Analysis of Neural Network Approaches for Nonlinear Modeling of Switched Reluctance Motor Drive

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Abstract – This paper attempts to employ and investigate neural based approaches as interpolation tools for modeling of Switched Reluctance Motor (SRM) drive. Precise modeling of SRM is essential to analyse the performance of control strategies for variable speed drive application. In this work the suitability of Generalized Regression Neural Network (GRNN) and Extreme Learning Machine (ELM) in addition to conventional neural network are explored for improving the modeling accuracy of SRM. The neural structures are trained with the data obtained by modeling of SRM using Finite Element Analysis (FEA) and the trained neural network is incorporated in the model of SRM drive. The results signify the modeling accuracy with GRNN model. The closed loop drive simulation is performed in MATLAB/Simulink environment and the closeness of the results in comparison with the experimental prototype validates the modeling approach.

Keywords: Switched reluctance motor, Finite element analysis, ANN, GRNN, ELM

1. Introduction

Switched reluctance motors (SRM) have been considered for variable speed applications because of its simple and rugged construction, low cost, absence of windings in the rotor, high speed, high power density and torque density [1-3]. The major drawback in SRM is torque ripple [4], because of its doubly salient structure. Various design and control approaches have been proposed to minimize torque ripple [5-8]. For analyzing the effectiveness of the control approaches on torque ripple and performance of SRM, effective modeling of the SRM drive is required. Researchers have proposed several mathematical models [9-13] and interpolation techniques to satisfactorily model SRM. The data required for modeling has been obtained by using analytical techniques [11-14], finite element analysis based techniques [15, 16] and methods based on experiments. FEA has been widely used to calculate the flux linkage characteristics of SRM from which the model is derived. Ahmed Khalil proposed [13] a model of SRM in which the non-linear characteristics are approximated by Fourier series representation. The model to determine rotor position is described in [17]. However this method is time consuming as the modeling involves more mathematical computations. Radial Basis Function Network-Based Adaptive Fuzzy System (RBFN-AFS) has been applied for modeling of SRM. The main drawback of this method is that it is computationally complex [18]. B-Spline Neural Network (BSNN) based online modeling of SRM is proposed in [19]. A nonlinear model of SRM based on an improved Least Square Support Vector Regression (LSSVR) algorithm optimized by Grid-Diamond Searching (GDS) method has been introduced by [20]. Least Square Support Vector Machine (LSSVM) based function estimation to model SRM has been proposed and the accuracy of the model depends on proper selection of the hyper parameters of LSSVM [21]. The main drawback of mathematical models is that they are less accurate and complex. On the other hand intelligent approximation algorithms are model free and are appropriate for modeling nonlinear systems [22]. Amidst validation of different algorithms in [23] the popularity of neural approaches in data interpolation and optimisation has made the authors explore the application of variants of neural network for SRM. The authors have presented neural based approaches in the perception for modeling of SRM and its experimental validation.

This paper is organized as follows, section 2 explains nonlinear modeling of SRM and section 3 presents the overview of Artificial Neural Network (ANN), Generalised Regression Neural Network (GRNN), Extreme Learning Method (ELM) approaches for modeling SRM. Finally the results are presented and discussed in the section 4.

2. Non-linear Modeling of SRM

The structure of SRM is shown in Fig. 1.

The following equations are used to model SRM in simulation environment for performance prediction. The instantaneous voltage applied across the terminal is given by

\[ V_j = R_j I_j + \frac{d\Psi_j}{dt} \]  

(1)
Where $V_j, I_j, R_j$ are the applied terminal voltage, phase current, resistance of $j$th phase respectively and $\Psi$ is the flux linkage between $j$th phase stator and rotor poles. From equation (1) flux can be calculated as

$$\Psi_j(i, \theta) = \int (V_j - I_j R_j) dt$$

(2)

The phase torque is given by the equation

$$T_{ph} = \frac{i^2}{2} \frac{dL_j}{d\theta} (i=\text{const})$$

(3)

The mechanical equation of the motor is represented by

$$T_E = J \frac{d\omega}{dt} + B\omega + T_L$$

(4)

Where $J$-Moment of inertia, $B$-Friction coefficient, $T_E$-Electrical torque developed, $\omega$ -Speed and $T_L$ -Load Torque.

