행열 부호 함수를 이용한 대규모 선형 시불변 계통의 준최적 합성 제어기의 설계에 관한 연구

천희영, 박귀태, 이창훈, °이동기
고려대학교 공과대학 전기공학과

On The Near Optimal Composite Regulator Problem For The Large Scale Linear Time Invariant System Using Matrix Sign Function

Hee Young Chun, Gwi Tae Park, Chang Hoon Lee, *Dong Ki Lee
Korea university

서 문

본 논문의 목적은 대규모 선형 시불변 계통에 대한 준최적 합성 제어기의 설계에 관한 새로운 방법을 제시하기 위한 것이다. 주어진 계통을 행열 부호 함수를 이용하여 그 고유치의 크기에 따라 블럭 대각 분해하고 각부 계통에 대한 최적 제어기및 전체계통에 대한 준최적 합성 제어기를 설계한다. 이 방법은 주어진 계통의 고유치를 미리 알 필요가 없으며 계통의 블럭 분해 과정에서 Riccati 방정식및 Lyapunov 방정식의 해를 구할 필요가 없고 특이섭동 기법이나 Two time scale seperation 방법에서의 제약조건에 관계없이 광범위하게 적용되는 장점이 있다.

I. Introduction

Optimization problem for the linear time invariant large scale system often gives rise to a computational impracticability for its computation time in digital computer. Consequently, near optimality of large scale system has been studied to solve this impracticability in various ways.(1)

In weakly coupled large scale systems, near optimality can be achieved by reduced order optimal controller with only its reduced order model which is constructed to contain dominant eigenvalues of original system through aggregation,(1) singular perturbation(2) or two-time scale seperation method.(1)

But in the case of strongly coupled large scale systems, the reduced order optimal controller based on only the reduced order model can't satisfy the given performance specifications due to the influence of the fast subsystem's neglected initial transiants. Hence, the neccesity of the near optimal composite controller is inevitable for the strongly coupled large scale systems.

Well known techniques such as two time scale separation method have been used for the block decomposition and the design of the near optimal composite controller for the large scale systems, but the solution of Riccati equation and Lyapunov equation in the two-time scale seperation method and the finding of eigenvalues, permutation in singular perturbation method must be carried out. Further more, the existence of nonzero off-diagonal terms when the singular perturbation method is applied make it complex to calculate the near optimal composite controller. And, in general, these two algorithms have restrictions(1) by their properties in thier applications to large scale systems

In this paper, another approch to near optimal regulator problem for the strongly coupled large scale systems is proposed using matrix sign function in model decomposition of original system. If a large scale linear time invariant system is block-decomposed in block diagonal form with its slow and fast modes, the near optimal composite controller can be designed with each subsystem's optimal regulator

2. Block-decomposition of a system via matrix sign function.

To develop the method for block-decomposition of a large scale system, it is reqired to review the property of sign function (3) as in the following definition.

Definition 1.

The scalar sign function of a complex variable with $Re(\lambda) \neq 0$ is defined by

$$sign(\lambda) = \begin{cases} +1 & ,\text{when } Re(\lambda) > 0 \\ -1 & ,\text{when } Re(\lambda) < 0 \end{cases}$$

From Definition 1., we can extend it to matrix sign function for the arbitrary reference line shifted from the imaginary axis to real value r, where r is chosen as $r = \frac{trace(A)}{n}$.

Let M be the modal matrix of A(nxn) and

$$J = M^{-1}(A - rI_n)M = \text{block diag}[J_+, J_-] \quad (1)$$

, where $J_+ \in C^{n1 x n1}$ and $J_- \in C^{n2 x n2}$, with $n1 + n2 = n$, are the collection of Jordan blocks associated with the spectrum $\partial(A) \in C^{r+}$ and $\partial(A) \in C^{r-}$, respectively, where C^{r+} and C^{r-} are the open right and left planes of C shifted as real value r. Then we can write the matrix sign function as follows;

$$sign(A - rI) = M[sign(J_+) \oplus sign(J_-)]M^{-1}$$
$$= M[I_{n1} \oplus (-I_{n2})]M^{-1} \quad (2)$$

And from the definition of a matrix sign function using projectors for mode decomposition (Riesz projector), we can write

$$sign^+(A - rI) = \tfrac{1}{2}\{I_n + sign(A - rI)\} \quad (3\text{-a})$$

$$sign^-(A - rI) = I_n - sign^+(A - rI) \quad (3\text{-b})$$

The computation of sign(A-rI) is possible by the algorithm proposed by Robert (3) and Shieh (4).

