# 3차원 유한요소 모델에서의 전류계산

°김 덕 원
연세대학교 의과대학 의용공학과

# Current Calculation in Three-Dimensional Finite Element Model

Deok-Won Kim
Dep't of Medical Engineering, Yonsei University

**Abstract**

An finite element code has been developed to calculate current flowing through an 8-node trilinear cubic element from the calculated potentials on the eight node. This code was implemented to the three-dimensional thoracic model for impedance cardiography to find the total currents in the z-direction flowing through the layers which are parallel to x-y plane.

The accuracy of the total current was estimated from its variation among the layers. It was found that the accuracy of the total currents in the layers was less than 0.6 %.

## 1. Introduction

There are many needs to simulate physical or biological system using computer model. Especially for human, three-dimensional model is necessary to obtain accurate result since human body is composed of many different organs and that the boundaries of the organs are very curved. Applying appropriate governing equation to the constructed model gives approximate solution, usually potential at each node. Using the calculated potential current flowing in an element can be obtained. In case of three-dimensional finite difference method, every cubic element has uniform size. Thus it is straightforward to calculate current. However in finite element method, every cubic element has different and irregular cross-section, which make area calculation complicated and so does the current calculation.

## 2 Current Calculation

The current flowing through an element in the z-direction can be found from Eq. (1).

$$I^e = \iint_\Gamma \mathbf{J} \cdot d\mathbf{s} \qquad (1)$$

where $\Gamma$ = middle plane to be integrated in Fig.1
$\mathbf{J}$ = current density (A/m$^2$)
$\mathbf{s}$ = normal vector of the plane $\Gamma$ in the (-z)- direction (m$^2$)
$I^e$ = current in an element in the (-z)-direction (A)

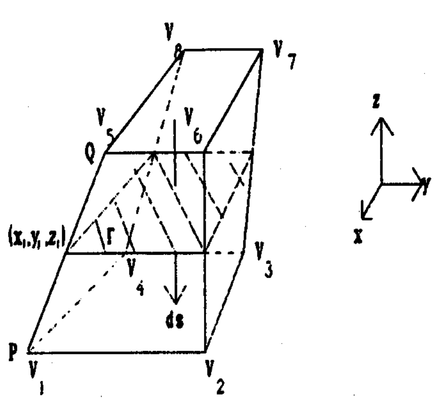


Fig.1. Current calculation in a typical element

Using the constitutive relation, $\mathbf{J} = \sigma \mathbf{E}$

$$I^e = \sigma\iint_\Gamma \mathbf{E}\cdot d\mathbf{s} = \sigma\iint_\Gamma E_z\, dxdy \qquad (2)$$

where $\sigma$ = conductivity of an element (S/m)
$\mathbf{E}$ = electric field intensity (V/m)
$E_z = -\partial V/\partial z$

Thus,

$$I^e = -\sigma\iint_\Gamma(\partial V/\partial z)\, dxdy \qquad (3)$$

where V = potential (V)

There exist special points at which extra good accuracy is usually found and this phenomenon of accuracy greater than average at these special points is called superconvergence [1]. The derivative term, $\partial V/\partial z$, is superconvergent at the middle of two nodes in a linear element while the solution, V, is superconvergent at the nodes. Therefore the integration was done on the shaded middle plane ($\Gamma$) in Fig.1.

By the interpolation rule, which is

$$V(x,y,z) = \sum_{i=1}^{8} V_i\,\psi_i(x,y,z) \qquad (4)$$

where $V_i = V(x_i,y_i,z_i)$

$\psi_i$ = element shape function

$x_i$, $y_i$, and $z_i$ were calculated by the interpolation between the node P and Q.

Substituting Eq. (4) into Eq. (3) yields

$$I^e = -\sigma\iint_\Gamma \partial/\partial z\Big(\sum_{i=1}^{8} V_i\psi_i\Big)\, dxdy$$
$$= -\sigma\iint_\Gamma \sum_{i=1}^{8} V_i(\partial\psi_i/\partial z)\, dxdy \qquad (5)$$

Eq. (5) is the final equation used in the finite element code in physical coordinates. For details regarding the transformation from the physical to master coordinates, please refer to Becker et al. [1].

## 3. Model Description

The model of the thorax and neck was constructed from horizontal cross-sections taken from anatomical maps [2] using eight-node trilinear cubic elements. The three-dimensional model consists of 22 elements on each of 29 layers. Fig.2 shows the grid of a typical cross-section. The layers were parallel to the x-y plane and nonuniformly spaced for a total of 880 nodes and 658 elements as shown in Fig.3. Each circumferential potential measuring electrode was simulated by ten thin elements having high conductivities of 100 S/m. Thus a layer containing a potential electrode has 10 electrode elements in addition to the 22 elements of other layers.

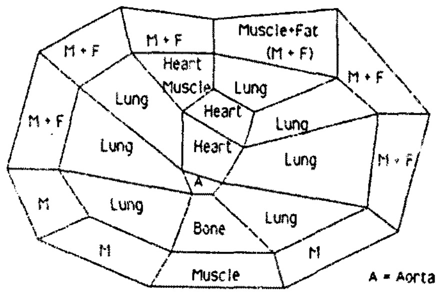


Fig. 2 A typical grid at the level of heart (G10 in Fig. 3)

The generalized form of Laplace's equation was solved subject to the following boundary conditions: a) All points of the body surface in contact with an electrode have the same potential as the electrode. ) At all other surface points the gradient normal to the surface will be zero. The constant potentials, 202 mV and 0 mV, measured between the current electrodes on the author using a conventional impedance cardiograph were assigned to the current electrodes to drive the solution.

