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# CONTROLLER DESIGN USING A REDUCED-ORDER MODEL

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Abstract This paper is concerned with the problem of designing satisfactory low-order controller starting with a high-order, state space model. The success of a design approach is rooted in the choice of a model reduction procedure. The powerful new reduction method of a modal approach was already evaluated /1/. Application of the technique to a simulated steam generator is demonstrated for the case of modal control with low-order controllers.

## Introduction

In many practical application of moddern control theory the most difficult problem is to obtain a suitable process model. An analytical approach using basic engineering principles often results in a dynamic model which consists of a large number of nonlinear differential equations. The model is usually too complicated for use in controller design or for implementation as part of the actual control system. Many ofthe multivariable control design techniques result in a control law that requires the availability of all elements of the state vector. Such control laws are frequently impractical because of their complexity and because in many applications it is not practical to measure or estimate all of the state variables. Hence there has been widespread interest in developing control laws that require only a subset of the state vector to be available. The resulting controllers are usually referred to as low-order controllers. This investigated is concerned with the problem of designing a satisfactory low-order, multivariable controller starting with a high -order, state-space model of a process. To be judged satisfactory, the low-order controller should perform almost as well as more complicated high-order state feedback controller.

## Problem formulation

Consider the original nth-order, time-invariant, state-space model

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} + D\underline{v} \qquad (1)$$

$$\underline{y} = C\underline{x} \qquad (2)$$

where the state vector $\underline{x}$, input vector $\underline{u}$, disturbance vector $\underline{v}$ and output vector are column vectors of dimensions n, p, l, and q, respectively. Matrices A, B, C and D are constants matrices of the approriate dimensions.

Let the mth-order (n<m) approximation of the original model as follows:

$$\dot{\underline{x}}_R = A_R\underline{x}_R + B_R\underline{u} + D_R\underline{v} \qquad (3)$$

$$\underline{y}_R = C_R\underline{x}_R \qquad (4)$$

The complete-state feedback and low-order controllers (incomplete state feedback controller) for the systems in Equations (1),(2),(3) and (4) are given by /2/

$$\underline{u} = G^{-1}KH^{-1}\underline{x} \qquad (5)$$

and

$$\underline{u} = G_R^{-1}KH_R^{-1}\underline{x}_R \qquad (6)$$

for the original and reduced-order state -variable models, respectively.
Here, G and $G_R$ are the elements of the canonical input column matrix corresponding to the stable mode:

$$G = (H^{-1}B) \qquad (7)$$

and

$$G_R = (H_R^{-1}B) \qquad (8)$$

Similarly, $H^{-1}$ and $H_R^{-1}$ represent the row of the corresponding left eigenvector matrices corresponding to the stable mode. K is the modal gain matrix. As mentioned above, $\underline{x}_R$ is simply chosen as a subset of the state vector $\underline{x}$. Hence, the control law expressed by Equation (6) is a good approximation of ideal control given by Equation (5) only if $H_R^{-1}\underline{x}_R$ approximates $H^{-1}\underline{x}$ fairly well.

## Application to a power plant steam generator

The controller design method described in the previous section is applied to the original 20th-order steam generator with five control inputs, four disturbances and six process outputs, and its reduced ninth-order model/3/.
Fig. 1 compares the response of the complete state feedback and low-order controllers to the heating value of fuel, a 10% step decrease. It is interesting to note that the enthalpy responses with two controllers are quite close.

## Conclusions

When an accurate reduced low-order model of the original large-scale is available, simple control approaches which obviate the need for state estimation can be used. In this study, a modal controller based on the reduced-order model with multi-inputs and only a few measurements proved to be quite successful in the steam generator model. The freedom to choose the measured variables is very important when low-order controller with no state reconstruction is used.
The design method resulting from this investigation should be of interest to that planning industial applications of multivariable control techniques.

## References


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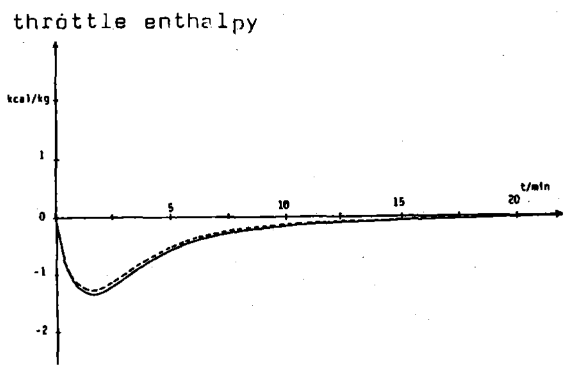


Fig. 1 Throttle enthalpy response to a 10% step decrease of the heating value of fuel.
Solid line denotes low-order controller. Broken line denotes complete state feedback.