}J_{2v}Hess^{V^2}f(e_2, e_1)\delta_{uv}\\
&= 0,
\end{aligned}$$

(3.8)
$$\begin{aligned}
&R(e_1, Je_2, e_1, Je_2)\\
&= R\big(e_1, \sum_a J_{2a}e_a + \sum_u J_{2u}e_u, e_1, \sum_b J_{2b}e_b + \sum_v J_{2v}e_v\big)\\
&= \sum_{a,b} J_{2a}J_{2b}R(e_1, e_a, e_1, e_b) + \sum_{a,v} J_{2a}J_{2v}R(e_1, e_a, e_1, e_v)
\end{aligned}$$

$$
\begin{aligned}
&+\sum_{b,u} J_{2b}J_{2u}R(e_1,e_u,e_1,e_b)+\sum_{u,v} J_{2u}J_{2v}R(e_1,e_u,e_1,e_v)\\
&=\frac{1}{f}\sum_{u,v} J_{2u}J_{2v}Hess^{V^2}f(e_1,e_1)\delta_{uv}\\
&=\frac{1}{f}(1-J_{12}^2)Hess^{V^2}f(e_1,e_1).
\end{aligned}
$$

Thus, from (3.3)∼(3.8) and Lemma 3.1, we have

$$
\begin{aligned}
0 &= R(e_1,e_2,e_1,e_2)+R(Je_1,Je_2,Je_1,Je_2)-2R(Je_1,Je_2,e_1,e_2)\\
&\quad -R(Je_1,e_2,Je_1,e_2)-2R(Je_1,e_2,e_1,Je_2)-R(e_1,Je_2,e_1,Je_2)\\
&= -\alpha-\alpha J_{12}^4+\frac{1}{f}J_{12}^2(1-J_{12}^2)\Delta^{V^2}f-\frac{1}{f^2}\big(\beta-|grad^{V^2}f|^2_{V^2}\big)(1-J_{12}^2)^2\\
&\quad +2\alpha J_{12}^2-\frac{1}{f}(1-J_{12}^2)\Delta^{V^2}f\\
&= -\alpha(1-2J_{12}^2+J_{12}^4)-\frac{1}{f}(1-J_{12}^2)^2\Delta^{V^2}f\\
&\quad -\frac{1}{f^2}\big(\beta-|grad^{V^2}f|^2_{V^2}\big)(1-J_{12}^2)^2\\
&= -(1-J_{12}^2)^2\Big(\alpha+\frac{1}{f}\Delta^{V^2}f+\frac{1}{f^2}\big(\beta-|grad^{V^2}f|^2_{V^2}\big)\Big)
\end{aligned}
$$

and hence,

$$
(1-J_{12}^2)^2\Big(\alpha+\frac{1}{f}\Delta^{V^2}f+\frac{1}{f^2}\big(\beta-|grad^{V^2}f|^2_{V^2}\big)\Big)=0. \tag{3.9}
$$

Here, since V^2 is compact, for any point $p_2\in S^4(\beta)$, there exists a point $p_1\in V^2$ such that the function f takes its minimum at p_1 and hence, $grad^{V^2}f=0$ and $\Delta^{V^2}f\ge 0$ at the point p_1. Thus, from (3.9), we have

$$
\alpha+\frac{1}{f}\Delta^{V^2}f+\frac{1}{f^2}\big(\beta-|grad^{V^2}f|^2_{V^2}\big)=\alpha+\frac{1}{f}\Delta^{V^2}f+\frac{\beta}{f^2}>0 \tag{3.10}
$$

along $\{p_1\}\times S^4(\beta)$. Thus, from (3.9) and (3.10), we see that $J_{12}^2=1$ holds along $\{p_1\}\times S^4(\beta)$ with respect to any local orthonormal frame field $\{e_i\}$ such that $\{e_1,e_2\}$ and $\{e_3,e_4,e_5,e_6\}$ are tangent to V^2 and $S^4(\beta)$, respectively. This means that the subspace $T_{p_1}V^2$ of $T_{(p_1,p_2)}M$ for any $p_2\in S^4(\beta)$ is J-invariant, and hence the subspace $T_{p_2}S^4(\beta)$ of $T_{(p_1,p_2)}M$ is also J-invariant for any $p_2\in S^4(\beta)$. But this is impassible. This completes the proof of Theorem 1.2.

From the discussion in the present paper, the following question will also naturally arise.

Question 2. Does there exist a warped product Riemannian manifold $S^4\times_f V^2$ of a round 4-sphere S^4 and an oriented closed surface V^2 admitting a complex structure?

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Yunhee Euh
Division of Mathematical Models
National Institute for Mathematical Sciences
Daejeon 34047, Korea
E-mail address: yheuh@nims.re.kr

Kouei Sekigawa
Department of Mathematics
Faculty of Science
Niigata University
Niigata 950-2181, JAPAN
E-mail address: sekigawa@math.sc.niigata-u.ac.jp