Stability of a Generalized Quadratic Functional Equation

일반화된 2차 범함수방정식의 안정성

김미혜
충북대학교 컴퓨터정보통신연구소
황인성
충북대학교 수학과

Mi-Hye Kim (mhkim@chungbuk.ac.kr)
RICIC, Chungbuk National University
In-Sung Hwang (buddaa@hanmail.net)
Dep. of Math., Chungbuk National University

중심어 : 안정성, 2차 범함수 방정식

Keyword : Stability, Quadratic Functional Equation

요 약

함수 방정식은 함수 자체의 정확한 형태를 가정하지 않고 단순히 기본적인 성질만을 언급하는 한정적이지 않은 방정식을 통하여 일반적인 관점의 수학적 형상화를 공식화하는데 매우 중요한 구실을 하기 때문에 실험적인 학문에서 유용하다. 본 논문에서는 범함수 방정식 문제 중 하나인 일반화된 2차 범함수 방정식

$$a^2 f(\frac{x-y-z}{a}) + f(x) + f(y) + f(z) = b^2\left[f(\frac{x-y}{b}) + f(\frac{y+z}{b}) + f(\frac{x-z}{b})\right]$$

의 해를 구하고 이 방정식의 안정성을 증명한다.

Abstract

Functional equations are useful in the expermental science because they play very important role to formulate mathematical models in general terms, through some not very restrictive equations, without postulating the forms of such functions. In this paper we solve one of a generalized quadratic functional equation

$$a^2 f(\frac{x-y-z}{a}) + f(x) + f(y) + f(z) = b^2\left[f(\frac{x-y}{b}) + f(\frac{y+z}{b}) + f(\frac{x-z}{b})\right]$$

and prove the stability of this equation.

I. Introduction

In 1940, S.M.Ulam [1] gave a wide ranging talk before a mathematical Colloquium at the University of Wisconsin in which *he discussed a number of important unsolved* problems. Among those was the following question concerning the stability of homomorphisms:

Let G_1 be a group and let G_2 be a metric group with a metric $d(\cdot,\cdot)$. Given $\varepsilon > 0$, does there exists a $\delta > 0$ such that if a function $h: G_1 \to G_2$ satisfies the inequality $d(h(xy), h(x)h(y)) < \delta$ for all $x, y \in G_1$ then there is a homomorphism $H: G_1 \to G_2$ with $d(h(x), H(x)) < \varepsilon$ for all $x \in G_1$?

For the case where the answer is affirmative, the functional equation for homomorphisms will be called stable.

The first result concerning the stability of functional equations was presented by D. H. Hyers. He has excellently answered the question of Ulam for the case where G_1 and G_2 are Banach spaces. In 1978, a generalized version of the theorem of Hyers for approximately liner mappings was given by Th. M. Rassias. During the last decades, the stability problems of several functional equations have been extensively investigated by a number of authors.

접수번호 : #030825-001
접수일자 : 2003년 8월 25일, 심사완료일 : 2003년 9월 18일
* 교신저자 : 김미혜, e-mail : mhkim@chungbuk.ac.kr

Y. W. Lee [2] showed the stability of the equation

$$9f\left(\frac{x-y-z}{3}\right)+f(x)+f(y)+f(z)$$
$$=4\left[f\left(\frac{x-y}{2}\right)+f\left(\frac{y+z}{2}\right)+f\left(\frac{x-z}{2}\right)\right].$$

The above equation has a solution of the form $Q(x)+B$, where Q is quadratic and B is a constant.

In this paper we deal with a generalized quadratic functional equation

$$a^2f\left(\frac{x-y-z}{a}\right)+f(x)+f(y)+f(z)$$
$$=b^2\left[f\left(\frac{x-y}{b}\right)+f\left(\frac{y+z}{b}\right)+f\left(\frac{x-z}{b}\right)\right].$$

In section II we solve a generalized quadratic functional equation. In section III we prove the stability of a generalized quadratic functional equation. Throughout this paper a and b are nonzero real constants.

II. A solution of a generalized quadratic functional equation

Throughout this section X and Y will be real vector spaces. Given a function $f: X\to Y$, consider the following equation.

$$a^2f\left(\frac{x-y-z}{a}\right)+f(x)+f(y)+f(z)$$
$$=b^2\left[f\left(\frac{x-y}{b}\right)+f\left(\frac{y+z}{b}\right)+f\left(\frac{x-z}{b}\right)\right] \quad (1)$$

for all $x,y,z\in X$. The following lemma is similar to S. H. Lee's result [3].

