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Optimal Output Feedback Design for Discrete Large Scale Systems with Two Time-Scale Separation Properties

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(Abstract)

Design problem of output feedback controllers for discrete large scale systems using simplified model is investigated. It is shown that neglecting fast modes does not generally guarantee the stability of the closed loop system. In this paper, the design procedure is proposed to stabilize the system by minmizing a quadratic cost function for the simplified model and a measure of stability for the neglected fast model.

1. Introduction

Output feedback schemes offer an important advantage in simplicity over using full state feedback when successfully employed. In often the case that all the state variables are not directly available for forming feedback signals, the use of output feedback schemes provides a more easily implemented alternative to the use of an observer or Kalman filter. Therefore, a lot of work is currently being done on the development and application of optimal output feedback schemes. It is well known that a major difficulty in designing output feedback system is to solve a set of coupled non-linear matrix equation. [1,2,3]

The large scale systems with time separation property by the presence of slow and fast modes give rise to ill-contioning in the dynamics, rendering the necessary condition to obtain optimal output feedback gain matrix quite difficult. Moreover, it is often the case that the designer does not have an accurate model for the high frequency behavior of the plant. Thus it is common practice to use a low frequency model for design purposes.

In the case continuous systems, the singular perturbation theory has been employed to separate the design by the application of full state feedback. However, in the case of output feedback, a two-part decomposition of the control design is not possible unless very restrictive conditions are placed on the structure of the output. [4] Thus , we consider only a single-stage design using the simplified low frequency model. In the case where the high frequency model is open loop stable, it is well known that state feedback stabilization of the simplified model will guarantee the stability of the full order system. [5,6]
Unfortunately, it has been known that in the case of output feedback, stabilization of the simplified model will not guarantee the stability, even if the high frequency dynamics are open loop stable. [7,8]

In the case of discrete systems with time separation property, we can obtain the similar properties with continuous systems. That is, in the case where high frequency model is open loop stable, the application of state feedback schemes for the simplified model does guarantee the stability of the full system. [9]
On the orther hand, output feedback stabilization of the simplified model does not guarantee the stability even if the high frequency dynamics are open loop stable. [10]

In this paper, we will propose a design schemes in the discrete systems that it does guarantee the stability of the full system by minimizing a quadratic cost in the states and control for the simplified system with an added term corresponding to the square of the sum of eigenvalues for the closed loop fast subsystem.

In most cases, the proposed design schemes will offer the designer a means for compromising the performance of the simplified system in return for increasing a measure of stability for the fast subsystem. It will be shown through the computer simulation.

2. Simplified Discrete Models and Problem Formulation

Consider the linear, time-invariant discrete system

$$X(K+1) = AX(K) + BU(K) \tag{1a}$$

$$Y(K) = CX(K) \tag{1b}$$

where $X(K)\in R^n$, $U(K)\in R^m$ and $Y(K)\in R^p$ are the state, control and output vectors respectively. we assume that the pair (A,B) is completely controllable, that is

$$\text{ranK}[B\ A^2B\cdots\cdots\cdots\ A^{n-1}B] = n \tag{2}$$

Furthermore, we consider the system (1) is asymptotically stable which means that all the eigenvalues of the system matrix A have moduli strictly less than one. In this paper, only the class of discrete systems which possess the time separation property is considered.

A simple characterization of this property is when the eigenspectrum of A is composed of n_1 eigenvalues distributed near the unit circle and the n_2 eigenvalues centered the origin in the complex plane. In order to exhibit this time separation, a suitable arrangement of the system (1) is often required and this can be done through pemutation and/or scaling of states. With this arrangement, the system (1) is transformed into

$$X_1(K+1)=A_1X_1(K)+A_2X_2(K)+B_1U(K) \tag{3a}$$

$$X_2(K+1)=A_3X_1(K)+A_4X_2(K)+B_2U(K) \tag{3b}$$

$$Y(K) = C_1X_1(K) + C_2X_2(K) \tag{3c}$$

where $X_1(K)\in R^{n_1}$ and $X_2(K)\in R^{n_2}$

The system (3) can be converted to the n_1th order simplified model using different methods like block-diagonalization, quasi-steady-state and modal aggregation. [11]

