Vibration Transmission of the Magnetically Coupled Rotating Devices

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1. Introduction

In many mordern power transmission systems, one of the principal concerns is the vibration transmission from the driving motor to the follower system. In some applications the reduction of the vibration level at the follower as low as possible is utmost important. The magnetically coupled torque transmission system is often used for this purpose. It has an advantage of transmitting power by non-contact means, thus reducing vibration transmission. Depending on the magnetic and electrical characteristics of the coupling halves, permanent magnet couplings can be divided into three categories ; eddy current, hysteresis, and synchronous couplings. The synchronous couplings can further be divided into two types ; coaxial and facial types. It is the purpose of this paper to report the transmission characteristics of the nonlinear torsional vibration of the rotating system with facial magnet couplings.

Kojima and Nagaya[1)] studied analytically the nonlinear torsional vibrations of a rotating shaft system with a coaxial magnet coupling. They obtained the solutions of the equation of motion including 1/2 and 1/3 subharmonics.

In this study, we included the torque loss due to the friction of the sliding bearings and obtained the solutions up to 3rd higher harmonics. The torque loss was treated as a damping in the boundary conditions. The theoretical results were compared with the experimental result for the rotating shaft system with facial magnet coupling.

2. Theoretical Analysis

The simplified model of a magnetically coupled rotating system is shown in Fig. 1. It consists of a driver and a follower, coupled by a pair of discs with an array of permanent magnets imbedded in them. Let θ denote the rotational angle of the shaft at the distance x, and T_ℓ be the load torque. When the shaft vibrates in a torsional mode, the governing equation of motion is given by

$$\frac{\partial^2\theta}{\partial t^2} = \frac{G}{\rho}\frac{\partial^2\theta}{\partial x^2} \qquad (1)$$

Here G and ρ are the shear modulus of elasticity and the density of the shaft, respectively. Assuming that the sinusoidal torsional vibration $\phi = \phi_u \cos \omega t$ is applied at driver, the transmitted torque of the magnet coupling is $T = T_m \sin \psi_m$ [2). Here T_m is the maximum torque transmitted by the magnet coupling, and $\psi_m = \phi - (\theta)_{x=0}$ is the relative angle between coupling halves.

In general the sliding bearings generate lower vibrations than the rolling element bearings, partly due to the damping provided by bearing oil. Such damping effect results in a slight loss in transmitted torque, and can be treated in the boundary conditions as frictional forces.

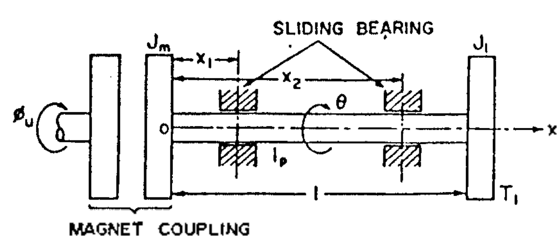

Fig. 1. Simplified model of the rotating shaft system with magnet coupling.

$$x = \ell : GI_p \frac{\partial\theta}{\partial x} = - J_\ell \frac{\partial^2\theta}{\partial t^2} - T_\ell \qquad (2)$$

$$x = 0 : GI_p \frac{\partial\theta}{\partial x} = J_m \frac{\partial^2\theta}{\partial t^2} - T_\ell - r^2 C_d A\{\frac{2\ell - x_1 - x_2}{\ell} \frac{\partial\theta}{\partial t}\Big|_{x=0}$$

$$+ \frac{x_1 + x_2}{\ell} \frac{\partial\theta}{\partial t}\Big|_{x=\ell}\} - T_m \cos\psi_\ell \left(\theta_m - \frac{\theta m^2}{2} \tan\psi_\ell - \frac{\theta m^3}{6}\right) \quad (3)$$

Here $\theta_m = \psi_m - \psi_\ell$, ψ_ℓ is the load angle, r is the radius of shaft, C_d is the frictional force per unit area, A is the frictional area, I_p is the polar moment of inertia, and J_m and J_ℓ are the mass moments of inertia of the discs at the ends of shaft. We assumed during the derivation of boundary conditions that the frictional force is proportional to the relative velocity.

