기계적 매니퓰레이터의 시변 물체 회피에서의 제약조건인식

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Constraints Identification in Time-varying Obstacle Avoidance for Mechanical Manipulators

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ABSTRACT

This paper addresses the identification of various constraints in time-varying obstacle avoidance for mechanical manipulators. The manipulator constraints include the smoothness constraint and torque constraint, while the environmental constraints include a motion priorty, a traveling time constraint, a path constraint, and a collison constraint. The inherent difficulties in combining these constraints are discussed with a suggestion for the purpose of time-varying obstacle avoidance.

1. INTRODUCTION

Most of the existing off-line path planning schemes which concern obstacle avoidance concentrate on the problem of avoiding fixed and stationary obstacles in a workspace. Since the locations of obstacles are fixed and stationary, the obstacle avoidance can be achieved through collision-free path planning schemes.

A time-varying obstacle is generally defined as an obstacle of which the position and orientation depend on time. There exist only few path planning schemes concerning the problem of avoiding the time-varying obstacles. E. Freund[3] analyzed the situation of two robots operating in a common workspace. Since one robot has the priority to move, the other may be considered as a time-varying obstacle. Motion commands for robots are stored in a database so that collision avoidance of two robots can be achieved. Fortune et al.[2] developed a useful algorithm for independent but synchronized motion of two Stanford arms. Erdmann et al.[1] used a configuration space-time technique to represent the constraints imposed on the moving object. The planner represented the space-time by using two dimensional slices which were then searched for a collision-free path. Tournassoud [6] presented a local method for collision avoidance based on the existence of separating hyperplanes between two manipulators. Its application was then extended to the coordinated motion of two manipulators. In this paper, various constraints for time-varying obstacle avoidance will be identified and the inherent difficulties in combining these will be discussed.

2. VARIOUS CONSTRAINTS

The prior knowledge required for solving the time-varying obstacle avoidance problem includes the description of the obstacle movement, the initial and final location of the manipulator, the phisical manipulator constraints and the various environment constraints. The manipulator constraints (MC) include the velocity, acceleration, jerk and torque limitations. The environment constraints (EC) include a motion priority, a traveling time constraint for the manipulator, a path constraint for the manipulator, and the collision constraint between the manipulator and obstacles.

2.1 Traveling Time Constraint

The traveling time constraint is a specified time period in which a manipulator must complete the desired motion from one location to another. If this constraint does not exist and there exists a potential collision, a collision-free trajectory can always be found. In order to increase the productivity, the minimum-time collision-free trajectory is desired for the manipulator movement.

2.2 Motion Priority Constraint

The motion priority constraint is defined as the moving priority. If this constraint is given to the obstacle, we interpret that the manipulator involved needs to change its motion strategy for avoiding the potential collision with the time-varying obstacle. Since it is generally difficult and impractical to modify the trajectory of the time-varying obstacle, the priority constraint should be given to the time-varying obstacle. Three possibilities may exist for avoiding a potential collision: changing the path and trajectory of the manipulator, rearranging the velocity of the time-varying obstacle during its movement, or modifying the path and trajectory for both the manipulator and the obstacle.

2.3 Manipulator Path Constraint

The manipulator path constraint is a constraint which restricts the maximum deviation from the pre-determined path. In order to simplify computation, we generally assume that a manipulator path and trajectory is composed of straight line segments. Thus, the initial location [$p(k_o)$, $\phi(k_o)$] and the final location [$p(k_f)$, $\phi(k_f)$] of a manipulator are given for each segment, where $p(k)$ and $\phi(k)$ specify the Cartesian position and the Euler angle of the manipulator hand at time k, respectively. Here, we used the index k to denote the time t=kT, where T is the sampling servo time period. The manipulator hand is required to move from the initial location to the final location in the Cartesian space along the straight line. The straight line equation that passes through these two locations is described by

$$p(k) = \lambda(k) \cdot (p(k_f) - p(k_o)) + p(k_o) \qquad (1)$$

$$\Phi(k) = \lambda(k) \cdot (\Phi(k_f) - \Phi(k_o)) + \Phi(k_o) \qquad (2)$$

where $0 \le \lambda(k) \le 1$, and k_o and k_f are the initial and the final discrete time indices, respectively.

