ON TWO-DIMENSIONAL LANDSBERG SPACE OF A CUBIC FINSLER SPACE

IL-YONG LEE AND DONG-GUM JUN

Abstract. In the present paper, we are to find the conditions that a cubic Finsler space is a Berwald space and a two-dimensional cubic Finsler space is a Landsberg space. It is shown that if a two-dimensional cubic Finsler space is a Landsberg space, then it is a Berwald space.

1. Introduction

In the Cartan connection $CT$, a Finsler space is called Landsberg space if the covariant derivative $C_{hijkl}$ of the $C$-torsion tensor $C_{hij} = \partial_h \partial_i \partial_j (L^2/4)$ satisfies $C_{hijkl}(x, y)y^k = 0$. A Berwald space is characterized by $C_{hijkl} = 0$. Berwald spaces are specially interesting and important because the connection is linear, and many examples of Berwald spaces have been known. But any concrete example of a Landsberg space which is not a Berwald space is not known yet. If a Finsler space is a Landsberg space and satisfies some additional conditions, then it is merely a Berwald space [3]. On the other hand, in a two-dimensional case, a general Finsler space is a Landsberg space if and only if its main scalar $I(x, y)$ satisfies $I_{11}y^1 = 0$ [7].

The purpose of the present paper is devoted to finding a two-dimensional Landsberg space with a cubic metric $L^3 = c_1 \alpha^2 \beta + c_2 \beta^3$, where $c_1$ and $c_2$ are constants. First we find the condition that
Finsler space with a cubic metric is a Berwald space (Theorem 3.2). Next we determine the difference vector and the main scalar of $F^2$ with the metric above. Finally we derive the condition that a two-dimensional Finsler space $F^2$ with a cubic metric is a Landsberg space (Theorem 4.1), and we show that if $F^2$ with the metric above is a Landsberg space, then it is a Berwald space (Theorem 4.2).

2. Preliminaries

Let $F^n = (M^n, L(\alpha, \beta))$ be an $n$-dimensional Finsler space with an $(\alpha, \beta)$-metric and $\bar{R}^n = (M^n, \alpha)$ the associated Riemannian space, where $\alpha^2 = a_{ij}(x)y^i y^j$ and $\beta = b_i(x)y^i$. In the following the Riemannian metric $\alpha$ is not supposed to be positive-definite and we shall restrict our discussions to a domain of $(x, y)$, where $\beta$ does not vanish. The covariant differentiation in the Levi-Civita connection $\gamma^i_jk(x)$ of $\bar{R}^n$ is denoted by the semi-colon. Let us list the symbols here for the late use:

\[
2r_{ij} = b_{i,j} + b_{j,i}, \quad 2s_{ij} = b_{i,j} - b_{j,i}, \quad r^i_j = \alpha^t r_{ij},
\]

\[
s^i_j = \alpha^t s_{ij}, \quad r_{i} = b_{i} r^i_{\,i}, \quad s_{i} = b_{i} s^i_{\,i}, \quad b^i = \alpha^t b_{i}, \quad b^2 = \alpha^t b_{i} b_{j}.
\]

\[
L_{\alpha} = \partial L / \partial \alpha, \quad L_{\beta} = \partial L / \partial \beta, \quad L_{\alpha \alpha} = \partial L_{\alpha} / \partial \alpha, \quad \text{and} \quad y_k = \alpha^k y^i.
\]

The Berwald connection $B_{\Gamma} = (G^i_j, G^i_{\,\cdot\,j})$ of $F^n$ plays one of the leading roles in the present paper. Denote by $B^i_j$ the difference tensor of $G^i_j$ from $\gamma^i_jk$ as follows [8].

\[
(2.1) \quad G^i_jk(x, y) = \gamma^i_jk(x) + B^i_jk(x, y)
\]

With the subscript 0, transvection by $y^i$, we have

\[
(2.2) \quad G^i_j = \gamma^i_{\cdot\,j} + B^i_j, \quad 2G^i = \gamma^i_{\cdot\,0} + 2B^i,
\]

and then $B^i_j = \partial_j B^i, \quad B^i_jk = \partial_k B^i_j, \quad \text{and} \quad \partial_j = \partial / \partial y^j$. On account of [8], the Berwald connection $B_{\Gamma}$ of a Finsler space with
(α, β)-metric $L(α, β)$ is given by (2.1) and (2.2), where $B_{j,i}^k$ are the components of a Finsler tensor of (1,2)-type which is determined by

\begin{equation}
L_α B_{j,i}^k y^j y_k = α L_β (b_{j,i} - B_{j,i}^k b_k) y^j.
\end{equation}