Then the block diagonalization of a system map can be accomplished via the matrix sign function as follows;

Let us rewrite Eq(2) as

$$sign(A - rI) = M[I_{n1} \oplus (-I_{n2})]W$$
$$= \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} [I_{n1} \oplus (-I_{n2})] \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix} \quad (4)$$

where $W \triangleq M^{-1}$

$$= \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}^{-1}$$

then from Eq(3-a,b), Eq(4) becomes

$$sign^+(A - rI) = M[I_{n1} \oplus 0_{n2}]W$$
$$= \begin{bmatrix} M_{11} \\ M_{21} \end{bmatrix} [W_{11} \quad W_{12}]$$
$$\triangleq M_1 W_1 \quad (5)$$

$$sign^-(A - rI) = M[0_{n1} \oplus (-I_{n2})]W$$
$$= \begin{bmatrix} M_{12} \\ M_{22} \end{bmatrix} [W_{21} \quad W_{22}]$$
$$\triangleq M_2 W_2 \quad (6)$$

Let matrices S_1 and S_2 be defined as

$$S_1 \triangleq ind\{sign^+(A - rI)\} = \{s_1, s_2, \ldots, s_{n1}\}$$
$$S_2 \triangleq ind\{sign^-(A - rI)\} = \{\bar{s}_1, \bar{s}_2, \ldots, \bar{s}_{n2}\}$$

where ind(.) indicates independent columns of $sign^+(A-rI)$ and $sign^-(A-rI)$, respectively. These matrices S_1 and S_2 can be written as follows;

$$S_1 = sign^+(A - rI)\, E_1$$
$$= M_1 W_1 E_1$$
$$\triangleq M_1 H_1 \quad , \quad \text{where } H_1 = W_1 E_1 \quad (7)$$

$$S_2 = sign^-(A - rI)\, E_2$$
$$= M_2 W_2 E_2$$
$$\triangleq M_2 H_2 \quad , \quad \text{where } H_2 = W_2 E_2 \quad (8)$$

where E_1 and E_2 are elementary column matrices.

According to the Sylvester's inequality, we obtain $rank(H_1) = n1$ and $rank(H_2) = n2$ or H_1 and H_2 are nonsingular. If we define the right block modal matrix M_r as

$$M_r \triangleq [S_1 \vdots S_2] = MH \quad (9)$$

where $H \triangleq [H_1 \oplus H_2]$

then from Eq(1) and Eq(9) we can get

$$AM_r = AMH$$
$$= M_r H^{-1}(J+rI)H$$
$$= M_r A_R \quad (10)$$

where $A_R \triangleq H^{-1}(J+rI)H$

Hence, we can write

$$A_R = M_r^{-1} A M_r$$
$$= \text{block diag}[A_{R1}, A_{R2}]$$

We can apply the previous results to block-decomposition of a system using the transformation matrix M_r as follows.

Let the linear time invariant system is given as

$$\dot{X} = AX + Bu \ , \ X \in R^{nx1}, u \in R^{mx1} \quad (11\text{-a})$$
$$y = CX \ , \ y \in R^{1x1} \quad (11\text{-b})$$

there exist a transformation matrix M_r, such as

$$X = M_r X_R = M_r \begin{bmatrix} X_{R1} \\ --- \\ X_{R2} \end{bmatrix} \quad (12)$$

then the system of Eq(11-a,b) can be block decomposed as follows.