$$Xs(K+1) = A_oX_s(K) + B_oU(K) \tag{4a}$$

$$Y(K) = C_oX_s(K) + D_oU(K) \tag{4b}$$

where

$$A_o = A_1 + A_2\ (I_2-A_4)^{-1}A_3 \tag{5a}$$

$$B_o = B_1 + A_2\ (I_2-A_4)^{-1}B_2 \tag{5b}$$

$$C_o = C_1 + C_2\ (I_2-A_4)^{-1}A_3 \tag{5c}$$

$$D_o = C_2(I_2-A_4)^{-1}B_2 \tag{5d}$$

I_j is the identity matrix of order n_j

The system (4) provides a first-order perturbation to the first n_1 dominant eigenvalues and the corresponding to the state and output trajectories.

Also it gives a good approximation to the steady-state behavior of the original system (1). We note that in the model (4) the output Y(k) is directly influenced by the input U(k) through a feedback matrix D_o which arises from the n_2 neglected non-dominant modes.

In frequently encountered case, the smplified model (4) is well known to the designer, but the full order system is not.

Therefore the feedback schemes for the simplified model are generally used.

We now introduce the constraint that the control law has a linear output feedback form

$$U(K) = -FY(K) \tag{6}$$

where F is an (mxp) constant feedback gain matrix designed for the simplified model (4).

Substituting (6) into (4) we find that the colsed-loop dynamics for the simplified model become

$$X_s(K+1) = WX_s(K),\ X_s(0) = X_{so} \tag{7}$$

where

$$W = A_o - B_o(I + FD_o)^{-1}\ FC_o \tag{8}$$

under the assumption that $(I+FD_o)$ has full rank.

The stability of the full order system using the simplified model can be stated by the following lemma.

Lemma: For the output feedback control (6), the necessary and sufficient conditions which the full order system (3) is asymptotically stable are;

$$|\lambda(W)|<1 \tag{9}$$
$$|\lambda(A_4-B_2FC_2)|<1 \tag{10}$$

From the above lemma, we can see that once the feedback gain matrix is selected to ensure the asymptotic stability of the closed loop dynamics for the simpified system, then condition (9) is always satisfied. But it does not guarantee that condition (10) is always satisfied.

3. Optimal Output Feedback Control

One approach to satisfying (9) is to formulate a standard optimal feedback control problem for the system (4), by defining a quadratic performance index of the form

$$J_o=\sum_{k=o}^{\infty}\{X_s^T(K)QX_s(K)+U^T(K)RU(K)\} \tag{11}$$

where Q is positive semidefinite matrix and R is positive definite matrix.
Substituting (6) into (4b) then

$$U(K) = -(I+FD_o)^{-1}FC_oX_s(K)=-F_oC_oX_s(K) \tag{12}$$

The necessary conditions for determining F_o that minimizes (11) are well known. [12]
The feedback matrix F can be determined from the identity in (12) as

$$F = F_o(I-D_oF_o)^{-1} \tag{13}$$

where $(I-D_oF_o)$ has full rank iff $(I+FD_o)$ has full rank. We now consider the condition (10). In the case where B_2 or C_2 is zero, the condition (10) is always satisfied.
But in the case where both of them are not zero, it does not guarantee that the condition (10) is always satisfied.
If the result is unsatisfactory, it becomes necessary to impose a constraint regarding the eigenvalues of the fast subsystem. Since the minimizing (11) subject to the constraints regarding to the all of the eigenvalues is quite intractible problem, we choose instead to constrain the square of the sum of eigenvalues sufficiently small.
This can most effectively be carried out by formulating

$$J = J_o + V_1\{tr(A_4-B_2F(2)\}^2+V_2tr\{FF^T\},V_i\geq 0 \tag{14}$$

and solving for F^* that minimizes J for increasing values of v until the desired degree of stability is reached.
The third term in (14) is added to avoid singular solutions that would arise if $(I-D_oF_o)$ lost full rank.