According to [8], $B'(x, y)$ is called the difference vector. If $β^2 L_α + αγ^2 L_{αα} \neq 0$, where $γ^2 = b^2 α^2 - β^2$, then $B^*$ is written as follows:

\begin{equation}
B^* = \frac{E}{α} y^* + \frac{α L_β}{L_α} s^* 0 - \frac{α L_{αα}}{L_α} C^* \left( \frac{1}{α} y^* - \frac{α}{β} b^* \right),
\end{equation}

where $E = β L_β C^*/L$ and

\[ C^* = \{αβ(r_{00} L_α - 2αs_0 L_β)\}/2(β^2 L_α + αγ^2 L_{αα}). \]

Furthermore, by means of [4] we have

\begin{equation}
α_{11} = -L_β L_{αα},
\end{equation}

\begin{equation}
β_{11} y^1 = r_{00} - 2b_r B^r,
\end{equation}

\begin{equation}
b_{11}^2 y^1 = 2(r_0 + s_0),
\end{equation}

\begin{equation}
γ_{11}^2 y^1 = 2(r_0 + s_0) α^2 - 2 \left( \frac{L_β}{L_α} b^2 α + β \right) (r_{00} - 2b_r B^r).
\end{equation}

The following Lemmas have been shown as follows.

**Lemma 2.1.** ([2]) If $α^2 \equiv 0 \pmod{β}$, that is, $a_{ij} y^i y^j$ contains $b_i(x) y^i$ as a factor, then the dimension $n$ is equal to two and $b^2$ vanishes. In this case we have $δ = d_i(x) y^i$ satisfying $α^2 = βδ$ and $d_i b^i = 2$. 
Lemma 2.2. ([4]) We consider the two-dimensional case.

1. If $b^2 \neq 0$, then there exist a sign $\varepsilon = \pm 1$ and $\delta = d_4(x)y^i$ such that $\alpha^2 = \beta^2/b^2 + \varepsilon \delta^2$ and $d_i b^i = 0$.

2. If $b^2 = 0$, then there exists $\delta = d_4(x)y^i$ such that $\alpha^2 = \beta \delta$ and $d_i b^i = 2$.

If there are two functions $f(x)$ and $g(x)$ satisfying $f\alpha^2 + g\beta^2 = 0$, then $f = g = 0$ is obvious, because $f \neq 0$ implies a contradiction $\alpha^2 = (-g/f)\beta^2$.

We shall state one more remark: Throughout the paper, we shall say "homogeneous polynomial(s) in $(y^i)$ of degree r" as $hp(r)$ for brevity. Thus $\gamma_0^i$ are $hp(2)$

3. Berwald spaces

In the present paper, we treat a condition that a Finsler space with a cubic metric is a Berwald space (cf.[11]). Then the so-called cubic metric on a differentiable manifold with the local coordinates $x^i$ is defined by

\begin{equation}
L(x, y) = (a_{ijk}(x)y^j y^y y^k)^{1/3} \quad (y^i = \dot{x}^i),
\end{equation}

where $a_{ijk}(x)$ are components of a symmetric tensor of $(0,3)$-type, depending on the position $x$ alone, and a Finsler space with a cubic metric is called the cubic Finsler space. The Finsler metric given by (3.1) was considered by Wegener (1935) [14] and by Kropina [6], and was also studied by M. Matsumoto [9], H. Shimada [13] and S. Numata [9]. It is regarded as a direct generalization of Riemannian metric in a sense. We quote from the proposition as follows:

Proposition 3.1. ([9]) Let $F^n$ be a Finsler space with a cubic metric $L(x, y)$.

1. In case of $n > 2$, if $L$ is an $(\alpha, \beta)$-metric where $\alpha$ is non-degenerate, then $L^3$ can be written in the form $L^3 = c_1 \alpha^2 \beta + c_2 \beta^3$ with two constants $c_1$ and $c_2$.

