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A SIMPLE PROOF OF THE NON-RATIONALITY OF A GENERAL QUARTIC DOUBLE SOLID

Yuri Prokhorov

Abstract. The aim of this short note is to give a simple proof of the non-rationality of the double cover of the three-dimensional projective space branched over a sufficiently general quartic.

1. Introduction

Throughout this work the ground field is supposed to be the complex number field $\mathbb{C}$.

A *quartic double solid* is a projective variety represented as a the double cover of $\mathbb{P}^3$ branched along a smooth quartic. It is known that quartic double solids are unirational but not rational [2], [8], [12], [14]. Moreover, a general quartic double solid is not *stably rational* [15]. There are also a lot of results related to rationality problems of *singular* quartic double solids see e.g. [1], [7], [5], [6], [10], [13].

The main result of this note is to give a simple proof of the following

1.1. Theorem. *Let X be the quartic double solid branched over the surface*

$$x_1^3x_2 + x_2^3x_3 + x_3^3x_4 + x_4^3x_1 = 0.$$

Then the intermediate Jacobian $J(X)$ is not a sum of Jacobians of curves. As a consequence, X is not rational.

1.2. Corollary. *A general quartic double solid is not rational.*

Our proof uses methods of A. Beauville [3], [4] and Yu. Zarhin [16]. The basic idea is to find a sufficiently symmetric variety in the family. Then the action of the automorphism group provides a good tool to prove non-decomposability the intermediate Jacobian into a sum of Jacobians of curves by using purely *group-theoretic* techniques. Since the Jacobians and their sums form a closed

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subvariety of the moduli space of principally polarized abelian varieties, this shows that a general quartic double solid is not rational[1].

2. Preliminaries

2.1. Notation. We use standard group-theoretic notation: if G is a group, then $\mathrm{z}(G)$ denotes its center, $[G,G]$ its derived subgroup, and $\mathrm{Syl}_p(G)$ its (some) Sylow p-subgroup. By ζ_m we denote a primitive m-th root of unity. The group generated by elements $\alpha_1, \alpha_2, \dots$ is denoted by $\langle \alpha_1, \alpha_2, \dots \rangle$.

2.2. Let X be a three-dimensional smooth projective variety with $H^3(X, \mathscr{O}_X) = 0$ and let $J(X)$ be its intermediate Jacobian regarded as a principally polarized abelian variety (see [9]). Then $J(X)$ can be written, uniquely up to permutations, as a direct sum

$$J(X) = A_1 \oplus \dots \oplus A_n, \tag{2.2.1}$$

where $A_1, \dots, A_p$ are indecomposable principally polarized abelian varieties (see [9, Corollary 3.23]). This decomposition induces a decomposition of tangent spaces

$$\mathrm{T}_{0,J(X)} = \mathrm{T}_{0,A_1} \oplus \dots \oplus \mathrm{T}_{0,A_n}. \tag{2.2.2}$$

Now assume that X is acted on by a finite group G. Then G naturally acts on $J(X)$ and $\mathrm{T}_{0,J(X)}$ preserving decompositions (2.2.1) and (2.2.2).

2.3. Lemma. *Let C be a curve of genus $g \ge 2$ and let $\Gamma \subset \mathrm{Aut}(C)$ be a subgroup of order $2^k \cdot 5$ whose Sylow 5-subgroup $\mathrm{Syl}_5(\Gamma)$ is normal in Γ. Then the following assertions hold*:

(i) *if $k = 2$, then $g \ge 3$,*
(ii) *if $k = 4$, then $g \ge 6$,*
(iii) *if $k = 5$, then $g \ge 11$.*

Proof. Let $C' := C/\mathrm{Syl}_5(\Gamma)$ and $g' := g(C')$. Let $P_1, \dots, P_n \in C'$ be all the branch points. By Hurwitz's formula

$$g + 4 = 5g' + 2n.$$

The group $\Gamma' := \Gamma/\mathrm{Syl}_5(\Gamma)$ of order 2^k faithfully acts on C' and permutes $P_1, \dots, P_n$. (i) Assume that $k = g = 2$. Then $g' = 0$, $C' \simeq \mathbb{P}^1$, and $n = 3$. At least one of the points P_1, P_2, P_3, say P_1, must be fixed by Γ'. But then Γ' must be cyclic (of order 4) and it cannot leave the set $\{P_1, P_2, P_3\} \subset \mathbb{P}^1$ invariant. This proves (i).

