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선형 이산치 시스템의 Robust Servomechanism 문제

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Robust Servomechanism Problem for Linear Discrete Systems

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Abstract

A method for designing a robust tracking controller for linear discrete systems is investigated. Only the observable variables are to be used in the controller synthesis. To insure the robustness, the system is augmented by a compensator at the output side. Then a feedback controller is designed using delayed values of the observable variables for the augmented system. The delay times are chosen to minimize the effect of measurement accuracy and/or noise.

1. Introduction

The main objective of the linear control theory is to design a controller which forces the system output to follow a given reference input with zero steady state error while satisfying certain transient conditions. There are several schemes available for solving the problem of tracking and disturbance rejection. Using the composite observer of the system and the disturbance, there is the so-called disturbance accomodating controller[1]. To assure that the controlled system is stabilizable and achieves robust control, Davison introduced two compensators called servocompensator and stabilizing compensator[2]. Ferreira and Desoer follow the design of Davison using the method of the state space in the frequency domain[3,4]. But, the above mentioned design methods do not provide means to prevent the problems of finite output measurement accuracy and excessively large control signals which may cause undesirable output responses due to saturation effect.

In this paper, the state variable reconstruction method[5] is used for designing a robust tracking and disturbance rejecting controller. The reference input and the disturbance input are assumed to be modeled by the state equations. The resulting controller is synthesized as a feedback of the available variables only.

2. Problem Statement

Consider a linear time invariant discrete-time control system

$$\begin{aligned} x(k+1) &= A\,x(k) + B\,u(k) + E\,d(k) \\ y(k) &= C\,x(k) \\ z(k) &= D\,y(k) \end{aligned} \qquad (1)$$

where the state vector $x(k)$, the observable vector $y(k)$, the output vector $z(k)$, the control vector $u(k)$ and the disturbance vector $d(k)$ have dimensions of n, m, q, r and m_d respectively. The constant matrices A, B, C, D and E have the appropriate dimensions. It is assumed throughout this paper that $\det(A) \neq 0$, the system is (A,B) controllable, (A,DC) observable and the matrix C has a full rank.

Let $z_{ref}(k)$ be the reference input vector which the output vector $z(k)$ is to follow. It is assumed that $z_{ref}(k)$ and $d(k)$ are given by the following state equations,

$$\begin{aligned} x_r(k+1) &= A_r\,x_r(k) \\ z_{ref}(k) &= C_r\,x_r(k) \end{aligned} \qquad (2)$$

$$\begin{aligned} x_d(k+1) &= A_d\,x_d(k) \\ d(k) &= C_d\,x_d(k) \end{aligned} \qquad (3)$$

where the reference state vector $x_r(k)$ and the disturbance state vector $x_d(k)$ have dimensions of n_r and n_d, (A_r,C_r) and (A_d,C_d) are observable pairs and $x_r(0)$ and $x_d(0)$ are not known a priori.

The problem is to synthesize a feedback controller, using only the observable variables, such that $z(k)$ of the system (1) follows $z_{ref}(k)$ without error in steady state for all $d(k)$. The controller should be robust in the sense that as long as the closed-loop system remains stable, the output response, with finite output measurement accuracy and the limit on control magnitude, tracks the reference input with zero steady state error for any variations in the system parameters.

3. Tracking and Disturbance Rejecting Controller Using Observable Variables

Let $e(k)$ be the error vector, that is, $e(k) = z(k) - z_{ref}(k)$. For $z_{ref}(k)$ and $d(k)$, let f_{Ar} and f_{Ad} be the minimal polynomials of A_r and A_d, and let f_{ArAd} be the least common multiple of f_{Ar} and f_{Ad}. Denote

$$f_{A_rA_d} = z^p + f_{p-1}z^{p-1} + \cdots + f_1 z + f_0.$$

Define the augmentation, which has the error vector as input, by

$$x_c(k+1) = A_c\, x_c(k) + B_c\, e(k) \qquad (4)$$

where

$$A_c = \begin{bmatrix} R_a & 0 & \cdot & \cdot & 0 \\ 0 & R_a & \cdot & \cdot & 0 \\ \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & & \cdot \\ 0 & 0 & \cdot & \cdot & R_a \end{bmatrix}$$