2. In case of $n = 2$, $L$ is always written in a generalized $(-1/3)$-Kropina type $L = \alpha^{2/3} \beta^{1/3}$, where $\alpha$ may be degenerate.
Now the cubic metric $L(\alpha, \beta)$ of Finsler space $F^n$ is given by

\begin{equation}
L^3(\alpha, \beta) = c_1 \alpha^2 \beta + c_2 \beta^3,
\end{equation}

where $c_1$ and $c_2$ are constants. In this case we have

\begin{equation}
\begin{aligned}
3L^2 L_\alpha &= 2c_1 \alpha \beta, \\
3L^2 L_\beta &= c_1 \alpha^2 + 3c_2 \beta^2 \\
9L^5 L_{\alpha\alpha} &= 2c_1 \beta^2 (3c_2 \beta^2 - c_1 \alpha^2), \\
w &= 2c_1 (3c_2 \beta^2 - c_1 \alpha^2), \\
C^a &= \frac{3L^3 [c_1 \beta r_{00} - (c_1 \alpha^2 + 3c_2 \beta^2) s_0]}{2c_1 \alpha \beta [3(c_2 \beta^2 + c_1) \beta^2 - c_2 \gamma^2]}.
\end{aligned}
\end{equation}

Substituting (3.3) into (2.3), we obtain

\begin{equation}
2c_1 \beta B_{jk}s^i y^j y^k = (c_1 \alpha^2 + 3c_2 \beta^2) (b_{j,s} - B_{jk}b^k) y^j,
\end{equation}

where $B_{jk} = \alpha_{kr} B_{j}^{r_1}$. Suppose that the cubic Finsler space $F^n$ is a Berwald space, that is, $B_{j^t k} = B_{j^t k}(x)$ and hence $b_{j,i}$ do not depend on $y^i$. Thus (3.4) leads to

\begin{equation}
B_{00} = (c_1 \alpha^2 + 3c_2 \beta^2) p_s,
\end{equation}

\begin{equation}
(b_{j,s} - B_{jk}b^k) y^j = 2c_1 \beta p_s,
\end{equation}

where we put $p_t = p_t(x)$. Making use of (3.5), we have

\begin{equation}
B_{jk} + B_{kj} = 2p_t (c_1 a_{jk} + 3c_2 b_j b_k).
\end{equation}

Since $B_{jk}$ is symmetric in $(j,i)$, (3.7) gives rise to

\begin{equation}
B_{jk} = c_1 (p_t a_{jk} + p_j a_{tk} - p_k a_{tj}) + 3c_2 (p_t b_j b_k + p_j b_k b_t - p_k b_t b_j).
\end{equation}

Therefore substitution of (3.8) in (3.6) yields

\begin{equation}
b_{j,i} = 3a p_t b_j + (3a - 2c_1) p_j b_i - p_b (c_1 a_{i j} + 3c_2 b_i b_j),
\end{equation}

where we put $a = c_2 b^2 + c_1$ and $p_b = p_k b^k$. 

ON TWO-DIMENSIONAL LANDSBERG SPACE

309
If \( \alpha^2 \equiv 0 \pmod{\beta} \), then Lemma 2.1 shows that \( n = 2, \alpha^2 = \beta \delta, \delta = d_1(x)y^1, b^2 = 0 \) and \( b^t d_i = 2 \). Thus (3.4) is of the form

\[
2c_1 B_{00t} = (c_1 \delta + 3c_2 \beta)(b_{j,t} - B_{jkt} b^k)y^1,
\]

which leads to

\[
B_{00t} = (c_1 \delta + 3c_2 \beta)u_{1t},
\]

(3.11)

\[
(b_{j,t} - B_{jkt} b^k)y^j = 2c_1 u_{1t},
\]

(3.12)

where \( u_{1t} = q_t u_k y^k \) are \( hp(1) \) and \( q_t = q_t(x) \). Then we have

\[
B_{jkt} + B_{kjt} = q_t (c_1 d_j + 3c_2 b_j) u_k + q_t (c_1 d_k + 3c_2 b_k) u_j,
\]

which lead to

\[
2B_{jkt} = c_t \{ q_t (d_j u_k + d_k u_j) + q_j (d_k u_t + d_j u_t) - q_k (d_t u_j + d_j u_t) \}
+ 3c_2 \{ q_t (b_j u_k + b_k u_j) + q_j (b_k u_t + b_t u_k) - q_k (b_t u_j + b_j u_t) \}.
\]