(ii) Assume that $k = 4$ and $g \le 5$. Then $g' \le 1$. If $g' = 0$, then $n \in \{3, 4\}$ and the group Γ' of order 16 acts on $C' \simeq \mathbb{P}^1$ so that the set $\{P_1, \dots, P_n\}$ is invariant. This is impossible. If $g' = 1$, then, as above, Γ' acts on an elliptic

[1] Recently V. Przyjalkowski and C. Shramov used similar method to prove non-rationality of some double quadrics [11].

curve C' leaving a non-empty set of $n \le 2$ points is invariant. This is again impossible and the contradiction proves (ii).

(iii) Finally, let $k = 5$ and $g \le 10$. Then $g' \le 2$ and $n \le 7$. If $g' \le 1$, then we get a contradiction as above. Let $g' = 2$, let $C' \to \mathbb{P}^1$ the canonical map, and let $\Gamma'' \subset \mathrm{Aut}(\mathbb{P}^1)$ be the image of Γ'. Since Γ'' is a 2-subgroup in $\mathrm{Aut}(\mathbb{P}^1)$, it is either cyclic or dihedral. On the other hand, Γ'' permutes the branch points $Q_1, \dots, Q_6 \in \mathbb{P}^1$ so that the stabilizer of each Q_i is a subgroup in Γ'' of index ≤ 4. Clearly, this is impossible. □

3. Symmetric quartic double solid

3.1. Let X be the quartic double solid as in Theorem 1.1. Then X is isomorphic to a hypersurface given by

$$y^2 + x_1^3 x_2 + x_2^3 x_3 + x_3^3 x_4 + x_4^3 x_1 = 0, \tag{3.1.1}$$

in the weighted projective space $\mathbb{P} := \mathbb{P}(1^4, 2)$, where $x_1, \dots, x_4, y$ are homogeneous coordinates with $\deg x_i = 1$, $\deg y = 2$.

Let α be the automorphism of X induced by the diagonal matrix

$$\mathrm{diag}(1,\ \zeta_{40}^{38},\ \zeta_{40}^{4},\ \zeta_{40}^{26};\ \zeta_{40}^{-1})$$

and let β be the cyclic permutation $(1, 2, 3, 4)$ of coordinates x_1, x_2, x_3, x_4. Since

$$\beta\alpha\beta^{-1} = \mathrm{diag}(\zeta_{40}^{26}, 1, \zeta_{40}^{38},\ \zeta_{40}^{4};\ \zeta_{40}^{-1}) = \mathrm{diag}(1,\ \zeta_{40}^{14},\ \zeta_{40}^{12},\ \zeta_{40}^{18};\ \zeta_{40}^{27}) = \alpha^{13},$$

these automorphisms generate the group

$$G = \langle \alpha,\ \beta \mid \alpha^{40} = \beta^4 = 1,\ \beta\alpha\beta^{-1} = \alpha^{13} \rangle \subset \mathrm{Aut}(X), \quad G \simeq \mathbb{Z}/40 \rtimes \mathbb{Z}/4.$$

3.2. Lemma. *Let G be as above. Then we have*

(i) $\mathrm{z}(G) = \langle \alpha^{10} \rangle$ *and* $[G, G] = \langle \alpha^4 \rangle$,
(ii) *the Sylow* 5*-subgroup* $\mathrm{Syl}_5(G)$ *is normal,*
(iii) *any subgroup in G of index* 10 *contains* $\mathrm{z}(G)$.

