행렬부호 함수에 의한 선형 이산치 대규모 계통의 블럭 삼각화 분해

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Block-triangular Decomposition of a Linear Discrete Large-Scale Systems via the Generalized Matrix Sign Function

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Abstract

An analysis and design of large-scale linear multivariable systems often requires to be block triangularized form for good sensitivity of the systems when their poles and zeros are varied. But the decomposition algorithms presented up to now need a procedure of permutation, rescaling and a solution of nonlinear algebraic equations, which are usually burden. To avoid these problem, in this paper we develop a newly alternative block triangular decomposition algorithm which used the generalized matrix sign function on the Z-plane. Also, the decomposition algorithm demonstrated using the fifth order linear model of a distillation tower system.

1. Introduction

The block-decomposition of linear discrete time-invariant system offer a major role in the analysis and design of large-scale systems. In particular, for a good sensitivity in physical system realization and modal control of multivariable systems, the system is often formed into a cascade structure which contains a block triangularized system map.

Because of that, in recent years, the decomposition methods of block-triangularization has been studied in state space by many reserchers.(1,2,3) Though the decomposition algorithms offer the major advantage of not having to solve the eigenvalue-eigenvector problem - which is necessary, for example, in modal decomposition technique - they nevertheless require the solutions of algebraic equations satisfy a necessary and sufficient condition.(1) Since these equations are generally nonlinear, recursive algorithms for solving them are highly desirable.(3) And these have to perform a permutation and rescaling procedure of system matrices.

To avoid these problems we develop a newly alternative block triangular algorithm of a discrete-time large-scale system, which used the generalized matrix sign function on the Z-plane.(4,5,6)

2. The matrix sign function on the Z-plane

We introduce the sign function on the Z-plane as follows:

(Definition 2.1)

The sign function of a complex variable λ with $|\lambda| \neq 1$ on the Z-plane is defined by

$$\mathrm{Sign}_{(r)}(\lambda)\ \mathrm{sign}\left(\frac{\lambda-r}{\lambda+r}\right) = \begin{cases} +1 & \text{when } |\lambda|>r \\ -1 & \text{when } |\lambda|<r \end{cases} \qquad (2\text{-}1)$$

(Definition 2.2)

Let M be the modal matrix of a square matrix $\Phi(n\mathrm{x}n)$ and

$$\Phi = M\left[J_+^r \oplus J^r\right]W \qquad (2\text{-}2)$$

where M and its inverse W are defined as

$$M \triangleq \left[M_1 \,\vdots\, M_2\right], \quad W \triangleq M^{-1} \triangleq \left[W_1^T \,\vdots\, W_2^T\right]^T \qquad (2\text{-}2b)$$

and $J_+^r \in C^{n_1 x n_1}$ and $J_-^r \in C^{n_2 x n_2}$, with $n_1+n_2=n$, are the collections of Jordan blocks associated with the spectrum $\sigma(\Phi)\subset Z_+^r$ and $\sigma(\Phi)\subset Z_-^r$, respectively, where Z_+^r and Z_-^r are the outside and inside planes of the reference circle with radius r on the Z-plane, respectively. Then

$$\mathrm{Sign}_{(r)}(\Phi) = M\left[\mathrm{sign}(J_+^r) \oplus \mathrm{sign}(J_-^r)\right]W$$
$$= M\left[I_{n_1} \oplus (-I_{n_2})\right] W \qquad (2\text{-}3)$$

The matrix sign function of a square matrix Φ, $\mathrm{sign}_{(r)}(\Phi)$, with the eigenvalue spectrum $\sigma(\Phi) \subset Z_+^r \cup Z_-^r$ can be expressed by using the Riesz projector[5] as follows:

$$\mathrm{sign}_{(r)}(\Phi) \triangleq 2\mathrm{sign}^+_{(r)}(\Phi) - I_n$$
$$= I_n - 2\mathrm{sign}^-_{(r)}(\Phi) \qquad (2\text{-}4a)$$

where

$$\mathrm{sign}^+_{(r)}(\Phi) \triangleq \tfrac{1}{2}\left[I_n + \mathrm{sign}_{(r)}(\Phi)\right] \qquad (2\text{-}4b)$$
$$\mathrm{sign}^-_{(r)}(\Phi) = \tfrac{1}{2}\left[I_n - \mathrm{sign}_{(r)}(\Phi)\right]$$
$$= \mathrm{sign}^+_{(r)}(\Phi) - \mathrm{sign}_{(r)}(\Phi) \qquad (2\text{-}4c)$$

To develop methods for the block triangular decomposition of a system, we present the following definition of the generalized matrix sign function on the Z-plane.

