증류탑에의 최적제어 응용연구

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APPLICATION OF OPTIMAL CONTROL TO A DISTILLATION COLUMN

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Abstract

The continuous time linear quadratic problem (LQP) has been applied to the control of a 8-tray distillation column using the code VASP. The weighting matrices for the state variables and control variables were adjusted iteratively.

The simulation results of the optimal control with 2 inputs and 2 outputs showed that the LQP method is very satisfactory for a rapid response and feedback control, and any desired response could be obtained by adjusting the weighting matrices $\underline{\underline{Q}}$ and $\underline{\underline{R}}$. The feedback gain matrix $\underline{\underline{K}}$ was also determined.

I. Introduction

Although the SISO (single input, single output) classical control scheme has been applied in most industrial operations, it is not quite satisfactory in the complicated MIMO (multi input, multi output) systems: in a distillation column a single loop control does not efficiently control both top and bottom compositions and the transfer functions of the decoupling controllers are quite complicated. Optimal control of multi-variable process, which is based on the degree of freedom of the process and the optimization theory, has received a great deal of attention during the last couple of decades due to increasing demand for systems of high performance and the ready availability of the digital computers. The application of optimization theory to system design and control, however, has been hampered due to the nonlinearity, incomplete measurements, and the conflict between analytical feasibility and practical utility in the selection of the performance index of the optimal control as noted in (1).

In this work, the continuous time linear quadratic programming, which is the only general optimal control problem that has an analytical representation in a closed form, has been applied to the design of optimal controller for a 8-tray distillation column with 2 inputs and 2 outputs. The optimal feedback gain matrix $\underline{\underline{K}}$ was determined via the code VASP developed in NASA (2).

II. Review of the Theory

The theory of the LQP has been well reviewed in the literature (1,3). Generally, the linear quadratic dynamic system of servo mechanism is represented by a state space form of

$$\dot{\underline{x}} = \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u} \qquad (1)$$

and the performance index for the LQP is defined as

$$J_{min} = \int_0^{t_f} (\underline{x}\underline{\underline{Q}}\underline{x} + \underline{u}\underline{\underline{R}}\underline{u})\,dt \qquad (2)$$

where x is the state variable vector, u is the control variable vector, and Q and R are the weighting matrices for the state variables and control variables, respectively. The performance index, which indicates the 'goodness' of the system performance, yields an optimal and determines the nature of the resulting optimal control.

By applying the Riccati transformation to the adjoint equations of the Euler-Lagrange equation obtained from equations (1) and (2), a nonlinear matrix Riccati equation is obtained, and the solution of the Riccati equation can be obtained analytically, numerically, or approximately.

The performance index of the LQP represents a PD control, and the deviations of the state variables for a system normalized about some steady state will be reduced to zero at $t_f \rightarrow \infty$ by the derivative action. Therefore, the feedback gain matrix coefficients of the LQP are independent of the initial conditions of the state variable vector, and the optimal control becomes a linear feedback of

$$\underline{u}(t) = -\underline{\underline{K}}\,\underline{x}(t) \qquad (3)$$

$$\underline{\underline{K}} = \underline{\underline{R}}^{-1}\underline{\underline{B}}^{T}\underline{\underline{P}} \qquad (4)$$

When the state coefficient matrix $\underline{\underline{A}}$ is constant as in this problem, the solution of the matrix Riccati equation $\underline{\underline{P}}$ is a constant matrix, and the feedback gain matrix $\underline{\underline{K}}$ is determined by equation (4).

III. Computational Approach

Figure 1 shows the schematic of the laboratory distillation column used in this work. The column consists of 8 trays, a total condenser, and a thermosyphen reboiler, and used to separate a binary mixture of methanol and water. The physical model was obtained from the mass and energy balance, and simplified resulting 10 nonlinear ordinary differential equations. The simplifications were justified experimentally (4). The parameters such as liquid holdup, plate efficiency, and correlation between the reflux ratio and heat input to the reboiler were determined using the pulse tests, but the time delay was not modeled so that the LQP could be used directly for the controller systhesis.

Since the optimal control of a distillation column is the steady state operation to get the maximum throughput maintaining the product quality within specifications, the model equations were linearized about the desired steady state conditions yielding 10 linear ordinary differential equations. Then, the model was expressed in a state form of equation (1), and reflux ratio and the heat input to the reboiler were the control variables and the vapor composition in each tray were the state variables.

For the 10th order linear model, the state coefficient matrix $\underline{\underline{A}}$ and the control coefficient matrix $\underline{\underline{B}}$ were supplied in the code, and initial conditions (i.e. pulse disturbance) were given as

$$\underline{x}^{T} = [0,0,0,-.17,-.17,-.15,-.14,0,0,0]$$

The initial weighting matrix Q for the state variables was heavily weighted as

$$Q_{1,1} = Q_{10,10} = 500$$

$$Q_{i,j} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

and the weighting matrix $\underline{\underline{R}}$ for the control variables was

$$\underline{\underline{R}} = \begin{bmatrix} 0.0001 & 0.0 \\ 0.0 & 0.5 \end{bmatrix}$$

In the code, the nonlinear matrix Riccati equation, which is obtained from the adjoint equations of the Hamiltonian was solved by the transition matrix approach, and the feedback matrix $\underline{\underline{K}}$ was calculated by equation (4). Then, the transient responses of x_1 and x_{10} and the control actions u_1 and u_2 were determined in the forward time direction, and plotted on the Tektronix terminal.