Since the dimension is equal to two and \((b_t, d_t)\) are independent pairs, we can put \( u_t = h b_t + k d_t \) and then \( u_t b^t = 2k \). Then we have

\[
B_{jkt} b^k
\]

(3.12)

\[
= c_t \left[ q_t (h b_j + 2k d_j) + q_j (h b_t + 2k d_t) - \frac{1}{2} q_b \{ h(b_t d_j + b_j d_t) + 2k d_t d_j \} \right]
+ 3c_2 \left[ k(q_t b_j + q_j b_t) - \frac{1}{2} q_b \{ 2h b_t b_j + k(b_t d_j + b_j d_t) \} \right],
\]

where \( q_b = q_k b^k \). Substituting (3.13) into (3.12), we have

\[
b_{t,j} = c_t \{ q_t (3 h b_j + 4 k d_j) + q_j (h b_t + 2 k d_t) \} + 3c_2 k (q_t b_j + q_j b_t)
- \frac{1}{2} q_b \{ (c_t h + 3c_2 k)(b_t d_j + b_j d_t) + 2(3c_2 h b_t b_j + c_t k d_t d_j) \}.
\]

In case of \( L_3 = \alpha^2 \beta \), substitution of \( c_t = 1 \) and \( c_2 = 0 \) in (3.14) leads to

\[
b_{t,j} = h(b_t q_j + 3b_j q_t) + 2k(d_t q_j + 2d_j q_t) - \frac{1}{2} q_b \{ h(b_t d_j + b_j d_t) + 2k d_t d_j \}.
\]

Summarizing up all the above, we have the following
Theorem 3.2. A cubic Finsler space with $L = c_1\alpha^2\beta + c_2\beta^3$, where $c_1$ and $c_2$ are constants, is a Berwald space if and only if

1. $\alpha^2 \not\equiv 0 \pmod{\beta}$: $b_{1,3}$ is of the form (3.9), where $a = c_2b^2 + c_1$ and $p_b = p_kb^k$.
2. $\alpha^2 \equiv 0 \pmod{\beta}$: $n = 2$, $b^2 = 0$ and $b_{4,3}$ is of the form (3.14), where $\alpha^2 = \beta\delta$, $\delta = d_s(x)y^i$ and $(h, k)$ are functions of $(x^i)$.

4. Two-dimensional Landsberg spaces

Now we are to find the necessary and sufficient conditions that a two-dimensional Finsler space with a cubic metric (3.2) is a Landsberg space (cf.[10]).

Because the difference vector $B^i$ and the main scalar $\varepsilon I^2$ play the leading roles, we have to determine the difference vector $B^i$. The difference vector $B^i$ of the Finsler space has been first given by [12]. Here, by means of (2.4) and reference of (3.3), we have

$$2B^i = \frac{2A}{\Omega} \left\{ y^i + \frac{(3c_2\beta^2 - c_1\alpha^2)}{2c_1\beta} b^i \right\} + \frac{(c_1\alpha^2 + 3c_2\beta^2)}{c_1\beta} s^i s^0,$$

where

$$A = c_1\beta r_{00} - (c_1\alpha^2 + 3c_2\beta^2)s_3,$$
$$\Omega = 3a\beta^2 - c_1\gamma^2.$$

It follows from (4.1) that

$$r_{00} - 2b_r B^r = \frac{2\beta A}{\Omega}$$

Now we deal with the necessary and sufficient conditions that a two-dimensional Finsler space $F^2$ with a cubic metric (3.2) is a Landsberg space. It is well known that in the two-dimensional case, a general Finsler space is a Landsberg space if and only if its main scalar $I_{y'y'} = 0$. Owing to [1], [5], the main scalar $I$ of a two-dimensional Finsler space $F^2$ with a cubic metric (3.2) and (3.3) is obtained as follows.

$$\varepsilon I^2 = \frac{c_1\gamma^2 Z^2}{2\Omega^3},$$
where \( Z = 9a\beta^2 + c_1\gamma^2 \).