Lemma 2.1. If an even function $f: X\to Y$ satisfies (1) for all $x,y,z\in X$, then $f(x)-f(0)$ is quadratic.

Proof. I. The case $a^2+3\neq 3b^2$

Putting $x=y=z=0$ in (1) we have $(a^2+3)f(0)=3b^2f(0)$. Hence $f(0)=0$ since $a^2+3\neq 3b^2$. Putting $z=x$ and $y=0$ in (1) we have

$$b^2f\left(\frac{x}{b}\right)=f(x) \quad (2)$$

for all $x\in X$. Putting $y=z=0$ in (1) and using (2) we have

$$a^2f\left(\frac{x}{a}\right)=f(x) \quad (3)$$

for all $x\in X$. From (1), (2) and (3) we have

$$f(x-y-z)+f(x)+f(y)+f(z)$$
$$=f(x-y)+f(y+z)+f(x-z) \quad (4)$$

for all $x,y,z\in X$. Putting $z=-y$ in (4) we deduce $2f(x)+2f(y)=f(x+y)+f(x-y)$ for all $x,y\in X$ since f is even. This show that f is quadratic.

II. The case $a^2+3=3b^2$.

Let $Q(x)=f(x)-f(0)$ for all $x\in X$. Then $Q(0)=0$ and Q satisfies (1). As a similar way to the case I, we have Q is quadratic. □

Lemma 2.2. If an odd function $f: X\to Y$ satisfies (1) for all $x,y,z\in X$, then f is additive.

Proof. Note that $f(0)=0$ since f is odd. As in the proof of Lemma 2.1, we have the equation (4). Putting $x=0$ in (4) we have

$$f(-y-z)+f(y)+f(z)$$
$$=f(-y)+f(y+z)+f(-z),$$

or

$$f(y+z)=f(y)+f(z) \text{ for all } y,z\in X.$$

Therefore f is additive. □

Theorem 2.3. If a function $f: X\to Y$ satisfies (1) for all $x,y,z\in X$ and $f(0)=0$, then there exists an additive function $A: X\to Y$ and a quadratic function $Q: X\to Y$ such that $f(x)=Q(x)+A(x)$ for all $x\in X$.

Proof. Let $A(x)=\frac{1}{2}(f(x)-f(-x))$ for all $x\in X$. Then $A(-x)=-A(x)$ and A satisfies

(1) for all $x, y, z \in X$. By lemma 2.2, A is additive.

Let $Q(x) = \frac{1}{2}(f(x)+f(-x))$ for all $x \in X$. Then $Q(0)=0$, $Q(-x)=Q(x)$ and Q satisfies (1) for all $x, y, z \in X$. By lemma 2.1, Q is quadratic. Clearly, we have

$$f(x) = Q(x)+A(x)$$

for all $x \in X$. □

III. Stability of a generalized quadratic functional equation

Let $\mathbb{R}^+$ denote the set of nonnegative real numbers. Recall that a function $H : \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ is homogeneous of degree $p > 0$ if it satisfies $H(tu, tv, tw) = t^p H(u, v, w)$ for all nonnegative real numbers t, u, v and w. Throughout this section X and Y will be a real normed space and a real Banach space, respectively. We may assume that H is homogeneous of degree p. Given a function $f : X \to Y$, we set

$$Df(x, y, z) = a^2 f\left(\frac{x-y-z}{a}\right) + f(x)+f(y)+f(z) - b^2\left[f\left(\frac{x-y}{b}\right) + f\left(\frac{y+z}{b}\right) + f\left(\frac{x-z}{b}\right)\right].$$

Theorem 3.1. Assume that $\delta > 0$, then $p \in (0, 1)$ and $\delta = 0$ when $p > 1$. Let an odd function $f : X \to Y$ satisfy

$$\|Df(x, y, z)\| \le \delta + H(\|x\|, \|y\|, \|z\|) \qquad (5)$$

for all $x, y, z \in X$. Then there exists a unique additive function $A : X \to Y$ such that

$$\|f(x) - A(x)\| \le \frac{3}{2}\delta + \frac{h(x)}{2|2-2^p|} \qquad (6)$$

for all $x \in X$, where

$h(x) = 2H(\|x\|, \|x\|, \|x\|) + H(\|2x\|, 0, \|2x\|)$.