From the above equation

$$\omega = \frac{1}{J} \left( T_E - T_L \right)$$

(5)

The total electrical torque is obtained from summation of individual phase torque and is given as

$$T_E(\theta, i) = \sum_{j=1}^{N_p} T_{ph}(\theta, i)$$

(6)

From the above equations, it is evident that the simulation accuracy of SRM drive depends on precise modeling and interpolation of non-linear flux linkage current characteristics. The flux linkage current characteristics of SRM are shown in Fig. 2, with dimensions as given in Table A1. From the graph it is evident that the flux linkage current characteristics is non-linear and the application of interpolation technique is inevitable to accurately model the SRM drive to analyse and evaluate the performance of the drive with respect to various control algorithms.

It is in this prelude this work attempts to highlight the advantages of neural approaches as interpolation tool for modeling and simulation of SRM drive.

3. Neural Approach for Non-linear Modeling of SRM

3.1 Artificial Neural Network (ANN)

In the early of 1940s McCulloch-Pitts built a simple neural network model for simple logic functions [24]. Artificial neural network is an electronic model of the neural structure of the human brain. It is a simulation of human brain system with artificial neurons and interconnections. Bio-logical neural networks are more complicated compared to the mathematical models used for ANNs. Neural networks can be applied to problems for which algorithmic solutions are too complex to be found or that do not have algorithmic solutions. In other words, the inputs and outputs are having the non-linear relationship for some systems. ANN uses the samples to generate the models for such system. The learning ability of the ANN from the samples makes it very flexible and powerful. Therefore ANN plays an intensive roll in solving regression and classification problems in many fields. ANN is nonlinear processes that perform learning and classification.

Neuron is the fundamental processing unit of ANN. A large amount of interconnected neurons accepts weighted set of inputs and responds with an output. The output of the neuron is the sum of the inputs as given,

$$n = \sum_{i=1}^{p} w_i x_i + b$$

(7)

Where, $P$ - number of elements, $x_i$ - input vector, $w_i$ - interconnection weight of $x_i$ and $b$ - bias for the neuron. Then $n$ is processed using an activation function $f$ [25].

$$f(n) = f\left( \sum_{i=1}^{p} w_i x_i + b \right)$$

(8)
Learning ability of neural network required to learn the system. This kind of training of the famous algorithms used for the correction of weights is the Back Propagation (BP) algorithm [27]. This training of the output until the network output matches the target. One of the powerful multilayer network is the perceptron. Increasing the number of hidden layers yields a more powerful multilayer network.

Neural networks are trained or the selected weights are adjusted to reach from a particular input to a specific target output until the network output matches the target. One of the famous algorithms used for the correction of weights is Back Propagation (BP) algorithm [27]. This training of neural network required to learn the system. This kind of learning is supervised learning. This learning ability depends on its applied algorithmic method and architecture during the training.

3.2 Generalized Regression Neural Network (GRNN)

One of the different forms of the radial basis network is GRNN which is based on kernel regression networks [28-30]. As back propagation algorithm, the GRNN does not require any iterative training procedure. From the sufficient number of distinct training data samples GRNN can approximate a continuous function to an arbitrary accuracy (universal approximation) between input and output vectors [31]. The error estimation will approach zero, when the training data set size is large with only mild restrictions on the function [30].

GRNN typically consists of three layers artificial neurons as input, hidden and output layers as shown in Fig. 4. The neurons in the output layer has a linear transfer function and the hidden layer has radial basis neurons and it is set to equal the number of training data samples.

If the training data samples are large, it results in a complex network with too many hidden neurons. To reduce the number of hidden neurons, first the training data samples are clustered to set the number of hidden neurons equal to the number of clusters [32]. The distance between the input vector and the training data sample is the input to each of the radial basis neurons. The output of the hidden layer is the RBF of the input scaled by the spread factor and it characterizes the closeness between inputs. In general the multivariate Gaussian function is used as the RBF and is given as

$$G(x,x_i) = \exp\left(-\frac{1}{2\sigma^2}||x-x_i||^2\right)$$

where $x_i$ center, $\sigma$, width.