The solution of Eq.(1), retaining up to 3rd harmonics, can be written as

$$\theta = \xi_o + \sum_{n=1}^{3} (\xi_n \cos n\omega t + \overline{\xi}_n \sin n\omega t) \quad (4)$$

The terms in the parentheses of Eq.(3) may be represented as follows.

$$\theta_m = \psi_o + \sum_{n=1}^{3} (\psi_n \cos n\omega t + \overline{\psi}_n \sin n\omega t)$$

$$\tfrac{1}{2}\theta_m{}^2 \tan\psi_\ell + \frac{1}{6}\theta m^3 = h_o + \sum_{n=1}^{3} (h_n \cos n\omega t + \overline{h}_n \sin n\omega t) \quad (5)$$

Substituting Eqs. (4) and (5) into Eqs. (1), (2) and (3), we obtain

$$W_o - g_o = 0$$

$$\{\chi_m n^2 H_{1n} (\lambda\ell)^2 + n\lambda\ell H_{2n} - \chi H_{1n} \cos\psi_\ell\} + \{D_1 n\omega H_{1n}$$

$$+ D_2 n\omega (H_{1n} \cos n\lambda\ell + H_{2n} \sin n\lambda\ell)\} \overline{W}_n - \chi H_{1n} g_n \cos\psi_\ell$$

$$= \{\chi_m n^2 H_{1n} (\lambda\ell)^2 + n\lambda\ell H_{2n}\} \delta_{1n} \quad (6)$$

$$\{D_1 n\omega H_{1n} + D_2 n\omega (H_{1n} \cos n\lambda\ell + H_{2n} \sin n\lambda\ell)\} W_n - \{\chi_m n^2 H_{1n} (\lambda\ell)^2$$

$$+ n\lambda\ell H_{2n} - \chi H_{1n} \cos\psi_\ell\} \overline{W}_n - \chi H_{1n} \overline{g}_n \cos\psi_\ell$$

$$= \{D_1 n\, \omega H_{1n} + D_2 n\omega (H_{1n} \cos n\lambda\ell + H_{2n} \sin n\lambda\ell)\} \delta_{1n}$$

Here

$H_{1n} = \cos n\lambda\ell - n\lambda\chi_\ell \ell \sin n\lambda\ell$, $H_{2n} = \sin n\lambda\ell + n\lambda\ell\chi_\ell \cos n\lambda\ell$

$W_o = \xi_o/\phi_u$, $W_n = \xi_n/\phi_u$, $\overline{W}_n = \overline{\xi}_n/\phi_u$, $g_o = h_o/\phi_u$, $g_n = h_n/\phi_u$, $\overline{g}_n = \overline{h}_u/\phi_u$ at $x=\ell$

$$\lambda = \omega\sqrt{\rho/G},\ \chi_\ell = J_\ell/J_s,\ J_s = \rho\ell I_p,\ \chi_m = J_m/J_s,\ \chi = T_m/ks,\ k_s = GIp/\ell \qquad (7)$$

$D_1 = r^2 C_d A\ (2\ell - x_1 - x_2)/GI_p$, $D_2 = r^2 C_d A\ (x_1 + x_2)/GI_p$

g's are complicated functions of W's and given in another paper written by authors[3].

The vibration amplitude magnification at the far end of the follower shaft is defined as

$$M = \left[\sum_{n=1}^{3} (M_n^{\ 2} + \overline{M}_n^{\ 2})\right]^{\frac{1}{2}} \qquad (8)$$

Where

$$M_n = (\delta_{1n} - W_n)/H_{1n},\quad \overline{M}_n = -\overline{W}_n/H_{1n} \qquad (9)$$

Eq.(8) gives the vibration amplitude magnification normalized to the excitation amplitude ϕ_u. Thus we can understand the torsional vibration transmission characteristics from the numerical values of W's. The Newton -Raphson method is used to solve the nonlinear simultaneous equations.

3. Numerical Results and Experiment

For the simple case of linear torsional vibration, Eq. (6) reduces to a set of 2 simultaneous equations. The numerical values of the parameters used in this study are chosen properly to fit our experimental system.