## 4. Accuracy of the Current

Table 1 shows the total current calculated for each layer (measured from the bottom upward in Fig.3). The z-coordinate of each layer is the middle of the element in the z-direction. The x-y plane in the middle of each element

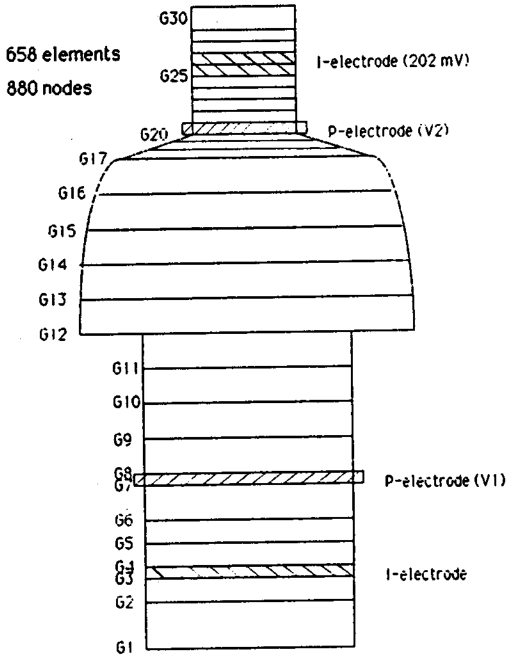


Fig.3 The levels of the 29 layers of the model

Table 1 Total current in each layer

| Z-coord.(cm) | No. of elem | I (uA) | |
|---|---|---|---|
| 1.88 | 22 | -0.7 | |
| 4.18 | 22 | .0 | |
| 5.20 | 22 | 1388.7 | I |
| 6.28 | 22 | 3619.8 | |
| 7.88 | 22 | 3620.2 | |
| 10.28 | 22 | 3619.7 | |
| 12.15 | 32 | 3619.8 | E |
| 13.75 | 22 | 3621.1 | |
| 16.25 | 22 | 3610.3 | |
| 18.75 | 22 | 3601.1 | |
| 21.25 | 22 | 3615.2 | |
| 23.75 | 22 | 3629.4 | |
| 26.25 | 22 | 3612.2 | |
| 28.75 | 22 | 3603.8 | |
| 31.25 | 22 | 3585.3 | |
| 33.75 | 22 | 3613.3 | |
| 35.50 | 22 | 3620.0 | |
| 36.38 | 22 | 3619.3 | |
| 37.13 | 22 | 3623.1 | |
| 37.85 | 32 | 3619.8 | E |
| 38.58 | 22 | 3619.8 | |
| 39.38 | 22 | 3624.2 | |
| 40.35 | 22 | 3614.3 | |
| 41.30 | 22 | 3619.8 | |
| 42.00 | 22 | 1542.1 | I |
| 42.60 | 22 | 897.0 | I |
| 43.20 | 22 | -.0 | |
| 44.25 | 22 | -4.2 | |
| 46.25 | 22 | -.0 | |

was chosen in calculating the current. The "No. of elem" in the second column indicates that the total number of elements in each layer. "I (uA)", is the total layer current in micro-amperes. The symbols of "E" and "I" indicate the levels which include the potential measuring and the current source electrodes respectively.

The two layers containing the potential electrodes have 10 more electrode elements than others. As seen in Table1, the total currents in the layers beyond the current electrodes are negligible since the net current should be zero in these regions. The fact that there are three layers where the total currents are virtually zero is one indication of the high accuracy of this finite element code. Likewise, the current in each layer between the current electrodes (I) should be the same and those calculated total currents are indeed quite consistent. The total current is within ± 0.7 % of 3619.8 uA. This is a vigorous test of accuracy since the accuracy of the calculated current is less than that of the potential. The total currents in the layers with the current electrodes are much less than the total currents in the layers without electrodes because a significant portion of the current out of the current electrodes does not intersect the middle x-y plane, where the current has been calculated.

The maximum and minimum currents are 3629.4 and 3585.3 uA respectively. Therefore the variation was calculated to be 1.2 %. However this 1.2 % variation resulted from not only the finite element code inaccuracy but the distortions of the elements in the particular layers. Indeed, elements in the two layers whose total currents were maximum and minimum were found to be more distorted than in other layers. This kind of element distortion is unique to the three-dimensional model and should be considered when a grid is being constructed. Excluding these two layers, the maximum and the minimum currents are 3623.1 and 3601.1 uA respectively, from which the variation is calculated to be 0.6 %.

Based on these observations, it can be said that the error of the calculated total current is less than 0.6 %. This 0.6 % of error estimation of the code is conservative since there is also some distortion of elements in other layers. The total current in each layer is also a valuable parameter in eliminating the badly distorted elements. From the table, there are five layers whose total currents are all the same, 3619.8 uA.

## 5. Discussion

In this study, a finite element code which is able to calculate the current in an 8-node trilinear cubic element was developed. The accuracy of the calculated total current in an layer was estimated 0.6 %, which is the variation of the total current among the layers. The total current in each layer is also a good indicator showing the degree of element distortions in the layer.

## Reference

[1] E.B. Becker, G.F. Carey, J.T.Oden, Finite Elements-An Introduction, Vol.1, Prentice-Hall, N.J., 1981.

[2] A.C. Eycleshymer, D.M. Shoemaker, A cross-section anatomy, Appleton-Century Crofts, 1911.