2.4 Collision Constraint

It is assumed that the wrist of a robot is modeled as a sphere. The radius of the sphere is determined from the wrist geometry and the size of the object grasped. Then the space x(k) assumed to be the wrist must satisfy;

$$\| x(k) - p(k) \| \le r \qquad (3)$$

where r is the radius of a sphere model. The obstacle considered is also modeled as a sphere in the same way.

We assume that point $p_o(k)$ is the position of the obstacle at time k, where the subscript o denote the obstacle. Point $p_r(k)$ is the position of the robot at time k, where the subscript r denote the robot. We assume that the robot should follow the straight line path. A potential collision may occur if the sphere of the robot wrist intersects the sphere of the obstacle during its motion. The distance between two set points on the robot and the obstacle path must be greater than $r_o + r_r$ for collision avoidance, where r_o is the radius of the sphere model of the obstacle and r_r is the radius of the sphere model of the robot wrist. The portion that must be avoided by the robot at time k is the sphere of radius of r_o , centered at the point on the path of the obstacle at time k [4].

The equation of the straight line path of the time-varying obstacle is denoted as:

$$p_o(k) = p_o(k_o) + \lambda(k) \cdot (p_o(k_f) - p_o(k_o)). \qquad (4)$$

where $0 \le \lambda(k) \le 1$. Then, the existence of a potential collision is found by solving the following equation:

$$(r_o + r_r)^2 = \| p_r(k) - p_o(k) \|^2 . \qquad (5)$$

Replacing $p_o(k)$ by using Eq.(4), we have:

$$(r_o + r_r)^2 = \| p_r(k) - p_o(k_o) \|^2 - 2\lambda(k) \cdot (p_r(k) - p_o(k_o)) \cdot (p_o(k_f) - p_o(k_o))^T + \lambda^2(k) \cdot \|p_o(k_f) - p_o(k_o)\|^2 \qquad (6)$$

Eq.(6) has three possible solutions: (1) Real roots don't exist; (2) Two real roots, $\lambda_1^c(k)$ and $\lambda_2^c(k)$ exist ($\lambda_1^c(k) > \lambda_2^c(k)$); and (3) Only one real double root $\lambda_1^c(k)$ exists. When no real root exists, there is no collision between the obstacle and the robot at time k . When two real roots exist, the collision exists and lengths range from $\ell_f \cdot \lambda_2^c(k)$ to $\ell_f \cdot \lambda_1^c(k)$, where ℓ_f is the total traveling length of the obstacles. When only one real double root exists, k marks the beginning or ending times of the collision. Thus, the constraint from the collision situation at time k can be written as:

$$\lambda_1^c(k) \le \lambda(k) \text{ or } \lambda_2^c(k) \ge \lambda(k). \qquad (7)$$

At time t = (k+1)T, Eq.(7) can be written as:

$$\lambda_1^c(k+1) \le \lambda(k+1) = \lambda(k) + \Delta\lambda(k). \qquad (8\text{-a})$$

or

$$\lambda_2^c(k+1) \ge \lambda(k+1) = \lambda(k) + \Delta\lambda(k). \qquad (8\text{-b})$$

Hence the collision constraint on $\Delta\lambda(k)$ can be expressed as:

$$\Delta\lambda(k) \ge \lambda_1^c(k+1) - \lambda(k) \text{ or } \Delta\lambda(k) \le \lambda_2^c(k+1) - \lambda(k). \qquad (9)$$

2.5 Smoothness Constraint

We first derive some useful definitions. The position p(k) and the Euler angle $\phi(k)$ of a manipulator can be represented by a 6 x 1 vector and described by:

$$\begin{pmatrix} p(k) \\ \phi(k) \end{pmatrix} = N(q(k)) = (N_1(q(k)), \ldots, N_6(q(k)))^T \qquad (10)$$

where N( ) is a 6 x 1 nonlinear vector function depending on the manipulator configuration. To initiate the discretized trajectory analysis, let us denote the sampling period for the servo control of the robot as T (usually 3 ms $\le$ T $\le$ 20 ms) and $\bar{q}(k)$ to represent angular displacement q(kT) :

