ADAPTIVE OPTIMAL OUTPUT FEEDBACK CONTROL

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Abstract

A practical and robust control scheme is suggested for MIMO discrete time processes with real simple poles. This type of control scheme, having the advantages of both the adaptiveness and optimality, may be successfully applicable to structured dynamic controllers for plants whose parameters are slowly time-varying. The identification of the process parameters is undertaken in ARMA form and the optimization of the feedback gain matrix is performed in the state space representation with regard to a standard quadratic criterion.

Summary

For single-input single output (SISO) systems, various adaptive direct control algorithms are proposed by many researchers in [12] , [13] , [14] but for general multi-input multi-output (MIMO) systems, adaptive control schemes are rare and further main emphasis is given on the stability property. For example, G. C. Goodwin and his co-workers [2] have recently established an globally convergent adaptive control algorithm, but the resulting controller may not be optimal. In this paper, a method of designing an adaptive and optimal controller is suggested for a class of MIMO systems. More specifically, for a given multivariable feedback control system with output proportional control structure, an on-line controller adjustment algorithm is given in which the adjustment is made to minimize a given cost functional. In the suggested adaptive control scheme, it is assumed that the changes of the process parameters are moderately slow so that an optimized feedback matrix at the i-th iteration stabilizes the (i+1)th identified system. This assumption is required to avoid the computation of an initial feedback matrix which stabilizes overall closed loop system for each iteration [3].