Proof. Putting $y = 0$, $z = x$ in (5), we have

$$\left\|2f(x) - 2b^2 f\left(\frac{x}{b}\right)\right\| \le \delta + H(\|x\|, 0, \|x\|) \qquad (7)$$

Replacing x by $2x$ in (7), we have

$$\|2f(2x) - 2b^2 f(\frac{2x}{b})\| \le \delta + H(\|2x\|, 0, \|2x\|) \qquad (8)$$

for all $x \in X$.

Putting $y = x$, $z = x$ in (5), we have

$$\left\|-a^2 f\left(\frac{x}{a}\right) + 3f(x) - b^2 f\left(\frac{2x}{b}\right)\right\| \le \delta + H(\|x\|, \|x\|, \|x\|) \qquad (9)$$

for all $x \in X$. Putting $y = -x$, $z = x$ in (5), we have

$$\left\|a^2 f\left(\frac{x}{a}\right) + f(x) - b^2 f\left(\frac{2x}{b}\right)\right\| \le \delta + H(\|x\|, \|-x\|, \|x\|) \qquad (10)$$

for all $x \in X$. By (9) and (10), we can see that

$$\left\|4f(x) - 2b^2 f\left(\frac{2x}{b}\right)\right\| \le 2\delta + 2H(\|x\|, \|x\|, \|x\|) \qquad (11)$$

for all $x \in X$. By (8) and (11), we also can see that

$$\|4f(x) - 2f(2x)\| \le 3\delta + 2H(\|x\|, \|x\|, \|x\|) + H(\|2x\|, 0, \|2x\|) \qquad (12)$$

for all $x \in X$. Hence

$$\left\|f(x) - \frac{1}{2}f(2x)\right\| \le \frac{3}{4}\delta + \frac{1}{4}h(x) \qquad (13)$$

for all $x \in X$, where $h(x) = H(\|x\|, \|x\|, \|x\|) + H(\|x\|, \|-x\|, \|x\|) + H(\|2x\|, 0, \|2x\|)$.

I. The case $0 < p < 1$.

Using (13) we have

$$\left\|\frac{f(2^n x)}{2^n} - \frac{f(2^{n+1}x)}{2^{n+1}}\right\| = \frac{1}{2^n}\left\|f(2^n x) - \frac{f(2 \cdot 2^n x)}{2}\right\| \le \frac{3}{4} \cdot \frac{1}{2^n}\delta + \frac{1}{4} \cdot 2^{n(p-1)} h(x) \qquad (14)$$

for all $x \in X$ and all positive integer n.

For nonnegative integers m and n with $m < n$,

$$\left\| \frac{f(2^m x)}{2^m} - \frac{f(2^n x)}{2^n} \right\|$$
$$\le \frac{3}{4}\delta \sum_{k=m}^{n-1} \frac{1}{2^k} + \frac{1}{4} \sum_{k=m}^{n-1} 2^{(p-1)k} h(x) \quad (15)$$

for all $x \in X$.

This show that $\left\{ \frac{f(2^n x)}{2^n} \right\}$ is a Cauchy sequence for each $x \in X$. Therefore, we can define a function $A : X \to Y$ by

$$A(x) = \lim_{n\to\infty} \frac{f(2^n x)}{2^n} \quad (16)$$

for each $x \in X$.

Since $f(-x) = -f(x)$, $A(-x) = A(x)$. And we have

$$\|DA(x,y,z)\|$$
$$= \lim_{n\to\infty} 2^{-n} \| Df(2^n x,\ 2^n y,\ 2^n z) \|$$
$$\le \lim_{n\to\infty} [2^{-n}\delta + 2^{(p-1)n} H(\|x\|, \|y\|, \|z\|)]$$
$$= 0$$

for all $x, y, z \in X$. By lemma 2, it follows that A is additive. By (13) and (15), we have

$$\left\| f(x) - \frac{f(2^n x)}{2^n} \right\|$$
$$\le \frac{3}{4}\delta \sum_{k=0}^{n-1} \frac{1}{2^k} + \frac{1}{4} h(x) \sum_{k=0}^{n-1} 2^{(p-1)k} \quad (17)$$

for all $x \in X$ and all positive integers n. Letting $n \to \infty$ in (17), we get (6).