$$q(kT) \cong \bar{q}(k) \ ; \ k = 0, 1, \ldots \qquad (11)$$

The velocity, the acceleration, and the jerk of a manipulator at time kT can be approximated respectively by:

$$\dot{q}(kT) \cong \frac{1}{T}(\bar{q}(k) - \bar{q}(k-1)) = \dot{\bar{q}}(k) \qquad (12)$$

$$\ddot{q}(kT) \cong \frac{1}{T^2}(\bar{q}(k) - 2\bar{q}(k-1) + \bar{q}(k-2)) = \ddot{\bar{q}}(k) \qquad (13)$$

$$w(kT) \cong \frac{1}{T^3}(\bar{q}(k) - 3\bar{q}(k-1) + 3\bar{q}(k-2) - \bar{q}(k-3)) = \bar{w}(K) \qquad (14)$$

where w(kT) denotes the jerk at time t= kT. For simplicity and brevity, we shall abuse notation and drop the super bar from the rest of the equations. That is,

$$q(k) \equiv \bar{q}(k) \ ; \ \dot{q}(k) \equiv \dot{\bar{q}}(k) \ ; \ \ddot{q}(k) \equiv \ddot{\bar{q}}(k) \ ; \ w(k) \equiv \bar{w}(k) \qquad (15)$$

All discretized control set points in

the joint variable space must be within certain limits to maintain the smoothness of the trajectory. The smoothness constraint on the joint trajectory set points can be stipulated by a velocity bound (VB), an acceleration bound (AB), and a jerk bound(JB). These three bounds are given respectively as [5] :

$$|\dot{q}_i(k)| \le \varepsilon_i^v \; ; \; \varepsilon_i^v > 0 \text{ and } i = 1, \ldots, 6 \quad (16)$$

$$|\ddot{q}_i(k)| \le \varepsilon_i^a \; ; \; \varepsilon_i^a > 0 \text{ and } i = 1, \ldots, 6 \quad (17)$$

$$|w_i(k)| \le \varepsilon_i^j \; ; \; \varepsilon_i^j > 0 \text{ and } i = 1, \ldots, 6 \quad (18)$$

where ε_i^v, ε_i^a, ε_i^j are the i^{th} element of 6-dimensional bound vectors for the manipulator (suppose the robot manipulator has 6 joints). The velocity bound (VB) and acceleration bound (AB) constrain the joint actuators from exceeding the maximum limits of the velocity and acceleration. The jerk bound (JB) reduces wear of joint actuators, and reduces excitation of vibrations. Hence, we impose VB, AB, and JB on the entire trajectory from one trajectory set point to another. Combining Eqs. (17) and (18), we have:

$$\ddot{q}_{i,min}(k) \le \ddot{q}_i(k) \le \ddot{q}_{i,max}(k) \; ; \; i=1,2, \ldots, 6 \quad (19)$$

where

$$\ddot{q}_{i,min}(k) = \max\left\{-\varepsilon_i^a, \; \ddot{q}_i(k-1) - \varepsilon_i^j \cdot T\right\} \quad (20)$$

$$\ddot{q}_{i,max}(k) = \min\left\{\varepsilon_i^a, \; \ddot{q}_i(k-1) + \varepsilon_i^j \cdot T\right\} \quad (21)$$

Similarly, we have the following equation from Eqs.(16) and (19) for the joint velocity constraint.

$$\dot{q}_{i,min}(k) \le \dot{q}_i(k) \le \dot{q}_{i,max}(k) \; ; \; i=1,2, \ldots, 6 \quad (22)$$

where

$$\dot{q}_{i,min}(k) = \max\left\{-\varepsilon_i^v, \; T \cdot \ddot{q}_{i,min}(k) + \dot{q}_i(k-1)\right\} \quad (23)$$

$$\dot{q}_{i,max}(k) = \min\left\{\varepsilon_i^v, \; T \cdot \ddot{q}_{i,max}(k) + \dot{q}_i(k-1)\right\} \quad (24)$$