Necessary conditions

In order to obtain the necessary condition, we adopt the approach in [12, 13, 14] where a constrained dynamic optimization problem is converted into a constrained static optimization problem.
This is done by recognizing that the performance index (11) can be expressed as

$$J_o = X_{so}^TKX_{so} \tag{15}$$

where K satisfies the matrix Lyapunov equation.

$$G(F_o,K)=(A_o-B_oF_oC_o)^TK(A_o-B_oF_cC_o)-K+S=0 \tag{16}$$
$$S = Q + C_o^TF_o^TRF_oC_o \tag{17}$$

Thus it can be summarized to the following problem that obtain the necessary conditions for minimizing

$$J= tr\{KX_{so}X_{so}^T\} + V_1\{tr(A_4-B_2FC_2)\}^2 + V_2tr\{FF^T\} \tag{18}$$

with respect to F,F_o and K subject to the constraint in (16) and the constraint

$$F = (I + FD_o)\ F_o \tag{19}$$

The necessary conditions are obtained by applying gradient matrix operations to the Lagrangian

$$\ell = tr\{KX_{so}X_{so}^T\} + V_1\{tr(A_4-B_2FC2)\}^2 + V_2tr\{FF^T\} + tr\{G(F_o,K)L_o^T\} + tr[\{F-(I+FD_o)F_o\}L^T] \quad (20)$$

where L_o and L are Lagrange multipliers.

The necessary conditions follow from:

$$\partial\ell/\partial F_o = \partial\ell/\partial F = \partial\ell/\partial K = 0 \quad (21a)$$

$$\partial\ell/\partial L_o = \partial\ell/\partial L = 0 \quad (21b)$$

Using the formulas

$$\partial tr\{NZ\}/\partial Z = N^T,\ \partial tr\{NZ^T\}/\partial Z = N \quad (22a)$$

$$\partial tr\{NZM\}/\partial Z = N^TM^T \quad (22b)$$

$$\partial tr\{NZMZ^T\}/\partial Z = N^TZM^T + NZM \quad (22c)$$

then the expression (21) can be expanded to give

$$F_o = \tfrac{1}{2}(B_o^TKB_o+R)^{-1}\ \{2B_o^TKA_oL_oC_o^T + (I+FD_o)^TL\}(C_oL_oC_o^T)^{-1} \quad (23)$$

$$L^T = (I-D_oF_o)^{-1}\ \{(2V_1 tr(A_4-B_2FC_2)C_2B_2 - 2V_2F^T\} \quad (24)$$

$$(A_o-B_oF_oC_o)L_o(A_o-B_oF_oC_o)^T - L_o + X_{so}X_{so}^T=0 \quad (25)$$

$$(A_o-B_oF_oC_o)^TK(A_o-B_oF_oC_o) - K+Q+C_o^TF_o^TRF_oC_o=0 \quad (26)$$

$$F = (I + FD_o)F_o \quad (27)$$

The dependence of (25) on the initial condition X_{so} can be removed by minimizing $E\{J\}$ with $E\{X_{so}X_{so}^T\} = I$. This amounts to simply replacing $X_{so}X_{so}^T$ by I.

A computational methods

This section provides a computational procedure for finding an F^* that satisfies (23)-(27).

For V_i=0, the usual necessary conditions for the optimal feedback problem result in terms of F_o,K and L_o

For $V_i \neq 0$, we have added constraint in (24) and feedback gain matrix is given by (13) with the underlying assumption throughout that $(I-D_oF_o)$ is invertible.

The solution for $V_i = 0$ can first be obtained. Then, choosing some $V_2 > 0$, V_1 should be gradually incremented, and a new solution obtained.