(Definition 2.3)

Let $\{|\sigma(\Phi)|\} \cap \{r_1, r_2\} = \emptyset$, where $0<r_1<r_2$ and $\{r_1, r_2\} \in R$. The generalized matrix sign function of Φ with respect to the circular stripe (r_1, r_2) on the Z-plane is defined as

$$\mathrm{Sign}_{(r_1,r_2)}(\Phi) \triangleq 2\mathrm{sign}^+_{(r_1,r_2)}(\Phi) + I_n$$
$$= I_n - 2\mathrm{sign}^-_{(r_1,r_2)}(\Phi) \qquad (2\text{-}5a)$$

where

$$\mathrm{sign}^+_{(r_1,r_2)}(\Phi) \triangleq \tfrac{1}{2}\left[\mathrm{sign}_{(r_1)}(\Phi) - \mathrm{sign}_{(r_2)}(\Phi)\right] \qquad (2\text{-}5b)$$

$$\mathrm{sign}^-_{(r_1,r_2)}(\Phi) \triangleq I_n - \mathrm{sign}^+_{(r_1,r_2)}(\Phi) \qquad (2\text{-}5c)$$

and from reference (7)

$$\mathrm{sign}_{(r_i)}(\Phi) = \mathrm{sign}\left[(\Phi - r_i I_n)(\Phi + r_i I_n)^{-1}\right]$$
$$r_i = 1 \text{ for } i=1,2 \qquad (2\text{-}5d)$$

3. The block-triangularization of a discrete multivariable systems

In this chapter we propose the block triangular decomposition algorithm for a discrete-time multivariable systems using by definitions of chapter 2.

Consider a linear discrete multivariable system as follows:

$$X(k+1) = \Phi X(k) + \Pi u(k) \qquad (3\text{-}1a)$$
$$y(k) = CX(k) \qquad (3\text{-}1b)$$

Assume that the state $X(k)$ is decomposed into the n_1 and n_2 vectors $X_1(k)$ and $X_2(k)$. In this system the class of eigenvalues located near the unit circle on the Z-plane are assigned to the slow mode and those located near the origin are assigned to the fast mode. Thus we need the reference circle[7] with radius r for block-decomposition. For system (3-1) the eigenspectrum $\sigma(\Phi)$ is arranged in decreasing order of absolute values:

$$|\lambda_1| > \cdots > |\lambda_{n_1}| > |\lambda_{n_1+1}| > \cdots > |\lambda_n| \qquad (3\text{-}2)$$

From Eq.(3-2), the positive real value $r \neq 1$ is chosen between $|\lambda_{n_1}|$ and $|\lambda_{n \neq 1}|$ arbitrary.

Now from Eqs.(2-4) and (2-3), we define two projection operators $P_s^r(\Phi)$ and $P_f^r(\Phi)$ as follows:[7]

$$P_s^r(\Phi) \triangleq \mathrm{sign}^+\left[(\Phi - rI_n)(\Phi + rI_n)^{-1}\right]$$
$$= M\left[I_{n_1} \oplus 0_{n_2}\right]W = M_1 W_1 \qquad (3\text{-}3a)$$
$$P_f^r(\Phi) \triangleq \mathrm{sign}^-\left[(\Phi - rI_n)(\Phi + rI_n)^{-1}\right]$$
$$= M\left[0_{n_1} \oplus I_{n_2}\right]W = M_2 W_2 \qquad (3\text{-}3b)$$

where $P_s^r(\Phi) + P_f^r(\Phi) = M_1W_1 + M_2W_2 = I_n$.

It is obvious that $\mathrm{Rank}(P_s^r(\Phi)) = n_1$ and $\mathrm{Rank}(P_f^r(\Phi)) = n_2$.