$$B_c = \begin{bmatrix} R_b & 0 & \cdot & \cdot & 0 \\ 0 & R_b & \cdot & \cdot & 0 \\ \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & & \cdot \\ 0 & 0 & \cdot & \cdot & R_b \end{bmatrix}$$

with

$$R_a = \begin{bmatrix} 0 & 1 & \cdot & \cdot & 0 & 0 \\ \cdot & \cdot & & & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & 0 & 0 \\ -f_0 & -f_1 & \cdot & \cdot & -f_{p-2} & -f_{p-1} \end{bmatrix} \qquad R_b = \begin{bmatrix} 0 \\ \cdot \\ 0 \\ 1 \end{bmatrix}$$

Then the augmented system (Fig. 1) is represented by

$$\begin{aligned} \bar{x}(k+1) &= \bar{A}\,\bar{x}(k) + \bar{B}\,u(k) + B_1\, z_{ref}(k) + B_2\, d(k) \\ \bar{y}(k) &= \bar{C}\,\bar{x}(k) \\ z(k) &= \bar{D}\,\bar{y}(k) \end{aligned} \qquad (5)$$

where

$$\bar{x}(k) = \begin{bmatrix} x(k) \\ x_c(k) \end{bmatrix}, \quad \bar{y}(k) = \begin{bmatrix} y(k) \\ x_c(k) \end{bmatrix}, \quad \bar{A} = \begin{bmatrix} A & 0 \\ B_cDC & A_c \end{bmatrix},$$

$$\bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \; B_1 = \begin{bmatrix} 0 \\ -B_c \end{bmatrix}, \; B_2 = \begin{bmatrix} E \\ 0 \end{bmatrix}, \; \bar{C} = \begin{bmatrix} C & 0 \\ 0 & 1 \end{bmatrix}, \; \bar{D} = [D \;\; 0]$$

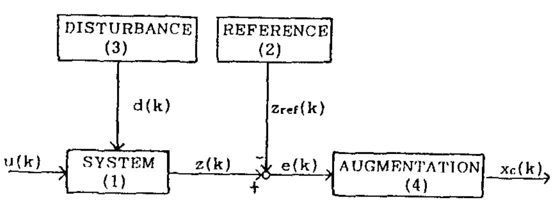

Fig. 1. Augmented System

It can be shown as in [3,4] with slight modification that the system(5) with $z_{ref}(k) = 0$ and $d(k) = 0$,

$$x(k+1) = A\, x(k) + B\, u(k), \qquad (6)$$

is controllable and, for a stable control law, asymptotic tracking and disturbance rejection hold if

$$\text{rank}\begin{bmatrix} zI-A & B \\ DC & 0 \end{bmatrix} = n+q, \qquad \forall z \in \sigma(A_c) \qquad (7)$$

where $\sigma(A_c)$ is the spectrum of A_c.

The stabilizing control

$$u(k) = K\, x(k) + K_c\, x_c(k) \qquad (8)$$

requires the feedback of the entire state vector. In most systems of interest, it is either impossible or impractical to measure the complete state vector directly. Therefore, $x(k)$ will be replaced by the available variables in the feedback controller. Generally the system can not be stabilized by a feedback controller using only the observable state variables. Therefore, delayed values of the observable variables are introduced in the feedback.

For the linear system

$$\begin{aligned} x(k+1) &= A\, x(k) + B\, u(k) \\ y(k) &= C\, x(k) \end{aligned} \qquad (9)$$

with the time delays of $0 < h_1 < h_2 < \ldots < h_l$, define

$$P = \begin{bmatrix} C \\ CA^{-h_1} \\ \cdot \\ CA^{-h_l} \end{bmatrix},$$

$$q(k) = \begin{bmatrix} y(k) \\ y(k-h_1) + \sum_{j=1}^{h_1} CA^{j-h_1-1}B\, u(k-j) \\ \cdot \\ \cdot \\ y(k-h_l) + \sum_{j=1}^{h_l} CA^{j-h_l-1}B\, u(k-j) \end{bmatrix}$$

where l is such that the rank of P is n. Then the results in [5] state that

$$x(k) = (P^TP)^{-1}P^T\, q(k).$$

Thus the control (8) can be expressed as

$$u(k) = K(P^TP)^{-1}P^T\, q(k) + K_c\, x_c(k) \qquad (10)$$

using a known vector $q(k)$ and an externally defined vector $x_c(k)$.