Proof. (i) can be proved by direct computations and (ii) is obvious because $\mathrm{Syl}_5(G) \subset \langle \alpha \rangle$. To prove (iii) consider a subgroup $G' \subset G$ of index 10. The intersection $G' \cap \langle \alpha \rangle$ is of index ≤ 4 in G'. Hence $G' \cap \langle \alpha \rangle$ is a 2-group of order ≥ 4 and so $\alpha^{10} \in G' \cap \langle \alpha \rangle$. □

3.3. Lemma (cf. [14, 0.1(b)]). *There exists a natural exact sequence*

$$0 \to H^2(X, \Omega_X^1) \to H^0(X, -K_X)^\vee \longrightarrow \mathbb{C} \to 0.$$

Proof. Since X is contained in the smooth locus of $\mathbb{P}$ and $\mathscr{O}_\mathbb{P}(X) = \mathscr{O}_\mathbb{P}(4)$, we have the following exact sequence

$$0 \longrightarrow \mathscr{O}_X(-4) \longrightarrow \Omega_\mathbb{P}^1|_X \longrightarrow \Omega_X^1 \longrightarrow 0,$$

and so

$$H^2(X, \Omega_\mathbb{P}^1|_X) \to H^2(X, \Omega_X^1) \to H^0(X, \mathscr{O}_X(2))^\vee \to H^3(X, \Omega_\mathbb{P}^1|_X) \to 0.$$

The Euler exact sequence for $\mathbb{P} = \mathbb{P}(1^4, 2)$ has the form

$$0 \longrightarrow \Omega^1_{\mathbb{P}} \longrightarrow \mathscr{O}_{\mathbb{P}}(-2) \oplus \mathscr{O}_{\mathbb{P}}(-1)^{\oplus 4} \longrightarrow \mathscr{O}_{\mathbb{P}} \longrightarrow 0.$$

Restricting it to X we obtain $H^2(X, \Omega^1_{\mathbb{P}}|_X) = 0$ and $H^3(X, \Omega^1_{\mathbb{P}}|_X) = \mathbb{C}$. □

3.4. Lemma. *We have the following decomposition of G-modules*:

$$\mathrm{T}_{0,J(X)} = V_4 \oplus V_4' \oplus V_2,$$

where V_4, V_4' are irreducible faithful 4-dimensional representations and V_2 is an irreducible 2-dimensional representation with kernel $\langle \alpha^8, \beta^2 \rangle$. Moreover, $\mathrm{z}(G)$ acts on V_4 and V_4' via different characters.

Proof. Clearly, $\mathrm{T}_{0,J(X)} \simeq H^0(J(X), \Omega_{J(X)})^\vee \simeq H^2(X, \Omega^1_X)$ and by Lemma 3.3 we have an injection $\mathrm{T}_{0,J(X)} \hookrightarrow H^0(X, -K_X)^\vee$. By the adjunction formula $K_X = (K_{\mathbb{P}} + X)|_X$ and so

$$H^0(X, -K_X) \simeq H^0(\mathbb{P}, \mathscr{O}_{\mathbb{P}}(-K_{\mathbb{P}} - X)).$$

Consider the affine open subset $U := \{x_1x_2x_3x_4 \neq 0\}$. Then $v = y/x_1^2$ and $z_i = x_i/x_1$, $i = 2, 3, 4$ are affine coordinates in $U \subset \{x_1 \neq 0\} \simeq \mathbb{A}^4$. Let ω be the 3-form

$$\omega := \frac{dz_2 \wedge dz_3 \wedge dz_4}{\partial\phi/\partial v} = \frac{dz_2 \wedge dz_3 \wedge dz_4}{2v},$$

where $\phi = v^2 + z_2 + z_2^3 z_3 + z_3^3 z_4 + z_4^3$ is the equation of X in U. It is easy to check that for any polynomial $\psi(z_2, z_3, z_4)$ of degree ≤ 2 the element $\psi \cdot \omega^{-1}$ extends to a section of $H^0(X, -K_X)$. Thus we have

$$H^0(X, -K_X) \simeq \{\psi(z_2, z_3, z_4) \cdot \omega^{-1} \mid \deg \psi \leq 2\}.$$