Now, let $A' : X \to Y$ be another additive function satisfying (6). Then we have

$$\|A(x) - A'(x)\| = 2^{-n}\|A(2^n x) - A'(2^n x)\|$$
$$\le 2^{-n}(\|A(2^n x) - f(2^n x)\|$$
$$+ \|A'(2^n x) - f(2^n x)\|)$$
$$\le 2^{-n}\left[3\delta + \frac{h(x)}{2-2^p}\right]$$

for all $x \in X$ and all positive integers n. By letting $n \to \infty$ we get $A(x) = A'(x)$ for any $x \in X$.

II. The case $p > 1$.

Replacing x by $\frac{x}{2}$ in (13) we have

$$\left\| f(x) - 2f\left(\frac{x}{2}\right) \right\| \le \frac{1}{2} 2^{-p} h(x) \quad (18)$$

for all $x \in X$. Using (18) we have

$$\|2^n f(2^{-n}x) - 2^{n+1} f(2^{-(n+1)}x)\|$$
$$\le \frac{2^{-p}}{2} \cdot 2^{(1-p)n} h(x) \quad (19)$$

for all $x \in X$ and all positive integers n. By (18) and (19) we have

$$\|f(x) - 2^n f(2^{-n}x)\|$$
$$\le 2^{-1-p} h(x) \sum_{k=0}^{n-1} 2^{(1-p)k} \quad (20)$$

for all $x \in X$ and all positive integers n. The rest of proof is similar to the corresponding part of the case $0 < p < 1$. □

Theorem 3.2. Assume that $\delta \ge 0$, then $p \in (0,2)$ and $\delta = 0$ when $p > 2$. Let an even function $f : X \to Y$ satisfy

$$\|Df(x,y,z)\| \le \delta + H(\|x\|, \|y\|, \|z\|) \quad (21)$$

for all $x, y, z \in X$ and $f(0) = 0$. Then there exists a unique quadratic function $Q : X \to Y$ such that

$$\| f(x) - Q(x) \| \le \frac{7}{6}\delta + \frac{1}{|4 - 2^p|} h(x) \quad (22)$$

for all $x \in X$,

where $h(x) = H(\|x\|, \|x\|, \|x\|) + H(0, \|x\|, 0)$
$+ (2^{p-1} + 1) H(\|x\|, 0, \|x\|)$

Proof. Putting $z = x$ in (21) we have

$$\left\| a^2 f\left(\frac{y}{a}\right) + 2f(x) + f(y) \right.$$
$$\left. - b^2\left[f\left(\frac{x-y}{b}\right) + f\left(\frac{x+y}{b}\right)\right] \right\|$$
$$\le \delta + H(\|x\|, \|y\|, \|x\|) \quad (23)$$

for all $x, y \in X$. Putting $y = x$ in (23) we have

$$\left\| a^2 f\left(\frac{x}{a}\right) + 3f(x) - b^2 f\left(\frac{2x}{b}\right) \right\|$$
$$\le \delta + H(\|x\|, \|x\|, \|x\|) \quad (24)$$

for all $x \in X$. Putting $x = 0$ and $y = x$ in (23) we have

$$\left\| a^2 f\left(\frac{x}{a}\right) + f(x) - 2b^2 f\left(\frac{x}{b}\right)\right\| \le \delta + H(0, \|x\|, 0) \quad (25)$$

for all $x \in X$. Putting $y = 0$ in (23) we have

$$\left\|2f(x) - 2b^2 f\left(\frac{x}{b}\right)\right\| \le \delta + H(\|x\|, 0, \|x\|) \quad (26)$$

for all $x \in X$. From (25) and (26) we have

$$\left\| a^2 f\left(\frac{x}{a}\right) - f(x)\right\| \le 2\delta + H(0, \|x\|, 0) + H(\|x\|, 0, \|x\|) \quad (27)$$

for all $x \in X$. Replacing x by $2x$ in (26) and then dividing both sides of (26) by 2 yields

$$\left\| f(2x) - b^2 f\left(\frac{2x}{b}\right)\right\| \le 2^{-1}(\delta + 2^p H(\|x\|, 0, \|x\|)) \quad (28)$$

for all $x \in X$. From (24), (27) and (28) we have

$$\| f(2x) - 4f(x) \| \le \frac{7}{2}\delta + h(x) \quad (29)$$

for all $x \in X$, where

$$h(x) = H(\|x\|, \|x\|, \|x\|) + H(0, \|x\|, 0) + (2^{p-1}+1) H(\|x\|, 0, \|x\|).$$