To find the control(10) one must determine the time delay values h_i. The only condition is that the rank of the matrix P is n. And such time delays exist whenever the given system is observable. However, if large time delays are chosen, then the system responses are slow. Also for h_l, there are h_l delayed control vector terms. On the other hand, in case of small sampling time, the control magnitude becomes large and feedback control tends to be sensitive to the measurement noise. Therefore, time delays should be chosen such that $|y(k) - y(k-h_i)|$ is large enough compared to the accuracy limit of the output measurement.

4. Example

For an a.c. motor driven third order position control system with a sampling period T = .001

$$x(k+1) = \begin{bmatrix} 1 & .1*10^{-2} & .5*10^{-6} \\ 0 & 1 & .1*10^{-2} \\ 0 & 0 & 1.0016 \end{bmatrix} x(k) + \begin{bmatrix} .198*10^{-7} \\ .595*10^{-4} \\ .1191 \end{bmatrix} u(k)$$

$$+ \begin{bmatrix} .5*10^{-7} \\ .1*10^{-3} \\ 0 \end{bmatrix} d(k)$$

$$\begin{aligned} y(k) &= [\,1 \;\; 0 \;\; 0\,]\; x(k) \\ z(k) &= y(k) \end{aligned}$$

assume that

$$z_{ref}(k) = r_0 + r_1 k$$
$$d(k) = d_0.$$

Then

$$f_{ArAd} = (z-1)^2,$$

and the augmentation $x_c(k+1) = A_c x_c(k) + B_c e(k)$ is given by

$$A_c = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}, \quad B_c = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Using an equivalent transformation, two discrete-time integrators

$$x_4(k+1) = x_4(k) + x_5(k)$$
$$x_5(k+1) = x_5(k) + z(k) - z_{ref}(k)$$

are added for the augmentation. The resulting system is given by

$$\bar{x}(k+1) = \begin{bmatrix} 1 & .1*10^{-2} & .5*10^{-6} & 0 & 0 \\ 0 & 1 & .1*10^{-2} & 0 & 0 \\ 0 & 0 & 1.0016 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix} \bar{x}(k)$$

$$+ \begin{bmatrix} .198*10^{-7} \\ .595*10^{-4} \\ .1191 \\ 0 \\ 0 \end{bmatrix} u(k) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} z_{ref}(k) + \begin{bmatrix} .5*10^{-7} \\ .1*10^{-3} \\ 0 \\ 0 \\ 0 \end{bmatrix} d(k)$$

To reconstruct the state variables x_2 and x_3, let $h_1 = 1$ and $h_2 = 2$. Then

$$x(k) = (P^T P)^{-1} P^T q(k)$$

$$= \begin{bmatrix} y(k) \\ .15*10^4 y(k) - .2*10^4 y(k-1) + .5*10^3 y(k-2) \\ + .3*10^{-4} u(k-1) + .1*10^{-4} u(k-2) \\ .1*10^7 y(k) - .2*10^7 y(k-1) + .1*10^7 y(k-2) \\ + .1u(k-1) + .02u(k-2) \end{bmatrix}$$

For the state feedback control

$$u(k) = \bar{K}\,\bar{x}(k)$$
$$J = \tfrac{1}{2} \Sigma_{k=0}^{\infty} [\ \bar{x}^T(k) Q \bar{x}(k) + u^T(k) R u(k)\ ]$$
$$Q = \mathrm{diag}\ [\ 1,\ 0,\ 0,\ .0001,\ 1000\ ]$$
$$R = 10,$$

the optimal control is found to be

$$u(k) = -[.85*10^6 y(k) - .17*10^7 y(k-1) + .82*10^6 y(k-2) + .08u(k-1) + .016u(k-2) + .095x_4(k) + 17.2x_5(k)].$$

The system response to the unit step reference input when $d(k) = .1$ is shown in Fig. 2. However, with the output measurement accuracy limit of .0001, the response to the unit step response is unsatisfactory as shown in Fig. 3. The output response with the increased time delays of $h_1 = 5$, $h_2 = 10$ is shown in Fig. 4. The system response and error to the reference input $z_{ref}(k) = 1 + k$ with the measurement accuracy of .0001, $h_1 = 5$, $h_2 = 10$ and $d(k) = .1$ are shown in Fig. 5 and Fig. 6.