It is easy to check that the forms

(3.4.1)
$$\omega^{-1}, z_2^2\omega^{-1}, z_3^2\omega^{-1}, z_4^2\omega^{-1}, z_2\omega^{-1}, z_2z_3\omega^{-1}, z_3z_4\omega^{-1}, z_4\omega^{-1}, z_3\omega^{-1}, z_2z_4\omega^{-1}$$

are eigenvectors for α and β permutes them. Clearly, the following subspaces

$$\begin{aligned} W_4 &= \langle \omega^{-1},\ z_2^2\omega^{-1},\ z_3^2\omega^{-1},\ z_4^2\omega^{-1} \rangle, \\ W_4' &= \langle z_2\omega^{-1},\ z_2z_3\omega^{-1},\ z_3z_4\omega^{-1},\ z_4\omega^{-1} \rangle, \\ W_2 &= \langle z_3\omega^{-1},\ z_2z_4\omega^{-1} \rangle \end{aligned}$$

are G-invariant in $H^0(X, -K_X)$. Moreover, in the basis (3.4.1) the element α acts diagonally:

$$\begin{aligned} \alpha|_{W_4} &= \mathrm{diag}(\zeta_{40}^{11},\ \zeta_{40}^{7},\ \zeta_{40}^{19},\ \zeta_{40}^{23}), \\ \alpha|_{W_4'} &= \mathrm{diag}(\zeta_{40}^{9},\ \zeta_{40}^{13},\ \zeta_{40},\ \zeta_{40}^{37}), \\ \alpha|_{W_2} &= \mathrm{diag}(\zeta_8^3,\ \zeta_8^7), \end{aligned} \tag{3.4.2}$$

and β acts on each of these subspaces permuting the eigenspaces of α cyclically. Thus α^{10} acts on W_4 (resp., W_4') via scalar multiplication by ζ_4^3 (resp., ζ_4). Put $V_4 := W_4^\vee$, $V_4' := W_4'^\vee$, $V_2 := W_2^\vee$. □

4. Proof of Theorem 1.1

4.1. Assume to the contrary to Theorem 1.1 that $J(X)$ is a direct sum of Jacobians of curves, i.e., in the unique decomposition (2.2.1) we have $A_i \simeq J(C_i)$, where C_i is a curve of genus ≥ 1 and $J(C_i)$ is its Jacobian regarded as a principally polarized abelian variety. Let G_i be the stabilizer of A_i. There is a natural homomorphism $\varsigma_i : G_i \to \mathrm{Aut}(C_i)$. By the Torelli theorem ς_i is injective and we have

$$\mathrm{Aut}(J(C_i)) \simeq \begin{cases} \mathrm{Aut}(C_i) & \text{if } C_i \text{ is hyperelliptic,} \\ \mathrm{Aut}(C_i) \times \{\pm 1\} & \text{otherwise.} \end{cases} \tag{4.1.1}$$

Let us analyze the action of G on the set $\{A_1, \dots, A_n\}$. Up to renumbering we may assume that subvarieties $A_1, \dots, A_m$ form one G-orbit (however, the choice of this orbit is not unique in general). Clearly, $m \in \{1, 2, 4, 5, 8, 10\}$. Denote the stabilizer of A_i by G_i. Consider the possibilities for m case by case.