The computational procedure is as follows

0: Choose any F_o^0 stabilizing the low frequency model in the closed loop and set i=0

1: Solve (24) - (27) for K^i, L^i_o, L^i and F^i, all functions of F^i_o.

2: Solve, from (23), for the gradient

$$\Delta F_o^i = \tfrac{1}{2}(B_o^TK^iB_o+R)^{-1}\ \{2B_o^TK^iA_oL_o^iC_o^T + (I+F^iD_o)^TL^i\}\ (C_oL_o^iC_o^T)^{-1} - F_o^i \quad (28)$$

3: Choose $\alpha\epsilon(0,1]$ so that

$$J(F_o^i+\alpha^i\Delta F_o^i) < J(F_o^i)=tr\{K^i\}+ V_1\{tr(A_4-B_2F^iC_2)\}^2 + V_2\{F^iF^{iT}\} \quad (29)$$

and set $F_o^{i+1} = F_o^i + \alpha^i\Delta F_o^i$

4: set i=i+1 and go to 1.

Remarks

1) It is necessary to ensure that $A_o-B_oF_o^iC_o$ is stable at each iteration
2) If Q>0 R>0 then the unique and positive definite solutions for K^i and L^i_o exist.
3) Unique solutions for L^i and F^i exist if $(I-D_oF_o^i)$ has full rank at each iteration.
4) If C2 or B_2 is zero, then the eigenvalues of the fast subsystem are unaffected by the choice of F.

4. Computer Simulations and Discussion

Consider the following third-order system.

$$X(K+1)=\begin{bmatrix} 0.90 & 0.00 & 0.10 \\ -0.10 & -0.30 & 0.00 \\ 0.00 & 0.30 & -0.23 \end{bmatrix} X(K)+\begin{bmatrix} 1.00 \\ -0.10 \\ -0.30 \end{bmatrix} U(K)$$

$$Y(K) = [-0.30 \quad 1.00 \quad 0.47]\ X(K)$$

Since the eigenvalues of the open loop system are (0.89778, -0.32560, -0.20218), the system is time separable to the first-order slow modes and second-order fast modes.

The system matrix for the simplified model is;

A_o = 0.898124, B_o = 0.973735, C_o = -0.385737, D_o = -0.200360 and the performance index is selected

$$J_o=\sum_{k=0}^{\infty}\{X_s^T(K)Q\,X_s(K) + U^T(K)RU(K)\},$$

where Q=1 and R=0.0001

For Vi=0, then, the optimal output feedback control for the simplified model is

$$U(K) = 3.37Y(K)$$

Since the eigenvalues of the fast subsystem when the feedback control is applied to the original system (3) are (-1.15535, -0.33378), the full order system is unstable.

The propsed algorithm is applied to make the unstable system stable and the results are shown in Fig. 1, 2 and 3

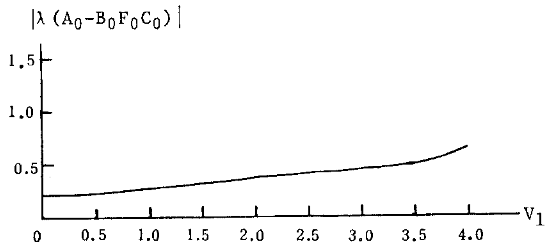

Fig. 1. Absolute value of eigenvalues for slow subsystem. $v_2 = 0.001$

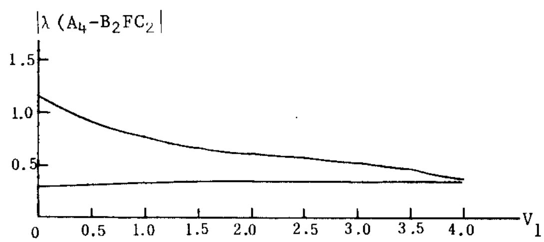

Fig. 2. Absolute value of eigenvalues for fast subsystem. $v_2 = 0.001$

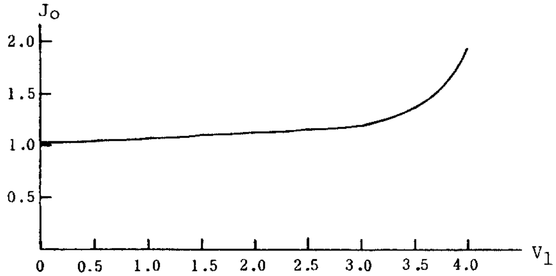

Fig. 3. Degradation of slow subsystem performance. $v_2 = 0.001$

We can recognize that the algorithm guarantees the stability of fast subsystem by compromising the results between low frequency performance and high frequency stability.