5. Conclusion

The robust tracking problem of linear discrete-time system is investigated. The method is based on the state variable reconstruction for the unobservable variables. The controller is robust in the sense that as long as the closed-loop system remains stable, the output response, with finite measurement accuracy and the limit on control magnitude, tracks the reference input with zero steady state error for any variations in the system parameters. When the responses are unacceptable due to the accuracy limit of the output measurement, increased time delays considerably lower the effect of measurement errors.

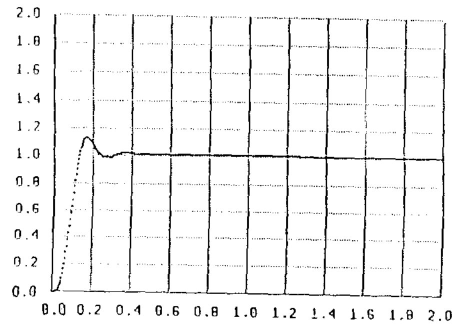

Fig. 2. System response for unit step reference ($d(k)=.1$)

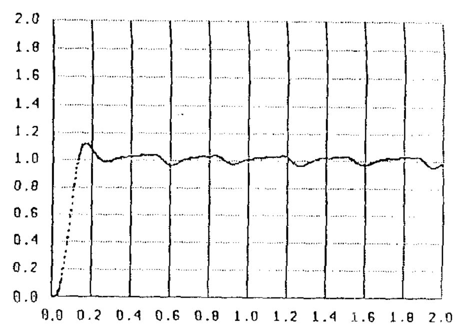

Fig. 3. System response for unit step reference (accuracy limit=.0001)

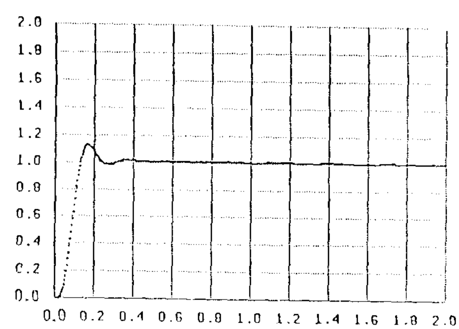

Fig. 4. System response for unit step reference (accuracy limit=.0001, $h_1=5$, $h_2=10$)

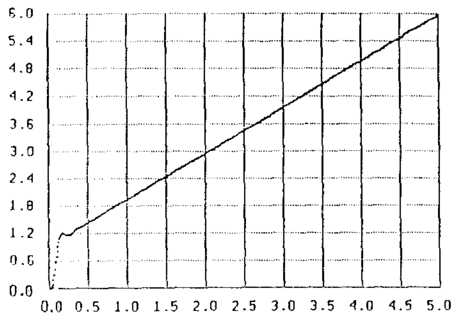

Fig. 5. System response for the reference input $z_{ref}(k)=1+k$

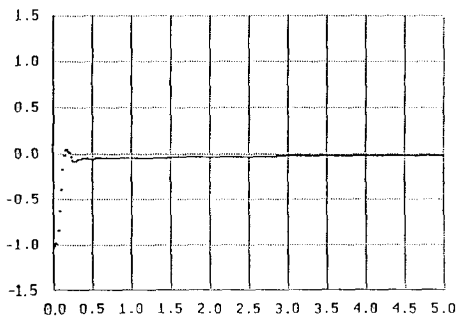

Fig. 6. System response error $e(k)=z(k)-z_{ref}(k)$ for input $z_{ref}(k)=1+k$

6. Reference

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