$$\dot{X}_R = A_R X_R + B_R u \quad (13\text{-a})$$
$$y = C_R X_R \quad (13\text{-b})$$

where

$$A_R = M_r^{-1} A M = \text{block diag}[A_{R1}, A_{R2}]$$
$$B_R = M_r^{-1} B = [B_{R1}^T \ B_{R2}^T]^T$$
$$C_R = C M_r = [C_{R1} \ C_{R2}]$$

3. Near optimal composite regulator problem

Let us rewrite Eq(13-a,b) as follows;

$$\dot{X}_{R1} = A_{R1} X_{R1} + B_{R1} u \ , \ X_{R1}(0) = X_{10} \quad (14\text{-a})$$
$$\dot{X}_{R2} = A_{R2} X_{R2} + B_{R2} u \ , \ X_{R2}(0) = X_{20} \quad (14\text{-b})$$
$$y = C_{R1} X_{R1} + C_{R2} X_{R2} \quad (14\text{-c})$$

where $\partial(A_{R1}) \in C^{r+}$, $\partial(A_{R2}) \in C^{r-}$

If the system Eq(14-a,b,c) is assymptotically stable, then the fast modes are important only during a short initial interval and decay rapidly. After that interval, they are negligible and the behaviour of the system Eq(14-a,b,c) can be descrived by its slow modes. When the transients of the fast modes are assumed to be instantaneous, then the system Eq(14-a,b,c) are reduced to

$$\dot{\bar{X}}_{R1} = A_{R1} \bar{X}_{R1} + B_{R1} \bar{u} \ , \ \bar{X}_{R1}(0) = X_{10} \quad (15\text{-a})$$
$$0 = A_{R2} \bar{X}_{R2} + B_{R2} \bar{u} \quad (15\text{-b})$$
$$\bar{y} = C_{R1} \bar{X}_{R1} + C_{R2} \bar{X}_{R2} \quad (15\text{-c})$$

where bar ("-") denotes that initial transients of fast modes are neglected. Assuming that A_{R2} is nonsingular, we can express $\bar{X}_{R2}$ as

$$\bar{X}_{R2} = -A_{R2}^{-1} B_{R2} \bar{u} \quad (16)$$

,and substituting Eq(16) into Eq(14-c), we can define the slow subsystem as

$$\dot{X}_s = A_{R1} X_s + B_{R1} u_s \quad (17\text{-a})$$
$$y_s = C_{R1} X_s + D u_s \quad (17\text{-b})$$

where $D = -C_{R2} A_{R2}^{-1} B_{R2}$ and $X_s = \bar{X}_{R1}$, $u_s = \bar{u}$, $y_s = \bar{y}$.

To derive the fast subsystem we assume that the slow modes are constant during the fast transient, that is $\dot{\bar{X}}_{R2} = 0$ and $X_{R1} = X_s =$ constant.

From Eq(14-b) and Eq(15-b), we obtain

$$(\dot{X}_{R2} - \dot{\bar{X}}_{R2}) = A_{R2}(X_{R2} - \bar{X}_{R2}) + B_{R2}(u - \bar{u}) \quad (18)$$

Letting $X_f = X_{R2} - \bar{X}_{R2}$, $u_f = u - \bar{u}_s$, $y_f = y - \bar{y}_s$, the fast subsystem of Eq(14-a,b,c) is defined as

$$\dot{X}_f = A_{R2} X_f + B_{R2} u_f \quad (19\text{-a})$$
$$y_f = C_{R2} X_f \quad (19\text{-b})$$

Suppose now that the subsystem controllers which are to be designed according to slow and fast performance specifications, have the form

$$u_s = G_s X_s \quad (20)$$
$$u_f = G_f X_f \quad (21)$$

, then we can define the composite controller u_c from Eq(16) and Eq(18) as

$$u_c = u_s + u_f$$
$$= G_s X_s + G_f X_f$$
$$= (G_s + G_f A_{R2}^{-1} B_{R2} G_s) X_{R1} + G_f X_{R2} \quad (22)$$

Eq(22) suggest that G_s and G_f are seperatly designed according to the slow and fast performance specifications, and implemented to the composite control.