In the simulation, each element in matrices $\underline{\underline{Q}}$ and $\underline{\underline{R}}$ was changed from one value to another fixing the other elements to see the weighting effects of each element, and the best control was obtained by adjusting the matrices $\underline{\underline{Q}}$ and $\underline{\underline{R}}$.

IV. Simulation Results and Discussion

The response curves were compared for the settling time to reach the steady state and the magnitude of maximum deviation. The following are the results extracted from the analysis:

1. When all the state variables from X_1 to X_{10} are evenly weighted, i.e. $\underline{\underline{Q}}=\underline{\underline{I}}$, the settling times and magnitudes of deviations of X_1 and X_{10} increase considerably with the loss of good control. Both X_1 and X_{10} should be weighted together even when X_1 is not important because of the interaction between X_1 and X_{10}.
2. Magnitudes of u_1 and u_2 could be reduced by adjusting R_{11} and R_{22}, and heavy weighting of $\underline{\underline{Q}}$ are approximately compensated for by heavy weighting of $\underline{\underline{R}}$.
3. Examining magnitudes of the components of the feedback matrix $\underline{\underline{K}}$, it was found that the control mainly depends on the top and bottom concentrations --- there are order of magnitude differences in the magnitudes between K_{11}, $K_{1,10}$ and $K_{1,j}$, and between K_{21}, $K_{2,10}$ and $K_{2,j}$, respectively. Where $j=2$ to 9.
4. Generally, the proportional action increases the deviation of the response and the derivative action brings the system to steady state in short time with least oscillation. Too much weighting of Q without enough weighting of R yields oscillatory responses in X_1 and X_{10}.
5. The dominant time constant can be estimated by the method proposed by Ziegler and Nichols. It was found that the dominant time constant varied from 5 to 10 time units depending on the weighting matrix $\underline{\underline{Q}}$, and everything settled down after about 5 times the dominant time constant -- the dominant time constant approaches the time constant(inverse of pole) of the most heavily weighted state variable.

Based on the analysis, the derivative action was added to the controller by increasing the weight in matrix $\underline{\underline{R}}$ and the best control was obtained when

$$Q_{1,1} = 700.$$
$$Q_{10,10} = 500.$$

and

$$\underline{\underline{R}} = \begin{pmatrix} 0.002 & 0.0 \\ 0.0 & 0.08 \end{pmatrix}$$

and Figure 2 shows the response curves for X_1, X_{10}, u_1, and u_2. The feedback gain matrix $\underline{\underline{K}}$ was determined as follows: (saturated)

$$\underline{\underline{K}}^T = \begin{pmatrix} -18.463822 & 56.583079 \\ -0.820136 & 2.679416 \\ -0.692640 & 2.385148 \\ -0.545967 & 2.084895 \\ -0.385234 & 1.830111 \\ -0.095166 & 1.414260 \\ -0.116208 & 1.196279 \\ +0.307473 & 1.216381 \\ +0.454391 & 1.347397 \\ +3.035943 & 11.606122 \end{pmatrix}$$

The better controller may be possible. However, this controller seems to be enough for the preliminary calculation of the optimal controller, because retuning will be necessary in an actual process due to the unmodeled time delay.

V. Conclusion

The LQP method was very satisfactory for a rapid response and optimal feedback, and any desired response could be obtained by adjusting the weighting matrices $\underline{\underline{Q}}$ and $\underline{\underline{R}}$. Therefoere, in a real process where only a few of many dependent variables are important, the response can be optimally controlled by heavily weighting the desired important variables. However, a reasonably accurate physical model including the time delay must be given, or retuning of the controller would be necessary to apply optimal control to the real system.

References

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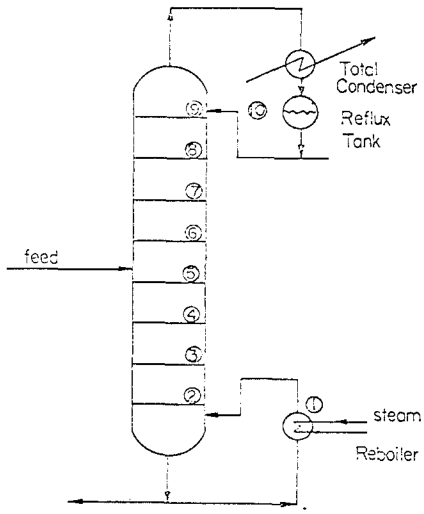
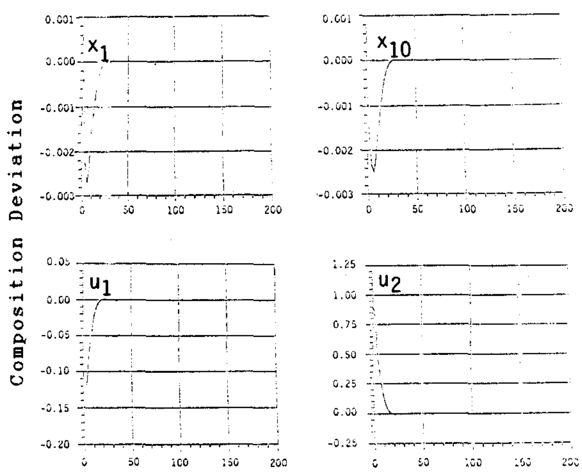


Figure 1. Schematic of a Pilot Distillation Column



Time (seconds)

Figure 2. Responces of x_1, x_{10}, u_1, and u_2 for the Proportional plus Derivative Controller