With N - distinct input output pairs $(x_i,t_i) \in \mathbb{R}^n \times \mathbb{R}^m$ $(i=1,2,3,\ldots,N)$ as without the clustering of data samples, the number of neurons in the hidden layer is equal ‘N’. For the test point $x \in \mathbb{R}^n$ the output is given as

$$\hat{y}(x) = \sum_{i=1}^{m} W_i y_i$$

$$w_i = \frac{\exp\left(-\frac{1}{2\sigma^2}||x-x_i||^2\right)}{\sum_{k=1}^{m} \exp\left(-\frac{1}{2\sigma^2}||x-x_k||^2\right)}$$

The validation of the GRNN depends on, the number of hidden units or radial basis function, Centroid of the hidden units and the tunable parameter ‘spread factor’. If the spread factor is small RBF is steep and minimum test samples closer to the input contribute the output and the response of the network will be less accurate. Increasing the spread factor [33] will increase the area around the input vector and first layer of neurons responds with significant outputs. Function of the neuron is depending on the slope of RBF. Increasing the spread factor smoothen the slope of the RBF which makes most of the neurons to respond for an input vector. At initial learning stage the Centroid of the RBF is randomly selected based on some input data sample. Then the weights between hidden and output layer is estimated based on the stochastic gradient.

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Image 1: Single neuron model

Image 2: GRNN model
approach. N. Umadevi et al addressed the application of GRNN used for data interpolation and design optimization for non-conventional motor applications in [34].

### 3.3 Extreme Learning Machine (ELM)

It is a novel learning algorithm proposed by Huang et al for Single-Hidden-Layer Feed Forward Neural Networks (SLFNs). A machine oriented approach for switched reluctance generator using ELM was proposed by Chao Wang et al in [35].

In this algorithm the randomly assumed learning parameters of hidden nodes, including input weights and biases need not be tuned. Using the simple generalized inverse operation the output nodes are analytically determined. The only parameter required to be defined is the number hidden layer. Comparing with the other learning algorithm ELM is extremely faster learning speed, efficient and effective. With N distinct training sample \((x_i, t_i) \in R^n \times R^m (i=1,2,3,.....,N)\) with N hidden RBF nodes the output can be represented as

\[
o_j = \sum_{i=1}^{N} \beta_i f_j(x_j) = \sum_{i=1}^{N} \beta_i f_j(x_j; a_i, b_i), \quad j=1, ..., N \quad (12)
\]

where \(o_j\), output vector, \(x_j\) input samples, \(f_j(x_j; a_i, b_i)\), activation function of the ELM [28]. Randomly generated learning parameters are \(a_i = [a_{i1}, a_{i2}, ..., a_{in}]^T\) and \(b_i\) of the jth hidden node. \(\beta_i = [\beta_{i1}, \beta_{i2}, ..., \beta_{im}]^T\) is the output weight that connects the jth node and the output nodes. By setting \(a_i, x_j\) as the inner product of \(a_i\) and \(x_j\) then the equation (12) becomes,

\[
H \beta = 0 \quad (13)
\]

where,

\[
H = \begin{bmatrix}
    f(a_1, x_1 + b_1) & \ldots & f(a_N, x_1 + b_N) \\
    \vdots & \ddots & \vdots \\
    f(a_1, x_N + b_1) & \ldots & f(a_N, x_N + b_N)
\end{bmatrix}_{N \times N}
\]

\[
\beta = \begin{bmatrix}
    \beta_1^T \\
    \vdots \\
    \beta_N^T
\end{bmatrix}_{N \times m}, \quad O = \begin{bmatrix}
    O_1^T \\
    \vdots \\
    O_N^T
\end{bmatrix}_{N \times m} \quad (14)
\]

By this way, the calculation of the output weights is based on simple mathematical transformation. Hence the iterative learning process is avoided. The inputs and output to the ELM algorithm is the training data samples \((x_i, t_i) \in R^n \times R^m (i=1,2,3,.....,N)\), the hidden node number N, activation function and the output weights \(\hat{\beta}\).

ELM calculates the output weights in the following three steps [36]

1. The parameters of the hidden nodes \((a_i, b_i), i=1, ..., N\) are assigned randomly,
2. The output matrix of the hidden layer H is calculated,
3. The output weights \(\hat{\beta}\) is calculated as \(\hat{\beta} = H^T T\)

### 4. Results and Discussions

To test the effectiveness of neural approaches in Non-linear modeling of SRM the following methodology is followed.

Step 1: FEA based modeling of SRM to obtain non-linear Flux linkage current characteristics

Step 2: With the data obtained from FEA train ANN, GRNN and ELM.

Step 3: Analyze the performance of different neural structures through error graph.

Step 4: Incorporate the best neural structure for Non-linear modeling in Matlab/Simulink environment.

Step 5: Hardware validation of the simulation results obtained.