I. The case $0 < p < 2$.

From (29) we have

$$\left\| f(x) - \frac{f(2x)}{4}\right\| \le \frac{7}{8}\delta + \frac{1}{4} h(x) \quad (30)$$

for all $x \in X$. Using (30) we have

$$\left\| \frac{f(2^n x)}{4^n} - \frac{f(2^{n+1}x)}{4^{n+1}} \right\| = \frac{1}{4^n}\left\| f(2^n x) - \frac{f(2\cdot 2^n x)}{4} \right\| \le \frac{7}{8}4^{-n}\delta + \frac{1}{4}2^{n(p-2)}h(x) \quad (31)$$

for all $x \in X$ and all positive integers n. From (30) and (31) we have

$$\left\|\frac{f(2^m x)}{4^m} - \frac{f(2^n x)}{4^n}\right\| \le \sum_{k=m}^{n-1} \frac{7}{8}4^{-k}\delta + \sum_{k=m}^{n-1}\frac{1}{4}2^{k(p-2)} h(x) \quad (32)$$

for all $x \in X$ and all nonnegative integers m and n with $m < n$. This show that $\left\{\frac{f(2^n x)}{4^n}\right\}$ is a Cauchy sequence for all $x \in X$. Consequently, we can define a function $Q: X \to Y$ by

$$Q(x) = \lim_{n\to\infty} \frac{f(2^n x)}{4^n}$$

for all $x \in X$. We have $Q(0) = 0, Q(-x) = Q(x)$ and

$$\|DQ(x,y,z)\| = \lim_{n\to\infty} 4^{-n}\| DQ(2^n x, 2^n y, 2^n z)\| \le \lim_{n\to\infty}(4^{-n}\delta + 2^{n(p-2)} H(\|x\|, \|y\|, \|z\|)) = 0$$

for all $x \in X$. By lemma 2.1, it follows that Q is quadratic. Putting $m = 0$ in (32) and letting $n \to \infty$ we have (22).

Now, let $Q': X \to Y$ be another quadratic function satisfying (22). Then we have

$$\begin{aligned}\|Q(x) - Q'(x)\| &= 4^{-n}\|Q(2^n x) - Q'(2^n x)\| \\ &\le 4^{-n}(\|Q(2^n x) - f(2^n x)\| + \|Q'(2^n x) - f(2^n x)\|) \\ &\le \frac{7}{3}\cdot 2^{-2n}\delta + 2\frac{2^{n(p-2)}}{|4-2^p|} h(x)\end{aligned}$$

for all $x \in X$ and all positive integers n. Since

$$\lim_{n\to\infty}\left(\frac{7}{3}\cdot 2^{-2n}\delta + 2\frac{2^{n(p-2)}}{|4-2^p|} h(x)\right) = 0,$$

we can conclude that $Q(x) = Q'(x)$ for all $x \in X$.

II. The case $p > 2$.

Replacing x by $\frac{x}{2}$ in (29) we have

$$\| 4f(2^{-1}x) - f(x) \| \le 2^{-p} h(x) \quad (33)$$

for all $x \in X$ and for all positive integers n. Using (33) we have

$$\| 4^n f(2^{-n}x) - 4^{n+1} f(2^{-(n+1)}x)\| \le 2^{-p} 2^{n(2-p)} h(x) \quad (34)$$

for all $x \in X$. From (33) and (34) we have

$$\| 4^n f(2^{-n}x) - f(x) \| \le \sum_{k=0}^{n-1} 2^{-p} 2^{k(2-p)} h(x)$$

for all $x \in X$ and all positive integers n. The rest of the proof is similar to the corresponding part of the case $0 < p < 2$.

Theorem 3.3. Let $\delta \ge 0$ and $p \in (0, \infty) - \{1, 2\}$. Assume that $\delta = 0$ if $p > 1$ and $\|(a^2+3-3b^2)f(0)\| = 0$ if $p > 2$. If a function $f : X \to Y$ satisfy (5) for all $x, y, z \in X$, then there exists a unique quadratic function $Q : X \to Y$ and a unique additive function $A : X \to Y$ such that

$$\|f(x) - f(0) - Q(x) - A(x)\| \le \frac{16}{6}\delta + \frac{7}{6}\| (a^2+3-3b^2) f(0) \| + \frac{1}{|4-2^p|} h_1(x) + \frac{h_2(x)}{2|2-2^p|}, \quad (35)$$

$$\left\| \frac{f(x)+f(-x)}{2} - f(0) - Q(x) \right\| \le \frac{7}{6}(\delta + \|(a^2+3-3b^2) f(0)\|) + \frac{1}{|4-2^p|} h_1(x), \quad (36)$$

and

$$\left\| \frac{f(x)-f(-x)}{2} - A(x) \right\| \le \frac{3}{2}\delta + \frac{h_2(x)}{2|2-2^p|} \quad (37)$$

for all $x \in X$, where $h_1(x) = H(\|x\|, \|x\|, \|x\|) + H(0, \|x\|, 0) + (2^{p-1}+1) H(\|x\|, 0, \|x\|)$ and $h_2(x) = 2H(\|x\|, \|x\|, \|x\|) + H(\|2x\|, 0, \|2x\|)$.