Before discussing our problem, we have to check the assumption \( \Omega \neq 0 \) and \( c_1 \neq 0 \) in the two-dimensional case because \( \Omega \) appears in the denominators in (4.1), (4.2) and (4.3), and \( c_1 \) also appears (4.1). Lemma 2.2 shows that \( \Omega = 0 \) if and only if \( c_1 = 0 \) and \( b^2 = 0 \), \( c_1 = 0 \) and \( c_2 = 0 \), namely, the space is \( L^3 = c_2\beta^2 \) and \( b^2 = 0 \), \( L = 0 \). Consequently, \( \Omega \neq 0 \), \( c_1c_2 \neq 0 \) and \( b^2 \neq 0 \) are a proper assumption in the present section.

The covariant differentiation of (4.3) yields

\[
\epsilon I^2 |_t = \frac{c_1 Z}{2\Omega^4} (\Omega Z \gamma^2 |_t + 2\gamma^2 \Omega Z |_t - 3 \gamma^2 Z \Omega |_t). \tag{4.4}
\]

Transvecting (4.4) by \( y^1 \), we get

\[
\epsilon I^2 |_{y^1} = \frac{27c_1b^2\beta(c_1\alpha^2 + c_2\beta^2)Z}{2\Omega^4}(a\beta\gamma^2 |_t y^1 - 2a\gamma^2 \beta |_t y^1 - c_2\beta\gamma^2 b^2 |_t y^1). \tag{4.5}
\]

Consequently, the cubic Finsler space is a Landsberg space if and only if

\[
27c_1b^2\beta(c_1\alpha^2 + c_2\beta^2)Z(a\beta\gamma^2 |_t y^1 - 2a\gamma^2 \beta |_t y^1 - c_2\beta\gamma^2 b^2 |_t y^1) = 0,
\]

which imply

\[
(c_1\alpha^2 + c_2\beta^2)Z(a\beta\gamma^2 |_t y^1 - 2a\gamma^2 \beta |_t y^1 - c_2\beta\gamma^2 b^2 |_t y^1) = 0
\]

because of \( c_1 \neq 0 \) and \( b^2 \neq 0 \). Thus the following three cases should be considered to find the condition:

1° \( c_1\alpha^2 + c_2\beta^2 = 0 \) : Lemma 2.2 shows a contradiction immediately; that is, we obtain \( c_1 \neq 0 \) and \( c_2 \neq 0 \).

2° \( Z = 9a\beta^2 + c_1\gamma^2 = 0 \) : This implies \( c_1 = 0 \) and \( c_2 = 0 \), which is a contradiction by Lemma 2.2, that is, \( Z \neq 0 \).

3° \( a\beta\gamma^2 |_t y^1 - 2a\gamma^2 \beta |_t y^1 - c_2\beta\gamma^2 b^2 |_t y^1 = 0 \) : By means of (2.6), (2.7) and (2.8) this equation is written as

\[
2c_1\beta(r_0 + s_0) - 3ab^2(r_{00} - 2b_r B^r) = 0.
\]
Substituting (4.2) in the above, we obtain

$$
\beta[c_1(3a + c_1)b r_0 + \{3(3a - 2c_1)a + c_1^2\}b s_0 - 3c_1 b^2 r_{00}] + c_1 b^2 a^2 \{(3a - c_1)s_0 - c_1 r_0\} = 0.
$$

(4.6)

First, this gives a condition \(c_1 b^2 a^2 \{(3a - c_1)s_0 - c_1 r_0\} = 0 \pmod \beta\).
Since \(b^2 \neq 0\) may be supposed in this case, Lemma 2.1 shows \(a^2 \neq 0 \pmod \beta\) and so there exists a function \(g(x)\) satisfying

$$
(3a - c_1)s_0 - c_1 r_0 = g \beta.
$$

(4.7)

Then (4.6) is reduced to

$$
[c_1(3a + c_1)r_0 + \{3(3a - 2c_1)a + c_1^2\} s_0]b + c_1 b^2 (g a^2 - 3r_{00}) = 0.
$$

This implies that there exists a 1-form \(\mu = m_t(x)y^4\) such that

$$
3ar_{00} = g a^2 - \beta \mu,
$$

(4.8)

and the above is reduced to

$$
c_1(3a + c_1)r_0 + \{3(3a - 2c_1)a + c_1^2\} s_0 + c_1 b^2 \mu = 0.
$$

Thus the above and (4.7) yield

$$
s_0 = \frac{(3a + c_1)g \beta - c_1 b^2 \mu}{6a(3a - c_1)},
$$

(4.9)

$$
r_0 = -\frac{\{(3a - c_1)g \beta + c_1 b^2 \mu\}}{6c_1 a}.
$$

(4.10)

Consequently, (4.8), (4.9) and (4.10) are attained from (4.6). Since (4.8) is written in the form

$$
6ar_{ij} = 2ga_{ij} - (b_i m_j + b_j m_i),
$$

the transvection by \(b^j y^j\) yields

$$
6ar_0 = 2g \beta - (b^2 \mu + m_0 \beta),
$$

(4.11)
where \(m_b = m_k b^k\).