Then, we can obtain the joint position constraint as:

$$q_{i,min}(k) \le q_i(k) \le q_{i,max}(k) \; ; \; i=1,2, \ldots, 6 \quad (25)$$

where

$$q_{i,min}(k) = T \cdot \dot{q}_{i,min}(k) + q_i(k-1) \quad (26)$$

$$q_{i,max}(k) = T \cdot \dot{q}_{i,max}(k) + q_i(k-1)$$

As used in Eqs.(1) and (2), we define

$$\bar{p}(k) \cong p(kT) \; ; \; \bar{\phi}(k) \cong \phi(kT) \quad (27)$$

and drop the super bar from $\bar{p}(k)$ and $\bar{\phi}(k)$ for notational consistency. Let us assume that at the time $t = (k-1)T$, $p(k-1)$ and $\phi(k-1)$ are given and that they are within the physical bounds from the straight line requirements , the smoothness and torque constraints at $t = (k-1)T$. Then, we would like to find the next set point, $[p(k), \phi(k)]$, such that it is again within the smoothness and torque constraints and must lie on the specified straight line path exactly. From Eq. (10),

$$\begin{pmatrix}\Delta p(k)\\ \Delta\phi(k)\end{pmatrix} \cong [\nabla N(q(k))] \cdot \Delta q(k) \quad (28)$$

where $\Delta p(k) = p(k) - p(k-1)$, $\Delta\phi(k) = \phi(k) - \phi(k-1)$, $\Delta q(k) = q(k) - q(k-1)$ and the elements of $[\nabla N(q(k))]$ are found to be

$$[\nabla N(q(k))]_{ij} = \frac{\partial N_i(q(k))}{\partial q_j(k)} \; ; \; i,j = 1,2,\ldots,6 \quad (29)$$

Combining Eqs. (1), (2) and (28) at time $t = (k-1)T$ and $t = kT$, we have:

$$[\nabla N(q(K))] \cdot \Delta q(K) \cong \Delta\lambda(k)\begin{pmatrix}p(k_f)-p(k_o)\\ \phi(K_f)-\phi(k_o)\end{pmatrix} \quad (30)$$

where $\Delta\lambda(k) = \lambda(k) - \lambda(k-1)$.
If $[\nabla N(q(k))]$ is non-singular at time $t = kT$, then

$$\Delta q(k) \cong \Delta\lambda(k) Q(k) \quad (31)$$

where

$$Q(k) = [\nabla N(q(k))]^{-1}\begin{pmatrix}p(t_f)-p(t_o)\\ \phi(t_f)-\phi(t_o)\end{pmatrix} = [Q_1(k), \ldots, Q_6(K)]^T \quad (32)$$

Physically, $Q(k)$ is the vector which relates the angular displacement of each joint with $\Delta\lambda(k)$ of a given straight line. Since the servo time interval T is very small, let us assume that, for the joint position at $t = kT$,

$$q(k) = q(K-1) + \Delta q(k) \quad (33)$$

Then using Eq. (31), we have,

$$q(k) \cong q(k-1) + \Delta\lambda(k) Q(k) \quad (34)$$

Combining Eqs. (25) and (34), we have:

$$\Delta\lambda_i^-(k) \le \Delta\lambda(k) \le \Delta\lambda_i^+(k) \quad (35)$$

where

$$\Delta\lambda_i^-(k) = \frac{T \cdot \dot{q}_{i,min}(k)}{Q_i(k)} \; ; \; \Delta\lambda_i^+(k) = \frac{T \cdot \dot{q}_{i,max}(k)}{Q_i(k)} \quad (36)$$

for i = 1, 2, ..., 6.

2.6 Torque Constraint

In general, the dynamic behavior of a robot can be described by the Lagrange-Euler equations of motion as

$$\tau(kT) \cong \bar{\tau}(k) = [D(q(k))]\ddot{q}(k) + h(q(k), \dot{q}(k)) + c(q(k)) \quad (37)$$

where $\bar{\tau}(k)$ is a 6x1 applied torque vector for joint motors, $c(q(k))$ is a 6x1 gravitational force vector, $h(q(k), \dot{q}(k))$ is a 6x1 Coriolis and centrifugal force vector, and $[D(q(k))]$ is a 6x6 acceleration related matrix. The approximate equality results from the discrete-time approximation of q, $\dot{q}$ and $\ddot{q}$. Hereafter we omit the super bar from $\bar{\tau}(k)$. If $q(k)$, $\dot{q}(k)$ and $\ddot{q}(k)$ are given, the required piecewise joint torques can be computed by treating the equations of motion as an inverse dynamics problem. In a simplified notation,