Fig. 2 shows the linear vibration amplitude magnification curve thus obtained. The first peak is due to the resonance of the magnet coupling itself, and the peaks at higher frequencies are due to the resonance of the shaft system including discs. The resonant frequency of the magnet coupling itself is very low as shown in Fig. 2. The resonant frequency varies with load angle ψ_ℓ, and it is possible to design the magnet coupling having even lower resonant frequency by proper choice of parameters.

Fig.3 shows the vibration amplitude magnification curves for the nonlinear case for 3 different values of frictional force per unit area, C_d. It also shows the curve for the linear case without damping for comparison purpose. The left ward bending of non-linear curve displays the typical behavior of the softening system, i.e., the larger the amplitude, the softer the stiffness. Such behavior results from the nature of the magnetic interaction between pairs of magnets imbedded in coupled discs. As the amplitude becomes large, the distance between magnet pairs facing each other increases. Thus the effective stiffness of the magnet coupling resulting from the magnetic forces decreases.

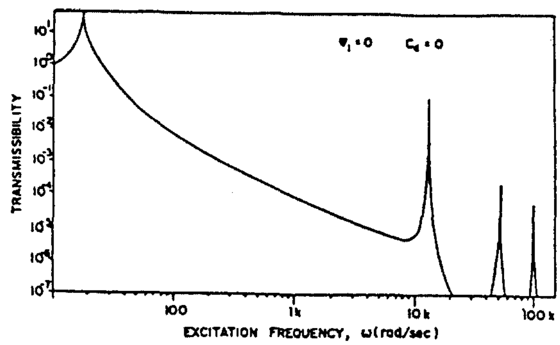

Fig. 2. Torsional vibration transmission characteristics of rotating shaft system with magnet coupling.

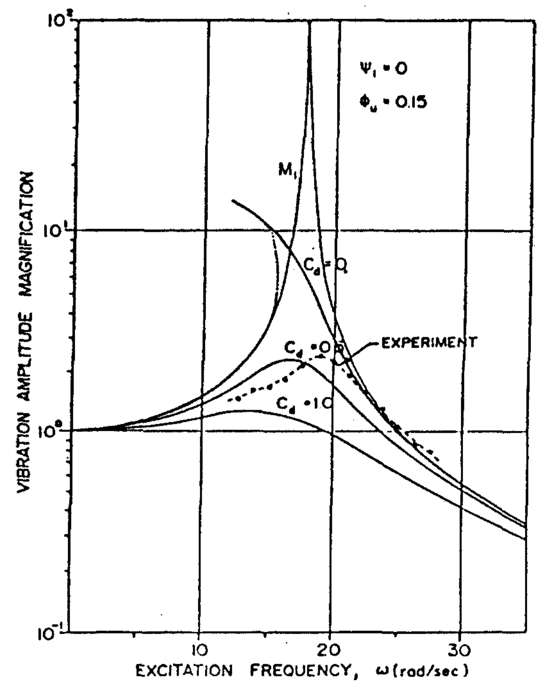

Fig. 3. Nonlinear torsional vibration transmission curves ; theory and experiment.

In order to verify theoretical analysis we constructed a system whose essential scheme is indentical as Fig. 1. The excitation was produced by applying sinusoidal power to the DC motor. The excitation amplitude was kept constant at ϕ_u = 0.15 rad for whole measured frequency range. The torsional vibration amplitude was measured by a pick-up installed on the circumference of the right-most disc in Fig.1. The vibration amplitude magnification was determined by dividing the measured torsional vibration amplitude with the constant exciting amplitude. The experimental result closely follows the theoretical curve for C_d = 0.5, indicating that the system we used has the corresponding frictional force.

The experiment also shows that the resonant frequency of our system is about 3 Hz, which is sufficiently low for general vibration reduction purposes.

4. Conclusions

The equation of motion for the torsional vibration of the rotating shaft system with magnet coupling was solved analytically with appropriate boundary conditions. The boundary conditions was formulated to reflect nonlinearity of the system as well as the transmission loss of torque due to the inherent system damping. The result showed the typical nonlinear behavior with softening restoring force characteristics. The analytical result was in good agreement with experimental results. The resonant frequency of the system decreases as the load troque increases. Also the resonant frequency of the magnetically coupled rotating system is sufficiatly low that it is possible to reduce effectively the vibration originating from the drivers. Such a low level vibration system can be used in the precision power transmission devices.

References

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