4.2. Case: $m = 1$**,** that is, $A_1 \subset J(X)$ is a G-invariant subvariety. Since $\mathrm{z}(G) = \langle \alpha^{10} \rangle$, the only normal subgroup of order 2 in G is $\langle \alpha^{20} \rangle$. Hence G cannot be decomposed as a direct product of groups of orders 2 and 80 (otherwise the order of α would be 20). If the action of G on $A_1 = J(C_1)$ is faithful, then by (4.1.1) so is the corresponding action on C_1. So, the curve C_1 of genus ≤ 10 admits faithful action of the group G of order $2^5 \cdot 5$. This contradicts Lemma 2.3(iii). Therefore the induced representation on T_{0,A_1} is not faithful. By Lemma 3.4 $\mathrm{T}_{0,J(C_1)} = V_2$. In this case $g(C_1) = 2$ and the action of G on $J(C_1)$ induces a faithful action of the group $\bar{G} := G/\langle \alpha^8, \beta^2 \rangle$ of order 16. Since C_1 is hyperelliptic, $\bar{G}$ is contained in $\mathrm{Aut}(C_1)$. If $\bar{G}$ contains the hyperelliptic involution τ, then τ generates a normal subgroup of order 2. In this case $\langle \tau \rangle = [\bar{G}, \bar{G}]$ and $\bar{G}/\langle \tau \rangle$ is an abelian non-cyclic group of order 8. But such a group cannot act faithfully on $C_1/\langle \tau \rangle \simeq \mathbb{P}^1$. Thus $\bar{G}$ does not contain the hyperelliptic involution. In this case the image of the induced action of $\bar{G}$ on canonical sections $H^0(C_1, \mathscr{O}_{C_1}(K_{C_1}))$ does not contain scalar matrices. Hence this representation is reducible and so it is trivial on $[\bar{G}, \bar{G}]$. On the other hand, the action of $\mathrm{Aut}(C_1)$ on $H^0(C_1, \mathscr{O}_{C_1}(K_{C_1}))$ must be faithful a contradiction.

From now on we may assume that the decomposition (2.2.1) contains no G-invariant summands.

4.3. Case: $m = 5$. The subspace $\mathrm{T}_{0,A_1} \oplus \cdots \oplus \mathrm{T}_{0,A_5} \subset \mathrm{T}_{0,J(X)}$ is a G-invariant of dimension 5 or 10. On the other hand, $\mathrm{T}_{0,J(X)}$ contains no invariant subspaces of dimension 5 by Lemma 3.4. Hence, $\mathrm{T}_{0,A_1} \oplus \cdots \oplus \mathrm{T}_{0,A_5} = \mathrm{T}_{0,J(X)}$, $\dim A_i = 2$, and $J(X) = \oplus_{i=1}^{5} A_i$. The stabilizer $G_i \subset G$ is a Sylow 2-subgroup that faithfully acts on C_i (because C_i is hyperelliptic, see (4.1.1)). Further, G_i permutes the Weierstrass points $P_1, \dots, P_6 \in C_i$. Hence a subgroup $G_i' \subset G_i$ of index 2 fixes one of them. In this situation, G_i' must be cyclic. On the other hand, it is easy to see that G does not contain any elements of order 16, a contradiction.

4.4. Case: $m = 10$**.** Then $A_1, \dots, A_{10}$ are elliptic curves and $G_i \subset G$ is a subgroup of index 10. By Lemma 3.2 each G_i contains $\mathrm{z}(G)$. Clearly, $\mathrm{z}(G)$ acts on T_{0,A_i} via the same character. Since the subspaces T_{0,A_i} generate $\mathrm{T}_{0,J(X)}$, the group $\mathrm{z}(G)$ acts on $\mathrm{T}_{0,J(X)}$ via scalar multiplication. This contradicts Lemma 3.4.

4.5. Case: $m = 8$**.** Then $A_1, \dots, A_8$ are elliptic curves and the stabilizer $G_1 \subset G$ is of order 20. In particular, the Sylow 5-subgroup $\mathrm{Syl}_5(G)$ is contained in G_1. Since $\mathrm{Syl}_5(G)$ is normal in G, we have $\mathrm{Syl}_5(G) \subset G_i$ for $i = 1, \dots, 8$. Since the automorphism group of an elliptic curve contains no order 5 elements, $\mathrm{Syl}_5(G)$ acts trivially on A_i. Therefore, $\mathrm{Syl}_5(G)$ acts trivially on the 8-dimensional G-invariant subspace $\mathrm{T}_{0,A_1} \oplus \dots \oplus \mathrm{T}_{0,A_8}$. This contradicts Lemma 3.4.