Now, in order to find each subsystem's optimal control, first we set the performance index for the slow subsystem as

$$J_s = \frac{1}{2}\int_0^\infty (y_s^T y_s + u_s^T R u_s)dt \qquad (23)$$

From Eq(17-b) we can write

$$J_s = \frac{1}{2}\int_0^\infty (X_s^T C_{R1}^T C_{R1} X_s + 2u_s D^T C_{R1} X_s + u_s^T R_o u_s)dt$$

where $R_o = R + D^T D$

Then the optimal control for the slow subsystem becomes

$$u_s^* = -R_o^{-1} (D^T C_{R1} + B_{R1}^T K_s) X_s$$
$$\triangleq G_s X_s \qquad (24)$$

where K_s is the positive semidefinite stabilizing solution of a Riccati equation,

$$0 = K_s(A_{R1} - B_{R1} R_o^{-1} D^T C_{R1}) + (A_{R1} - B_{R1} R_o^{-1} D^T C_{R1})^T K_s + C_{R1}^T (I_{n1} - D R_o^{-1} D^T) C_{R1} - K_s B_{R1} R_o^{-1} B_{R1}^T K_s$$

Second, let us define performance index for the fast subsystem as follows

$$J_f = \frac{1}{2}\int_0^\infty (y_f^T y_f + u_f^T R u_f)dt \qquad (25)$$

Then the fast subsystem's optimal control can be obtained as

$$u_f^* = -R^{-1} B_{R2}^T K_f X_f$$
$$\triangleq G_f X_f \qquad (26)$$

where K_f is the positive stabilizing solution of the following Riccati equation

$$0 = K_f A_{R2} + A_{R2}^T K_f + C_{R2}^T C_{R2} - K_f B_{R2} R^{-1} B_{R2}^T K_f$$

From Eq(22) and Eq(24),Eq(26), the near optimal composite controller can be obtained as

$$u_c^* = (G_s + G_f A_{R2}^{-1} B_{R2} G_s) X_{R1} + G_f X_{R2}$$
$$= G_o X_{R1} + G_f X_{R2} \quad , \text{ where } G_o = G_s + G_f A_{R2}^{-1} B_{R2} G_s$$
$$= \begin{bmatrix} G_o & G_f \end{bmatrix} \begin{bmatrix} X_{R1} \\ X_{R2} \end{bmatrix} \qquad (27)$$

And to express Eq(27) as variables of original system in Eq(11-a,b) we use the transformation given in Eq(12), then the Eq(27) can be expressed as the near optimal composite controller for the system in Eq(11-a,b) as follows

$$u_c^* = [G_o \vdots G_f] M_r^{-1} X$$

4. Numerical examples

To illustrate the responses by the near optimal composite control u_c^*, consider following weakly coupled and strongly coupled systems.

4.1 Weakly coupled system (1)

$$\dot{X} = \begin{bmatrix} -.2 & .5 & 0. & 0. & 0. \\ 0. & -.5 & 1.6 & 0. & 0. \\ 0. & 0. & -14.28 & 85.71 & 0. \\ 0. & 0. & 0. & -25. & 75. \\ 0. & 0. & 0. & 0. & -10. \end{bmatrix} X + \begin{bmatrix} 0. \\ 0. \\ 0. \\ 0. \\ 30. \end{bmatrix} u$$

$$y = \begin{bmatrix} 1. & 0. & 0. & 0. & 0. \end{bmatrix} X$$

The optimal control u_{opt}^* ,near optimal composite control u_c^* and reduced order control u_r^* (7) can be obtained as