Let matrix S be defined as

$$S \triangleq \mathrm{ind}\left[P_s^r(\Phi)\right] = \{s_1 s_2 \cdots s_{n_1}\} \in C^{n \times n_1} \qquad (3\text{-}4)$$

where S is a monic map which contains n_1 independent column vectors of $P_s^r(\Phi)$. These independent column vectors are selected from the n column vectors of $P_s^r(\Phi)$ and the number of independent vectors, n_1 is equal to the trace of $P_s^r(\Phi)$. Thus we obtain the results as follows:

(Lemma 3.1)

There exists a nonsingular matrix $H \in C^{n_1 \times n_1}$ such that

$$S = M_1 H \tag{3-5}$$

where M_1 are defined as in Eq.(2-2b).

Proof) Assume that S contains n_1 columns of $P_s(\Phi)$ with column indices k_i for $i=1,2,\ldots,n_1$. Then, from Eq.(3-4), we obtain

$$S = P_s(\Phi)E_1 = M_1 W_1 E_1 \triangleq M_1 H,\ E_1 \triangleq \left[e_n^{k_1},\ e_n^{k_2},\ \ldots,\ e_n^{k_n},\ \ldots,\ e_n^{k_{n_1}}\right] \in C^{n \times n_1} \tag{3-6}$$

where e_n is elementary column vectors for $i=1,2,\ldots,n_1$. According to Sylvester's inequality, we obtain Rank(H)=n_1 or H is nonsingular. (Q.E.D.)

The matrix S defined in Eq.(3-4) can be used for block triangular decomposition of the system (3-1) as follows:

(Theorem 3.1)

Let $\Phi \in C^{n \times n}$ and $\{|\sigma(\Phi)|\} \cap \{r_1, r_2 |\ r_1 < r_2\} = \emptyset$, where $\{r_1,\ r_2\} \in R$. Define

$$\bar{S}_1 \triangleq \bar{S}_{(r_1,r_2)} \triangleq \mathrm{ind}\left[P_f(\Phi)\right] = \mathrm{ind}\left[I_n - P_s(\Phi)\right] \in C^{n \times n_1} \tag{3-7a}$$

$$V_2 \triangleq V_{(r_1,r_2)} \triangleq \left\{\mathrm{ind}\left[P_s^T(\Phi)\right]\right\}^T \in C^{n_2 \times n} \tag{3-7b}$$

Assume that $n_1 + n_2 = n$ and $n_1 n_2 \neq 0$. Let

$$T \triangleq \begin{bmatrix} \bar{S}^{\#}_{(r_1,r_2)} \\ \hdashline V_{(r_1,r_2)} \end{bmatrix} \in C^{n \times n} \tag{3-8}$$

where $\bar{S}^{\#}_{(r_1,r_2)}$ is the left pseudo-inverse of $\bar{S}_{(r_1,r_2)}$. Then

$$T^{-1} = \left[\bar{S}_{(r_1,r_2)} \vdots\ V^{\#}_{(r_1,r_2)}\right] \tag{3-9}$$

where $V^{\#}_{(r_1,r_2)}$ is the right pseudo-inverse of $V_{(r_1,r_2)}$. Let

$$Z(k) = TX(k) \tag{3-10}$$

Then the block-triangularized system of (3-1) using Eq.(3-10) becomes

$$Z(k+1) = \begin{bmatrix} \Phi_R & \Phi_{RL} \\ 0_{n_2 \times n_1} & \Phi_L \end{bmatrix} Z(k) + \begin{bmatrix} \Theta_1 \\ \Theta_2 \end{bmatrix} u(k) \tag{3-11a}$$

$$y(k) = \left[C_1 \vdots C_2\right] Z(k) \tag{3-11b}$$

where

$$\Phi_R = \bar{S}^{\#}_{(r_1,r_2)} \Phi\ \bar{S}_{(r_1,r_2)},$$
$$\Phi_L = V_{(r_1,r_2)} \Phi\ V^{\#}_{(r_1,r_2)} \tag{3-11c}$$
$$\Phi_{RL} = \bar{S}^{\#}_{(r_1,r_2)} \Phi\ V^{\#}_{(r_1,r_2)},$$
$$\Theta_1 = \bar{S}^{\#}_{(r_1,r_2)} \Theta$$
$$\Theta_2 = V_{(r_1,r_2)} \Theta,\quad [C_1\ C_2] = C\left[\bar{S}_{(r_1,r_2)} V^{\#}_{(r_1,r_2)}\right]$$