4.6. Case: $m = 4$**.** The intersection $G_1 \cap \langle \alpha \rangle$ is a subgroup of index ≤ 4 in both G_1 and $\langle \alpha \rangle$. Hence, $G_1 \ni \alpha^4$ and so $G_1 \supset [G, G]$. In particular, G_1 is normal and $G_1 = \dots = G_4$. If $\dim A_1 = 1$, then the element α^8 of order 5 must act trivially on elliptic curves $A_i \in 0$, $i = 1, \dots, 4$. Therefore, α^8 acts trivially on the 4-dimensional space $\mathrm{T}_{0,A_1} \oplus \dots \oplus \mathrm{T}_{0,A_4}$. This contradicts Lemma 3.4.

Thus $\dim A_1 = 2$. Then $\mathrm{T}_{0,A_1} \oplus \dots \oplus \mathrm{T}_{0,A_4} = V_4 \oplus V_4'$. An eigenvalue of α on $\mathrm{T}_{0,A_1} \oplus \dots \oplus \mathrm{T}_{0,A_4}$ must be a primitive 40-th root of unity (see (3.4.2)). Hence the group $G_1 \cap \langle \alpha \rangle$ acts faithfully on T_{0,A_1} and C_1 (see (4.1.1)). By Lemma 2.3(i) $G_1 \cap \langle \alpha \rangle$ is of order 10, i.e., $G_1 \cap \langle \alpha \rangle = \langle \alpha^4 \rangle$ and the kernel $N := \ker(G_1 \to \mathrm{Aut}(C_1))$ is of order 4. Thus $G_1 = \langle \alpha^4 \rangle \times N$. In particular, G_1 is abelian. But then the centralizer $\mathrm{C}(\alpha^8)$ of α^8 contains N and $\langle \alpha \rangle$. Therefore, $\mathrm{C}(\alpha^8) = G$ and $\alpha^8 \in \mathrm{z}(G)$. This contradicts Lemma 3.2(i).

Thus we have excluded the cases $m = 1, 4, 5, 8, 10$. The only remaining possibility is that all the orbits of G on $\{A_i\}$ are of cardinality 2.

4.7. Case: $m = 2$**.** Then $\dim A_1 \leq 5$ and G_1 is a group of order 80. By replacing the orbit $\{A_1, A_2\}$ with another one we may assume that $\mathrm{T}_{0,A_1} \oplus \mathrm{T}_{0,A_2} \not\subset V_2$ and so $\mathrm{T}_{0,A_1} \oplus \mathrm{T}_{0,A_2}$ coincides with either V_4, V_4', or $V_4 \oplus V_4'$. In particular, $g(C_1) \geq 2$. Clearly, $G_1 \cap \langle \alpha \rangle$ is of order 40 or 20. Hence, $\alpha^2 \in G_1$ and so the group G_1 cannot be decomposed as a direct product $G_1 = \langle \alpha^{20} \rangle \times H$. By the Torelli theorem G_1 faithfully acts on C_1. This contradicts Lemma 2.3(ii).

Proof of Theorem 1.1 is now complete.

Proof of Corollary 1.2. The Jacobians and their sums form a closed subvariety of the moduli space of principally polarized abelian varieties. By Theorem 1.1, in our case, this subvariety does not contain the subvariety formed by Jacobians of quartic double solids. Therefore a general quartic double solid is not rational. □

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Yuri Prokhorov
Steklov Mathematical Institute
and
Department of Algebra
Moscow State University
and
National Research University Higher School of Economics
Russia
E-mail address: `prokhoro@mi.ras.ru`