ON THE SELECTION OF INPUT VARIABLES TO BE RETAINED IN A REDUCED-ORDER MODEL

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Abstract. This paper presents the choice of appropriate sets of input variables for large-scale linear multivariable systems. It is shown that the selection of a good set of input variables for control may become important when both strong and weak input variables are available. The transmission of information from the inputs to the outputs is investigated and appropriate scaling procedures to derive a scaled input matrix are proposed. Singular value decomposition methods facilitate the quantification of the system's excitation stemming from the various input variables, and thus the selection of an appropriately strong and orthogonal set of input variables. The need for and the implementation and benefits of reducing the number of input variables are illustrated by a large-scale steam generator model of a real process.

Introduction

The number of input variables in large-scale systems is often determined by modeling or instrumentation considerations. With a larger number of input variables there seem to be more independent possibilities of exciting the system or correcting deviations between set-point and controlled variables. The situation is more clear cut with respect to order reduction. The presence of strong and weak input variables in a model is often the cause of difficulties in the application of reduced order models for control purposes/1/. The coupling between a weak input and the various state or output variables can be poorly approximated in the reduced-order model because strong input variables receive preferential treatment with a criterion, for example, trying to minimize the output error. The presence of poorly approximated input-output couplings in a reduced-order model may have disastrous consequences when the controller is designed on the basis of the lower-order model/2/. Furthermore, with many reduction methods/3/ requiring static agreement between original and reduced models, the greater the number of input variables, the lesser the degrees of freedom for the reduction process. Finally, a larger number of input variables requires that more couplings between inputs and outputs be described. This usually leads to larger model models. Hence, a reduction in the number of input variables seems desirable with systems possessing a relatively large number of inputs of various strengths. This paper discusses the need for analyzing all potential inputs in order to detect both weak,and similarly acting inputs. Since a direct inspection of input/output gains is not appropriate,this study proposes a careful investgation of the systems excitation through the various inputs. The objective is to reduce the number of input variables while still preserving the controllability of the system. Ideally,the retained most efficient input variables are strong and such that each of them excites the modes in a way orthogonal to the others. It may be worth mentioning here that a reduction in the number of input variables is not aimed at squaring down the plant,but rather at improving the quality of reduced-order models and the performance of closed-loop systems.

Selection of Strong Input Variables

An attempt is made to quantify the importance of the various input variables in multivariable systems. Consider the linear deterministic,time-invariant state-space model,

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} \qquad (1)$$

$$\underline{y} = C\underline{x} \qquad (2)$$

with $\underline{x} \in R^n$, $\underline{u} \in R^p$ and $\underline{y} \in R^q$. The system given by equations(1) and (2) is assumed to be both controllable and obserable. It is furthermore assumed that the input and output variables have been scaled in such a way that the numerical values of the deviation variables are representative of their relative importance as judged by the final user of the system. The scaled variables may represent a cost term or a conserved quantity,such as energy or mass and therefore can be compered numerically to each other. That assumption is primordial in this study and will be used again below.

The dynamic system given by equations (1) and (2) can be described by the transfer function matrix:

$$T(s) = C\ (sI-A)^{-1}B \qquad (3)$$

or in modal form

$$T\ (s) = \tilde{C}\ (sI-\Delta)^{-1}\tilde{B} \qquad (4)$$

with $\tilde{C} = CM$, $\tilde{B} = M^{-1}B$ and $\Delta = M^{-1}AM$. Equation(4) clearly shows that the transmission of information from the inputs to the outputs.

The matrix B indicates both the magnitude and the direction in modal space of the excitation from the various input variable. However,the analysis of the inputa by considering the matrix $\tilde{B}$ alone requires the dynamics and the composition to be similar for all the modes,i.e.the modal basis must be scaled. Inorder to find a set of input variables which contains the strongest input variables in terms of the norm of T(s), the scaling procedure is proposed:The mode dynamics are normalized,so as to exhibit a unity steady-state gain. This is justified by the fact that,for most dynamic systems,the transmission of information through the modal space is strongest at low frequencies. For highly oscillatory systems, the resonance frequency may be chosen for purpose of normalizing the dynamics.