As example, in the case of $V_1 = 2$ and $V_2 = 0.001$, $|\lambda(A_0 - B_0F_0C_0)| = 0.4007$, $|\lambda(A_4 - B_2FC_2)| = (0.6259, 0.3396)$, $J_0 = 1.1908$ and $F^* = -1.80$

5. Conclusion

In often the case, the designer has a good model for the slow behavior of the system, and a rough approximation for the fast behavior.

Thus the simplified models is used to describe the dynamic behavior of large scale systems. Therefore the basic problem using the simplified model for output feedback control will guarantee the stability of the closed loop original system. The design procedure proposed in this paper allows designer to trade-off between a measure for the slow subsystem performance and the degree of stability for the fast subsystem.

References

1. W.S. Levine and M. Athans, "On the Determination of the Optimal constant Output Feedback Gains for Linear Multivariable Systems", IEEE Trans. Automat. Contr., AC-15, pp. 44-48, Feb. 1970.
2. P.M. Makila, "On the Anderson-Moore Method for Solving the Optimal Output Feedback Problem", IEEE Trans.Automat. Contr., Vol. AC-29, pp. 834-835, Sep. 1984.
3. J.M. Mendel, "A Concise Derivation of Optimal Constant Limited State Feedback Gains", IEEE Trans. Automat. Contr., AC-19, pp. 447-448, Aug. 1974.
4. P. Chemoul and A.M. Wahdan, "Output Feedback Control Systems with Slow fast modes", Large Scale System, 1, 257, 1980.
5. J.H. Chow and P.V. Kokotovic, "A Decomposition of near Optimum Regulators for Systems with Slow and Fast modes", IEEE Trans. Automat.

Contr., AC-21, pp. 701-705, Oct. 1976.

6. P.V. Kokotovic and R.A. Yackel, "Singular Perturbation of Linear Regulators: Basic theorem", IEEE Trans. Automate Contr., Vol. AC-17, pp. 29-37, Feb. 1972.
7. H.K. Khail, "On the Robustness of Output Feedback Control Methods to Modeling Errors" IEEE Trans. Automat. Contr., Vol. AC-26, pp. 524-526, Apr. 1981.
8. J. O'Reilly, "Robustness of linear feedback control systems to unmodelled high-frequency dynamics" Int. J. Cont., Vol. 44, pp. 1077-1088, 1986.
9. M.S. Mahmoud, "Design of Observer-based Controllers for a class of Discrete Systems", Automatica, Vol. 18, pp. 323-328, 1982.
10. M.S. Mahmoud and M.G. Singh, "On the Use of Reduced Order Models in Output Feedback Design of Discrete Systems" Automatica, Vol. 21, pp. 485-489, 1985.
11. M.S. Mahmoud and M.G. Singh, "Large Scale Systems Modeling", Pergamon Press, pp. 265-295, 1981.
12. C.S. Berger "An Algorithm for Designing Sub-optimal Dynamic Controller", IEEE Trans. Automat. Contr., pp. 596-597, Oct. 1974.
13. C.M. Ermer and V.D. Vandelinde, "Output Feedback Gains for a Linear-Discrete Stochastic Control Problem", IEEE Trans. Automat. Contr., pp. 154-157, Apr. 1973.
14. H.T. Toivonen, "A globally convergent algorithm for the optimal constant output feedback problem", Int. J. Control., Vol. 41, pp. 1589-1599, 1985.