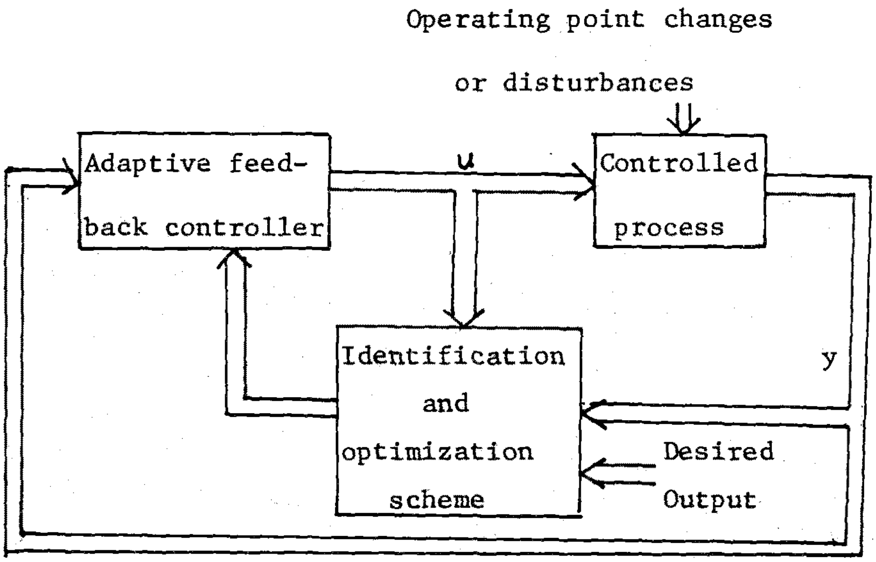

Fig. 1 Adaptive optimal control

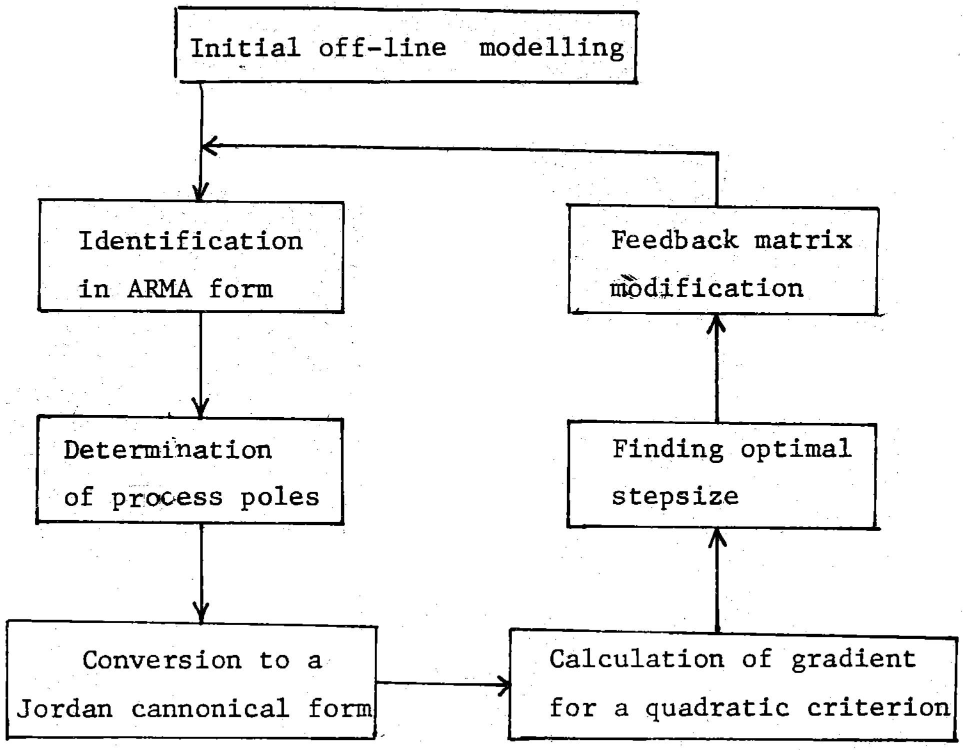

Fig. 2 Flow diagram of adaptive optimal control

Consider a linear time-invariant process whose n-dimensional state vector x(k), m-dimensional control vector u(k), and r-dimensional output vector y(k) are related by

$$x(k+1) = A\,x(k) + B\,u(k),\ x_o = x(o) \qquad (1)$$

$$y(k) = C\,x(k). \qquad (2)$$

From the system equations (1) and (2), using unit delay operator q^{-1}, one can obtain the following matrix transfer function whose entries are to be determined by using identification algorithms.

$$G(g^{-1}) = q^{-1}\begin{bmatrix} B_{11}(q^{-1})/A_1(q^{-1}) & \cdots & B_{1m}(q^{-1})/A_1(q^{-1}) \\ \vdots & & \vdots \\ B_{r1}(q^{-1})/A_r(q^{-1}) & \cdots & B_{rm}(q^{-1})/A_r(q^{-1}) \end{bmatrix} \quad (3)$$

Let all the poles of $G(q^{-1})$ in Eq. (3) be $\lambda_1, \lambda_2, \ldots, \lambda_{n_o}$ where $\lambda_1 > \lambda_j$ if $i > j$. Also, let P_i be the rank of M_i, where

$$M_i = \lim_{q\ \lambda_i} \left\{(q-\lambda_i) \cdot G(q^{-1})\right\} \quad (4)$$

Then $\sum_{i=1}^{n_o} P_i = n$ = number of states.

Theorem II-1

Suppose $G(q^{-1})$ in Eq. (3) has real poles with simple order. Then the Jordan representation (A, B, C) of $G(q^{-1})$ is given as follows :

$$A = \mathrm{diag}\Big(\overbrace{\lambda_1, \ldots, \lambda_1}^{P_1}\ \overbrace{\lambda_2, \ldots, \lambda_2}^{P_2}, \ldots, \lambda_{n_o}\Big) \quad (5)$$

$$B = \begin{bmatrix} B_1 \\ \hline B_2 \\ \hline \vdots \\ \hline B_{n_o} \end{bmatrix}$$

$$C = \left[C_1 \,\vdots\, C_2 \,\vdots\, \cdots \,\vdots\, C_{n_o} \right]$$

where (P_i x m) matrix B_i and (r x P_i) matrix C_i satisfy

$$C_i B_i = M_i$$

for i=1, 2, ..., n_o.

Theorem II-2

Let B_i, C_i, M_i be p x m, r x p, r x m matrices respectively, where $P \leq P_i$ = rank of M_i. Then

(1) for a fixed B_i,

$C_i = M_i B_i' [B_i B_i']^{-1}$ minimizes $||M_i - C_i B_i||$.

(2) For a fixed C_i

$B_i = [C_i' C_i]^{-1} C_i' M_i$ minimizes $|| M_i - C_i B_i||$.

The algorithm can be described as follows.

1. Find all roots of the equation $q^{n_i} A_i(q^{-1})=0$ for i=1, 2, ..., r.

   There will be $n_t = \sum_{i=1}^{r} n_i$ roots.

   Set $\omega_i=1$ for i=1, w, ..., n_t.

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2. Check if $|\lambda_i - \lambda_j| < \varepsilon$, for every $i \neq j$

   If above condition is satisfied, set λ_i as $(\omega_i \lambda_i + \lambda_j)/(\omega_i + 1)$ and remove λ_j and set ω_i as $\omega_i + 1$.

3. For each λ_i, calculate M_i, using (4).

4. For each M_i, determine the minimal rank P_i of C_i and B_i to satisfy the following criterion.

$$\frac{\| M_j - C_i B_i \|}{1 - | i |} < \delta \qquad (6)$$

5. Calculate the system matrices (A, B, C)

In step:2, one should set small enough so that Eq. (6) can be satisfied for the given δ.

With this algorthm the maximum number of eigenvalues is n_t, and the number of states is the total sum of P_i's.

For optimization of a feedback matrix, gradient method is used [15]. In finding an optimal stepsize, a rough and fast algorithm is suggested which is similar to that in [10] but is far more simple.

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