Comparing (4.11) with (4.10), we have
\[
c_1 m_b = (3a + c_1)g.
\]
Thus we get the condition in the form
\[
(4.8')
\]
\[
\begin{align*}
 r_{00} &= \frac{c_1 m_b}{3a(3a + c_1)} \alpha^2 - \frac{1}{3a} \beta \mu, \\
 s_0 &= \frac{c_1 (m_b \beta - b^2 \mu)}{6a(3a - c_1)}.
\end{align*}
\]
Eliminating \(\mu\) from (4.8') and (4.9'), we have
\[
(4.12)
\]
\[
\begin{align*}
 r_{00} &= \frac{c_1 f}{3a(3a + c_1)} \alpha^2 - \frac{f}{3b^2 a} \beta^2 + \frac{2(3a - c_1)}{c_1 b^2} \beta s_0, \\
 \end{align*}
\]
where \(f(x) = m_b\).

Thus we have the following

**Theorem 4.1.** The necessary and sufficient condition for a two-dimensional cubic Finsler space with \(c_1 c_2 \neq 0\) and \(b^2 \neq 0\) to be a Landsberg space is that (4.12) is satisfied.

Now we shall prove the reduction theorem:

**Theorem 4.2.** Let \(F^2\) be a two-dimensional cubic Finsler space with \(c_1 c_2 \neq 0\) and \(b^2 \neq 0\). If \(F^2\) is a Landsberg space, then \(F^2\) is a Berwald space.

**Proof.** The condition that (3.9) be a Berwald space may be rewritten in the form
\[
(4.13)
\]
\[
\begin{align*}
 r_{ij} &= (3a - c_1)(b_i p_j + b_j p_i) - p_b(c_{12} a_{ij} + 3c_2 b_i b_j), \\
 s_{ij} &= c_1 (b_i p_j - b_j p_i).
\end{align*}
\]

Now let \(F^2\) be a Landsberg space, that is, suppose that (4.12) holds. Then the system of linear equations
\[
b^1 p_1 + b^2 p_2 = -\frac{f}{3a(3a + c_1)}, \quad -b^2 p_1 + b^1 p_2 = \frac{s_{12}}{c_1}
\]
for \((p_1, p_2)\), where \(f\) is the one in (4.12), determines unique solution \((p_1, p_2)\) because of \(c_1 b^2 \neq 0\). The above are written as

\[ f + 3a(3a + c_1)p_b = 0, \quad s_{ij} = c_1(b_ip_j - b_gp_i). \]

The latter is nothing but (2) of (4.13). Then we obtain \(s_0 = c_1(b^2 \phi - p_b \beta)\), \(\phi = p_4(x)y^t\), and (4.12) is now written in the form

\[ r_{00} = 2(3a - c_1)\beta \phi - p_b(c_1 \alpha^2 + 3c_2 \beta^2), \]

which is nothing but (1) of (4.13). Thus the proof is completed. \(\square\)

REFERENCES


[13] H Shimada, On Finsler spaces with the metric $L^m = a_{11} \cdots a_{mm} y^1 y^2 \cdots y^m$, Tensor, N S 33 (1979), 365-372.

[14] J. M Wegener, Untersuchungen der zwei- und dreidimensional Finslerschen Raume mit der Grundform $L = \sqrt[3]{a_{kli} x^i x^j x^k}$, Koninkl Akad Wetensch, Amsterdam, Proc 38 (1935), 949-955

Department of Mathematics
Kyungsung University
Busan 608-736, Korea
E-mail: iylee@star.ks.ac.kr

Department of Mathematics
SoonChunHyang University
Asan 337-830, Korea
E-mail: jundk@sch.ac.kr