$$\tau(k) = [D_k]\ddot{q}(k) + h_k + c_k \quad (38)$$

where $D_k = D(q(k))$, $h_k = h(q(k), \dot{q}(k))$ and $c_k = c(q(k))$, we assume:

$$\dot{q}(0) = 0 \; ; \; \ddot{q}(0) = 0 \; ; \; w(0) = 0 \quad (39)$$

at $k_o = 0$. Let us further assume that the torques generated from Eq. (38) are constrained by limits that are dependent on the joint position (due to the manipulator actuator geometry) and on the joint velocity (due to the back electromotive force terms or other actuator effects) as,

$$\tau_{i,min}(q(k),\dot{q}(k)) \le \tau_i(k) \le \tau_{i,max}(q(k),\dot{q}(k)) \qquad (40)$$

or in a simplified notation as,

$$\tau_{i,min}(k) \le \tau_i(k) \le \tau_{i,max}(k). \qquad (41)$$

Since the joint torque is represented by Eq. (38), we have

$$\tau_{i,a}^{-}(k) \le D_{i,k}\ddot{q}(k) \le \tau_{i,a}^{+}(k) \qquad (42)$$

where $D_{i,k}$ represents the i^{th} row of the matrix D_k, and $\tau_{i,a}^{-}(k)$ and $\tau_{i,a}^{+}(k)$ are written as:

$$\tau_{i,a}^{-}(k) = \tau_{i,min}(k) - h_{i,k} - c_{i,k} \qquad (43)$$

$$\tau_{i,a}^{+}(k) = \tau_{i,max}(k) - h_{i,k} - c_{i,k} \qquad (44)$$

where $h_{i,k}$ and $c_{i,k}$ are the i^{th} element of the vectors h_k and c_k, respectively.
Combining Eqs. (34) and (42), we have:

$$\Delta\lambda_{i,min}(k) \le \Delta\lambda(k) \le \Delta\lambda_{i,max}(k) \; ; \; i = 1,\ldots,6 \qquad (45)$$

where

$$\Delta\lambda_{i,min}(k) = \frac{T^2\tau_{i,a}^{-}(k) + D_{i,k}q(k-1) - D_{i,k}q(K-2)}{D_{i,k}Q(k)} \qquad (46)$$

$$\Delta\lambda_{i,max}(k) = \frac{T^2\tau_{i,a}^{+}(k) + D_{i,k}q(k-1) - D_{i,k}q(k-2)}{D_{i,k}Q(k)} \qquad (47)$$

$\Delta\lambda_{i,min}(k)$ and $\Delta\lambda_{i,max}(k)$ are the maximum and minimum constraints for $\Delta\lambda(k)$ from the i^{th} joint torque constraint.

3. TIME-VARYING OBSTACLE AVOIDANCE

When there is only a time-invariant obstacle, the existence of a collision-free path only depends on the geometry of the obstacle and the manipulator movement. However, when a time-varying obstacle exist, the existence of collision-free path depends not only on the geometry of the obstacle and the manipulator movement, but also on the various constraints and dynamic information of the time-varying obstacle. Hence, determining whether a trajectory satisfies all the constraints or not becomes the difficult part in finding a collision-free trajectory. It is notable that the traveling time constraint is mainly from the job requirement of a manipulator. Thus, if the manipulator is allowed to travel the same path for a longer period of time than the initially specified traveling time, it may move very slowly on the path without violating both the smoothness constraint and the torque constraint. Then, the collision constraint is the only one to be satisfied for the purpose of collision avoidance. In the limiting case, the manipulator may delay its movement until the scheduled path is cleared from any obstacles. This case corresponds to the situation that the manipulator stays at the starting location in order to make the collision constraint to be the whole straight line path. Thus, a simple delay of the manipulator motion could be a solution which avoids the expense of enormous computational burden.

4. CONCLUSION

Various constraints necessary for avoiding a collision between a manipulator and a moving obstacle were investigated and identified. It was recognized that the collision avoidance problem associated with the time-varying obstacle is not always solvable. A practical suggestion was discussed finally with the inherent difficulties in time-varying obstacle avoidance.

5. REFERENCES

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