Proof. Let $q_1(x) = \frac{f(x)+f(-x)}{2}$ for all $x \in X$. Then $q_1(0) = f(0)$, $q_1(-x) = q_1(x)$ and

$$\|Dq_1(x, y, z)\| \le \delta + H(\|x\|, \|y\|, \|z\|)$$

for all $x, y, z \in X$.

Let $q(x) = q_1(x) - q_1(0)$ for all $x \in X$. Then $q(0) = 0$, $q(-x) = q(x)$ and

$$\begin{aligned} &\|Dq(x, y, z)\| \\ &= \| Dq_1(x, y, z) - (a^2+3-3b^2) q_1(0) \| \\ &\le \| Dq_1(x, y, z) \| + \|(a^2+3-3b^2) q_1(0) \| \\ &\le \delta + \| (a^2+3-3b^2) q_1(0) \| \\ &\quad + H(\|x\|, \|y\|, \|z\|) \end{aligned}$$

for all $x, y, z \in X$.

By Theorem 3.2, there exists a unique quadratic function $Q : X \to Y$ satisfying (36).

Let $g(x) = \frac{f(x)-f(-x)}{2}$ for all $x \in X$. Then $g(-x) = -g(x)$ and

$$\| Dg(x, y, z) \| \le \delta + H(\|x\|, \|y\|, \|z\|)$$

for all $x, y, z \in X$. By Theorem 3.1, there exists a unique additive function $A : X \to Y$ satisfying (37). Clearly, we have (35) for all $x \in X$. □

References

[1] S. M. Ulam, Problems in Mordern Mathematics, Chapter VI, Science Editions, Wiley, New York 1964.

[2] Y. W. Lee and S. H. Park, "On the Hyers-Ulam-Rassias stability of a quadratic functional equation," Korean J. Comput. & Appl. Math.(Series A) 9, pp. 371-380, 2002.

[3] S. H. Lee, "Stability of some generalized quadratic functional equation," J. Chungcheong Math. Soc., 14, pp. 1-6, 2001.

[4] D. H. Hyers, "On the stability of the linear functional equation," Proc. Nat. Acad. Sci. U.S.A. 27, pp. 222 - 224, 1941.

[5] D. H. Hyers and Th. M. Rassias, "Approximate homomorphisms," Aeq. Math. 44, pp. 125-153, 1992.

[6] S. M. Jung, "Hyers-Ulam-Rassias stability of Jensen's equation and its application," Proc. Amer. Math.Soc. 126, pp. 3137-3143, 1998.

[7] S. M. Jung, "Quadratic functional equation of Pexider type," Internat. J. Math. & Math. Sci. 24, pp. 351-359,

2000.
[8] Y. H. Lee and K. W. Jun, "A generalization of the Hyers-Ulam-Rassias stability of Jensen's equation," J. Math. Anal. Appl. 238, pp. 305-315, 1999.
[9] J. C. Parnami and H. L Vasudeva, "On Jensen's functional equation," Aeq. Math. 43, pp. 211-218, 1992
[10] Th. M. Rassias, "On the stability of the linear mapping in Banach spaces," Proc. Amer. Math. Soc., 72, pp. 297-300, 1978.

김 미 혜(Mi-Hye Kim) 정회원

1992년 2월 : 충북대학교 수학과 (이학사)
1994년 2월 : 충북대학교 대학원 수학과(이학석사)
2001년 2월 : 충북대학교 대학원 수학과(이학박사)
2001년 4월 ~ 현재 : 충북대학교 전기전자컴퓨터공학부 초빙교수
<관심분야> : 퍼지 이론, 금융 수학

황 인 성(In-Sung Hwang) 정회원

2001년 2월 : 충북대학교 수학과 (이학사)
2003년 2월 : 충북대학교 대학원 수학과(이학석사)