The matrix $\hat{C}$ is post-multiplied with a diagonal matrix D whose objective is to scale the modes so as to make them equally well observed from the output vector $\underline{y}$.

The matrix $\tilde{C}$ can be normalized to make its codition number minimal,i.e.

$$\hat{C} = \tilde{C}\ D \qquad (5)$$

Minimizing the condition number of $\hat{C}$ makes the effects of the different modes on the outputs as comparable as possible.

Equation (4) can be extended by the normalizing matrices D and $(-\Delta)$ as follows:

$$T(s) = \tilde{C}DD^{-1}(sI-\Delta)^{-1}(-\Delta)(-\Delta)^{-1}\tilde{B} \qquad (6)$$

Let

$$\hat{B} = D^{-1}(-\Delta)^{-1}\tilde{B} \qquad (7)$$

Therefore,

$$T(s) = \hat{C}\ (sI-\Delta)^{-1}(-\Delta)\hat{B} \qquad (8)$$

The matrix $\hat{B}$ (eq.(7)) describes the excitation of the normalized modes. The modes are called normalized because they all exhibit the same maximal contribution to the dynamics of the system and the modal space is scaled as proposed above. Consequently $\hat{B}$ contains information about the magnitude and direction of the excitation of the normalized modes from the input variables.

An analysis of the singular values and singular vectors of $\hat{B}$ will be used to select the dominant input variables:

$$\hat{B} = U\Lambda W^H = U \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} w_1^H \\ w_2^H \end{bmatrix} \qquad (9)$$

where λ_1 contains the dominant singular values of $\hat{B}$. The column vectors of W provide an orthonormal coordinate system for viewing the input variables. The element w_{ij} of W represents the projection of the i-th input along the j-th coordinate. The strongest input direction is the one associated with the largest singular value. w_1 spans the subspace of the input space R^p which is mapped on to the normalized modes via the dominant singular values.

Steam Generator Model as an Example

The 20th-order model of a 500 MW steam generator with nine inputs and nine outputs can be found in /1/. Following the procedure derived above,and using the mode weighting that equalizes the norm of the column vectors in $\hat{C}$,the inputs were analyzed through decomposition of the $\hat{B}$ matrix. The singular values of $\hat{B}$ read : 54.2,50.1,24.5 9.2,5.7,3.8,1.2,and 0.8×10^{-5},indicating that the poorly conitioned input space has three dominant input variables: u_8,u_2 and u_7, four less important u_4,u_3,u_5 and u_1, and two weak ones u_9 and u_6.

7th-order reduced models derived using modal method and retaining only three or five input variables were compared to the original 20th-order model. Figures 1 and 2,indicate the response of x_1 to a unit step perturbation of a strong and weak input,u_8 and u_6,respectively. It is seen that the strong coupling x_1/u_8 is relatively better approximated than the weak coupling x_1/x_6. Another interesting feature is the fact that the approximations tend to worsen as the number of inputs considered in the models gets larger the number of degrees of freedom for order reduction is decreased as more stationarily conditions need to be satisfied. These results lead one to expect that a controller designed on the basis of a seventh-order model with three input variables/4/is more appropriate to control the 20th-order model than when five input variables are considered.

Conclusions

This paper has demonstrated the need for analyzing the potential input variables of a large-scale linear multivariable system for both order reduction and controller design. It was shown that the number of input variables can often be reduced without affecting much the controllability and observability properties of the process. Furthermore,a large reduction in the condition number of a plant can be achieved by discarding nondominant input variables. This may lead to improved robustness characteristics for the controlled plant since the controller is based on uncertain models. The importance of reducing the number of input variables in some cases was also clearly demonstrated with regard to order reduction. The dominant input variables can be selected according to a well defined procedure which attempts to keep the systeḿs controllability as strong as possible.

References

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