and

$$\sigma(\Phi_L) = \{\lambda \,|\, \lambda \in \sigma(\Phi),\ r_1 < |\lambda| < r_2\} \tag{3-12a}$$

$$\sigma(\Phi_R) = \{\lambda \,|\, \lambda \in \sigma(\Phi),\ \lambda \in (\Phi_L)\} \tag{3-12b}$$

Proof) Theorem 3.1 can be shown in Ref.(8).

If more than two blocks are desired for the block-triangularization of Φ, an another sequential method is considered in the following theorem.

(Theorem 3.2)

Let $\Phi \in C^{n \times n}$ and $\{|\sigma(\Phi)|\} \cap \{r_i \in R;\ i=0,1,\ldots,k\} = 0$, where $r_0 < r_1 < r_2 \cdots < r_k$.

Also, let n_i be the number of $\sigma(\Phi)$ lying within the circular stripe (r_{i-1}, r_i) for $1 \le i \le k$ and $\sum_{i=1}^{k} n_i = n$. Further, define

$$m_i \triangleq n - \sum_{j=i+1}^{k} n_j\ ;\quad m_1 = n_1\ ;\quad m_k = n \tag{3-13}$$

$$\Phi_{R,k} \triangleq \Phi \tag{3-14}$$

$$\bar{S}_i \triangleq \mathrm{ind}\left[I_{m_{i+1}} - \mathrm{sign}^+_{(r_i, r_{i+1})}(\Phi_{R,i+1})\right] \in C^{m_{i+1} \times m_i} \tag{3-15a}$$

$$V_{i+1} \triangleq \{\mathrm{ind}\left[(\mathrm{sign}^+_{(r_i, r_{i+1})}(\Phi_{R,i+1}))^T\right]\}^T \in C^{n_{i+1} \times m_{i+1}} \tag{3-15b}$$

for $i=k-1, k-2, \ldots, 1$

Then, the cascade transformation matrices T_{k-1}, T_{k-2}, ... , T_1 can be formed as

$$T_1 = \begin{bmatrix} \bar{S}^{+}_{k-1} \\ \hdashline V_k \end{bmatrix} \in C^{n\times n} \;;\; \bar{S}^{\#}_{k-1} \in C^{m_{k-1}\times m_k} \;;\; V_k \in C^{n_k \times m_k} \qquad (3\text{-}16a)$$

$$T_2 = \begin{bmatrix} \bar{S}^{\#}_{k-2} & \mathbb{O}_{m_{k-2}\times n_k} \\ \hdashline V_{k-1} & \mathbb{O}_{n_{k-1}\times n_k} \\ \hdashline \mathbb{O}_{n_k \times m_{k-1}} & I_{n_k} \end{bmatrix} \in C^{n\times n}$$

$$;\; \bar{S}^{\#}_{k-2} \in C^{m_{k-2}\times m_{k-1}}$$

$$;\; V_{k-1} \in C^{n_{k-1}\times m_{k-1}} \qquad (3\text{-}16b)$$

$$T_{k-1} = \begin{bmatrix} \bar{S}^{\#}_{1} & \mathbb{O}_{n_1 \times (n-n_1-n_2)} \\ \hdashline V_2 & \mathbb{O}_{n_2 \times (n-n_1-n_2)} \\ \hdashline \mathbb{O}_{(n-n_1-n_2)\times m_2} & I_{(n-n_1-n_2)} \end{bmatrix}$$