$$u_{opt}^* = -.92438X_1 - .17109X_2 - .01612X_3 - .04924X_4 - .26441X_5$$

$$u_c^* = -.91366X_1 - .19165X_2 - .0201X_3 - .0673X_4 - .49774X_5$$

$$u_r^* = -.52342X_1 - .1098X_2 - .01059X_3 - .03285X_4 - .17655X_5$$

The response curves y for each cases are plotted in Fig.1

4.2 Strongly coupled system (1)

$$\dot{X} = \begin{bmatrix} -.2 & .5 & 0. & 0. & 0. \\ 0. & -.5 & 1.6 & 0. & 0. \\ 0. & 0. & -2.85 & 17.14 & 0. \\ 0. & 0. & 0. & -5. & 15. \\ 0. & 0. & 0. & 0. & -2. \end{bmatrix} X + \begin{bmatrix} 0. \\ 0. \\ 0. \\ 0. \\ 6. \end{bmatrix} u$$

$$y = \begin{bmatrix} 1. & 0. & 0. & 0. & 0. \\ 0. & 1. & 0. & 0. & 0. \end{bmatrix} X$$

The optimal control u_{opt}^* , near optimal composite control u_c^* and reduced order optimal control u_r^* can be obtained as

$$u_{opt}^* = -.6267X_1 - .93871X_2 - .35458X_3 - .7894X_4 - .6811X_5$$
$$u_c^* = -.5081X_1 - 1.015X_2 - .06219X_3 + .41672X_4 - 25.44X_5$$
$$u_r^* = -.0027X_1 - .0054X_2 - .0033X_3 - .00122X_4 - 0.3039X_5$$

The response curves for y_2 in each cases are plotted in Fig.2.

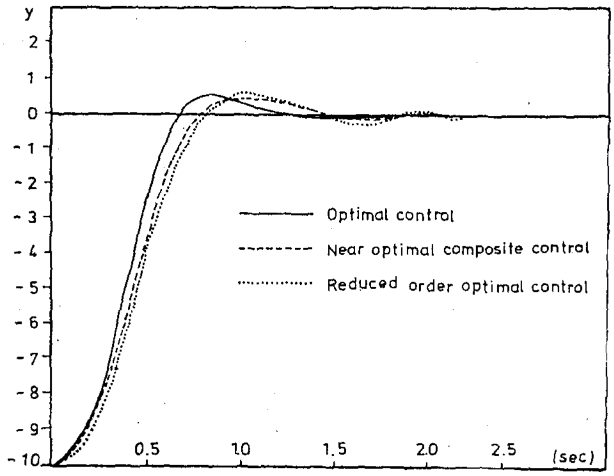

Fig.1 Output responses y for the weakly coupled system

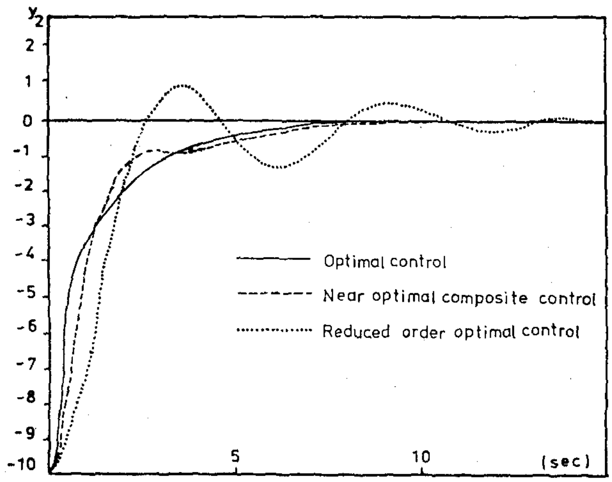

Fig.2 Output responses y_2 for the strongly coupled system

5. Conclusions

In this paper, the near optimal composite regulator for the strongly coupled large scale linear time invariant system has been proposed by using matrix sign function in model decomposition.

As illustrated in numerical examples, the near optimal composite regulator shows the better performances than the ones by the reduced order optimal regulator, and the superiority of the composite controller to the reduced order controller is clear in the strongly coupled system.

The completely block-decomposed form by the proposed algorithm makes it more easy to design composite controller than by the singular perturbation method and the requirement of the two-time scale property and the procedures of solving Riccati equation and Lyapunov equation for the block decomposition in the two-time scale seperation method can be eliminated by using proposed algorithm via matrix sign function in model decomposition.

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