This work engages the use of MagNet software for modeling and simulation of SRM. The electromagnetic characteristics obtained from FEA are depicted in figure.5

The training data set consist of magnetization data of SRM

![Fig 5. Flux Linkage-Current-Rotor Position characteristics of SRM](image)

<table>
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<tr>
<th>Table 1. Neural Network Parameters</th>
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<td>Neural Network techniques</td>
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<tr>
<td>ANN</td>
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<td>GRNN</td>
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<td>ELM</td>
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in which flux linkage and rotor position serve as inputs and the corresponding current as output.

The simulation parameters for training ANN, GRNN and ELM are tabulated in Table 1. In GRNN interpolation technique the number of neurons is equal to the number of training samples.

The error between the actual and neural networks interpolated currents with respect to flux linkage and rotor position are shown as surface error maps in Fig. 6, 7, 8. The minimum and maximum modeling errors for different neural network techniques are presented in Table 2.

From the results it is observed that the GRNN model results in low modeling current error. It is further validated from the closeness of GRNN and FEA inductance plots as shown in Fig. 9. The results encourage integration of GRNN in the system simulation model of SRM drive.

The performance of the GRNN approach for modeling and simulation of SRM drive is analyzed by performing system simulation of SRM drive in MATLAB/Simulink environment.

The single phase MATLAB model of SRM incorporating the equations as discussed in section 2 is shown in Fig. 10. The $V_{out}$ (theta) block ensures the availability of the voltage at correct communication angles $\theta_{on,off}$. 

The Current_GRNN_Block computes the value of current from the value of flux linkage and position. Electromagnetic torque is calculated with the values of current and position. From the periodicity $(2\pi / Nr)$ nature of the phase inductance, it is appropriate to transform the rotor position angle coming from the mechanical equation so that it is performed by the block Theta Mod pi/6.

Fig. 11 shows the close loop SRM drive system. The speed controller generates the reference current for a particular speed and computes the position of the rotor. Current control block sets the amplitude of the current as

![Fig. 9. Phase inductance profile](image)

![Fig. 10. One phase simulation model of SRM](image)

![Fig. 11. Closed loop SRM drive system](image)
required for the speed with the pulse generator. The gating
signals of IGBTs are switched by the drive controller based
on the rotor position the current pulse

The simulated phase voltage and current for the speed of
1300 rpm with a load of 0.3 Nm shown in Fig. 12. The
accuracy of modeling is verified through the experimental
investigations. The experimental setup to validate system
simulation model of SRM is shown in Fig. 13.

The experimental setup consists of the following units:
(1) A four phase 8/6 pole SRM along with loaded DC
shunt generator,
(2) The Xilinx Nexyxx-4 FPGA controller is used for
developing a control program and executing the control
algorithm
(3) A four-phase split DC converter with its drive
circuits is used as the power converter.
(4) The terminal measurements such as phase current,
voltage, signals are measured by corresponding sensors
and DSO with corresponding probes.

Fig. 14 shows the experimental phase voltage and current
waveform for 1300 rpm. The closeness of the simulated
and experimental results validates the modeling and
performance evaluation of SRM drive using GRNN

5. Conclusion

This work envisages the application of variants of
neural network namely GRNN and ELM for modeling and
simulation of SRM drive. The data for training the network
is the flux linkage characteristic data obtained using FEA.
The results indicate that the modeling accuracy is
improved with GRNN based modeling. The closed loop
drive simulation is validated with the experimental setup
and the results illustrate the suitability of GRNN in
accurate modeling of SRM and performance enhancement
with novel control strategies.

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Appendix

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<tr>
<th>Table A1. Specifications of SRM</th>
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<tr>
<td>Design Parameter</td>
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<tr>
<td>Stator pole arc $\beta_s$</td>
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<tr>
<td>Rotor pole arc $\beta_r$</td>
</tr>
<tr>
<td>Air gap length $g$</td>
</tr>
<tr>
<td>Stator diameter $D_s$</td>
</tr>
<tr>
<td>Bore diameter $D$</td>
</tr>
<tr>
<td>Stack length $L$</td>
</tr>
<tr>
<td>Shaft diameter $D_{sh}$</td>
</tr>
<tr>
<td>Back iron thickness $C$</td>
</tr>
<tr>
<td>Height of stator pole $h_s$</td>
</tr>
<tr>
<td>Height of rotor pole $h_r$</td>
</tr>
<tr>
<td>Turns Per Phase</td>
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<tr>
<td>Rated current</td>
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<td>Turns Per Phase</td>
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References


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