$$;\; \bar{S}^{\#}_{1} \in C^{m_1 \times m_2}$$

$$;\; V_2 \in C^{n_2 \times m_2} \qquad (3\text{-}16c)$$

$$Z(k) = T_{k-1}T_{k-2} \cdots T_1 X(k) \qquad (3\text{-}17)$$

Then, the block-triangularized system of Eq.(3-1) using Eq.(3 -17) becomes

$$Z(k+1) = \Phi_T Z(k) + \Theta_T u(k) \qquad (3\text{-}18a)$$

$$y(k) = C_T X(k) \qquad (3\text{-}18b)$$

where

$$\Phi_T = T_{k-1} \cdots T_2 T_1 \Phi T_1^{-1} T_2^{-1} \cdots T_{k-1}^{-1}$$

$$= \begin{bmatrix} \Phi_{R,1} & \Phi_{R,12} & \Phi_{R,13} & \cdots & \Phi_{R,1k} \\ 0 & \Phi_{L,2} & \Phi_{L,23} & \cdots & \Phi_{L,2k} \\ 0 & 0 & \Phi_{L,3} & \cdots & \Phi_{L,3k} \\ \hdashline 0 & 0 & 0 & \cdots & \Phi_{L,k} \end{bmatrix} C^{n\times n} \qquad (3\text{-}18c)$$

$$\Phi_{R,1} = \bar{S}^{\#}_1 \Phi_{R,2} \bar{S}_1 \;,\; \Phi_{L,i} = V_i \Phi_{R,i} V^{\#}_i$$

for $i=k, k-1, \ldots, 2$

$$\Theta_T = T_{k-1} \cdots T_2 T_1 \Theta$$

$$= [\Theta_1 \; \Theta_2^T \cdots \Theta_k^T]^T \qquad (3\text{-}18d)$$

$$C_T = C T_1^{-1} T_2^{-1} \cdots T_{k-1}^{-1}$$

$$= [\, C_1 C_2 \cdots C_k \,] \qquad (3\text{-}18e)$$

and

$$\sigma(\Phi_{R,1}) = \{\lambda \mid \lambda \in \sigma(\Phi),\ r_0 < |\lambda| < r_1 \}$$

$$\sigma(\Phi_{L,i}) = \{\lambda \mid \lambda \in \sigma(\Phi),\ r_{i-1} < |\lambda| < r_i,\ 2 \le i \le k\}$$

Proof)

Theorem 3.2 can be proved using Theorem 3.1 and the definitions of S_i and V_i in Eq.(3-15).

4. Numerical Example

The fifth-order linearlized model of a distillation tower system was discretized with sampling time of 2 sec, and it can be represented by:

$$\Phi = \begin{bmatrix} 0.86849 & 0.03754 & 0.01485 & 0.00523 & 0.00824 \\ 0.04172 & 0.03519 & 0.05052 & 0.05059 & 0.85352 \\ 0.71374 & 0.00054 & 0.08031 & 0.03819 & 0.1346 \\ 0.46328 & 0.13092 & 0.10921 & 0.05842 & 0.33378 \\ 0.28531 & 0.10947 & 0.10454 & 0.06559 & 0.56948 \end{bmatrix}$$

$$\Theta = \begin{bmatrix} 0.00534 & 0.06989 & 0.09867 & 0.09085 & 0.02699 \\ -0.00334 & -0.05273 & -0.13139 & -0.14296 & -0.04867 \end{bmatrix}^T$$

$$C = [\, 1. \; 0. \; 0. \; 0. \; 0. \,]$$

The eigenvalues of Φ are $\lambda_1=0.00022$, $\lambda_2=0.00337$, $\lambda_3=0.1662$, $\lambda_4=0.85946$ and $\lambda_5=0.9797$. Find the block-triangularized decomposition of this system map in which the first block matrix contains λ_1 and λ_2, the second consists of λ_3, and the third contains λ_4 and λ_5. Thus we choose $r_1=0.05$ and $r_2=0.5$.

Since the number of submatrices to be determined is three, k=3. According the Theorem 3.2, the first transformation matrix T_1 Eq.(3-16a) is

$$T_1 = \begin{bmatrix} \bar{S}^{\#}_2 \\ \hdashline V_3 \end{bmatrix}$$

where from Eq.(3-15a) and (3-15b), we obtain

$$\bar{S}_2 = \begin{bmatrix} 0.058 & 0.02314 & 0.04528 \\ 0.11149 & 0.08364 & 0.04558 \\ -0.86152 & -0.17632 & 0.94757 \\ -0.58807 & -0.41985 & -0.05248 \\ -0.31993 & -0.66517 & -0.0529 \end{bmatrix}$$

$$V_3 = \begin{bmatrix} 0.942 & -0.2314 & 0.04528 & 0.01899 & 0.00586 \\ -0.11149 & 0.91636 & 0.04558 & 0.06391 & 0.05892 \end{bmatrix}$$

Thus we obtain

$$\Phi_{R,2} = \begin{bmatrix} 0.17937 & 0.09142 & -0.11126 \\ 0.05369 & 0.03227 & -0.02719 \\ 0.06845 & 0.03528 & -0.04184 \end{bmatrix},$$

$$\Phi_{L,3} = \begin{bmatrix} 0.91612 & 0.02727 \\ 0.13078 & 0.92241 \end{bmatrix}$$

$\Phi_{L,3}$ is desired third block matrix which contains λ_4 and λ_5.

Using similar procedures to the reduced-order model, $\Phi_{R,2}$, we obtain k=2. Thus

$$\mathrm{sign}_{(r_1)}(\Phi_{R,2}) = \begin{bmatrix} 1.1332 & 1.1171 & -1.2869 \\ 0.68896 & -0.6392 & -0.41564 \\ 0.81877 & 0.42878 & -1.494 \end{bmatrix}$$

$$\mathrm{sign}_{(r_2)}(\Phi_{R,2}) = -I_3$$

$$\tilde{S}_1 = \begin{bmatrix} -0.06658 & -0.34448 & -0.40939 \\ -0.55856 & 0.8196 & -0.21439 \end{bmatrix}^T$$

$$V_2 = \begin{bmatrix} 1.0666 & 0.55856 & -0.64346 \end{bmatrix}$$

$$\Phi_{R,1} = \tilde{S}_1^{\#}\Phi_{R,2}\tilde{S}_1 = \begin{bmatrix} 0.00103 & -0.0005 \\ -0.00391 & 0.00258 \end{bmatrix}$$

$$\Phi_{L,2} = V_2\Phi_{R,2}V_2^{\#} = \begin{bmatrix} 0.16619 \end{bmatrix}$$

and

$$T_2 = \left[\begin{array}{c|c} \tilde{S}_1^{\#} & \Phi\ 2\times 2 \\ \hline V_2 & \Phi\ 1\times 2 \\ \hline \Phi_{2\times 3} & I_2 \end{array}\right]$$

Thus, the block-triangularized system map becomes

$$\Phi_T = T_2 T_1 \Phi T_1^{-1} T_2^{-1}$$

$$= \left[\begin{array}{cc|c|cc} 0.00103 & -0.0005 & -0.24339 & 0.46449 & 0.60042 \\ -0.00391 & 0.00258 & -0.10232 & 0.29244 & -0.71849 \\ \hline 0. & 0. & 0.16619 & -0.92495 & -0.43124 \\ \hline 0. & 0. & 0. & 0.91612 & 0.02727 \\ 0. & 0. & 0. & 0.13078 & 0{:}92241 \end{array}\right]$$

$$\Theta_T = T_2 T_1 \Theta$$

$$= \left[\begin{array}{cc|c|cc} 0.18432 & 0.24649 & -0.12955 & 0.00998 & 0.03898 \\ -0.29368 & 0.01265 & 0.14811 & -0.00774 & -0.06345 \end{array}\right]^T$$

$$C_T = C T_1^{-1} T_2^{-1}$$

$$= \left[\begin{array}{cc|c|cc} 0.00671 & -0.00373 & 0.05577 & 1.61 & 0.02157 \end{array}\right]$$

Note that $\Phi_{R,1}$ is the first block diagonal matrix which contains λ_1 and λ_2, while the second block diagonal matrix is λ_3.

5. Conclusions

In this paper, a block triangular decomposition algorithm for a linear large-scale multivariable discrete system is proposed by the matrix sign function and generalized matrix sign function on the Z-plane. The proposed decomposition algorithm will be facilitate the study of the effects on the system when the poles and zeros are varied.

The main contributions of this paper are:

(1) There is no need to find the eigenvectors of system matrix in this algorithm.

(2) This algorithm do not need a permutation and rescaling procedure.

(3) There is no need to solve algebraic equations.

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