# 전류제어형 입력보상 및 최적 출력제어를 적용한 직류전동기 구동시스템의 정밀 제어 방법에 관한 연구 고정호, 손승걸, 최영호, 안태영

영오, 존승물, 최영오, 안태? 대 전 기 계 창

A Study on the Precision Control of DC Motor Driving System using Current-controlled Feedforward Compensation & Optimal Output Feedback

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# Abstract

In this paper, a controller design method for the DC motor driving system is described emphasizing the specified degree of accuracy undergoing large time varying disturbances, coulomb friction and arm-load resonance. A feedforward compensation technique using the current controller is proposed, and resulting in the performance improvement as well as the implementational simplicity. A timeweighted quadratic performance index is used in the optimization of the controller, which is a salient way of obtaining better closed-loop performance in a simple manner. Computer simulations are also given to show the usefulness of the proposed techniques.

# I. Introduction

In recent years, there have been a number of researches on the DC motor driving system(1)-(6) due to the increasing demands in the industrial applications. In the design of the control system, it is generally known that the control systems with large loads are more difficult to design rather than those with smaller ones because of the inherently existing resonance problem, and practical hardware implementation problems

(1)-(2). It is needed, however, to satisfy the performance requirements to a specified degree of accuracy under the large disturbances caused by more severe operational conditions. However, little has been known about the design of control system especially engaged with large loads. As a way of large disturbance rejection. an anticipatory signal method has been attempted(1). However, the design of the compensator has some limitations such as the noise problems in the differentiators eventhough it is partially overcome by the approximation, and thus additional tedious simulation required. In designing the controller, it has mainly been handled in the frequency-domain using the conventional Bode plot or Root-locus analysis(2)-(3), however, optimization of feedback gains comes up as a rather difficult problem. More recently, a modern digital control theory has been applied to deal with those problems(4)-(5), neverthless, there exists some limitations such as implementation complexity and extensive computations.

In this paper, a design method of the feedforward compensator and feedback controller for the precision control of DC motor driving system is presented. In order to effectively minimize the disturbance and coulomb friction effect in a simple manner, a feedforward compensation technique using the current controlled

inner loop is used, thus resulting in the performance improvement in the steady state as well as the implementational simplicity. In the optimization of the controller, a time-weighted quadratic performance index is used to obtain better closed-loop performances, and its advantages are also discussed with the simulation results.

# II. Problem formulation

The control system operating in a stationary or moving status is desired to satisfy the given time domain specifications such as the specified settling time, overshoot, and steady state error to a velocity(or a position) command in spite of the practical hardware limitations such as the bounded input, and the saturation problems in the sub-systems. Furthermore, implementation difficulties should also be considered in the design stage. To effectively deal with those problems, a feedforward control concept is applied for the effective rejection of the large disturbances before designing the feedback controller.

# o Feedforward control problem

It is assumed that the disturbances affecting the motor torque can be measured by using the properly chosen velocity and acceleration sensing units with some signal processing network. The disturbance can not be directly rejected at the motor shaft, but it is possible to compensate effective magnitude in advanced phase at the side of the motor input(Voltage or current). It is known as the anticipatory signal method(1).

The difficulties in the realization of the feedforward compensator are noise problems which are frequently occurred when implementing the inverse of the transefer function of the electrical part, P(s)<sup>-1</sup> in Fig.1, using the diffentiators and other passive elements. In the anticipa-

tory signal method, these have been partially solved by the proper approximation, however, additional tedious simulation work was required to effectively minimize the disturbances. Now, the problem can be formulated as finding a way of simple realization of P(s) in the compensator without noise problems. Note that a rejection of the coulomb friction is also included in Fig.1.

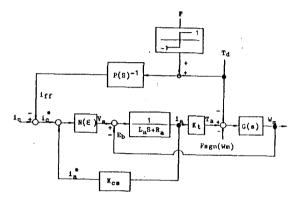


Fig.1 Block diagram of the feedforward compensator

# o Feedback control problem

Using the feedforward control technique stated above, it is assumed, hereafter, that there is little influence of disturbances in the control system. We take the following PIM controller to meet the specified degree of accuracy without complicating the implementation of the controller;

 $u(t)=-k_{1}z(t)-k_{p}(y(t)-y_{s}(t))-k_{me}y_{me}(t)$  (1) where,  $\dot{z}(t)=y(t)-y_{s}(t)$ . This type of the controller provides no steady state error for a step response, and better dynamic responses are expected using the measurable variable. Now, we take a TWPI(time-weighted quadratic performance index) for the optimization of the feedback gains.

$$J = \int_{0}^{\infty} ((t/t_{r})^{N} \cdot (y-y_{s})^{*}Q(y-y_{s}) + (u-u_{s})^{*}R(u-u_{s}))dt$$
(2)

where  $k_p$ ,  $k_i$ ,  $k_{me}$  are to be determined as minimizing the above performance index.

#### III. Main results

In order to obtain an analytical tool for analysis, the control system modelling is given in the following. Typical governing equations, known as the torque equation and voltage equation, are easily obtained as (3) and (4). Current control and main feedback control law are also described in (5). A block diagram expressing the following governing equations is given in Fig.2.

o Governing equations Torque equations;

$$T_a = J_m \dot{w}_m + B_m w_m + F_m sgn(w_m) + T_I / N + T_T = K_t i_a$$
 (3-a)

$$T_L = J_L \dot{w}_L + B_L w_L + F_L sgn(w_L) = K_L(\theta_m - N\theta_L)$$
 (3-b)

$$T_T = J_T \dot{w}_T = K_T (\Theta_m - \Theta_T)$$
 (3-e)

Voltage equations ;

$$V_a = L_a i_a + R_a i_a + E_b + \Delta e_b, E_b = K_v w_m$$
 (4)

where  $T_T$  denotes torque for tachometer which is negligible when compared with  $T_a$ , and  $\Delta e_b$ , also neglected, refers to brush drop of motor.

Control laws ;

$$i_c = -k_p e - k_i \int_0^\infty e dt - k_m e w_T$$
 (5-a)

$$V_a = N(\epsilon)(i_c - i_a)$$
, or (5-b)

$$k_{pc}(i_c-i_a)+k_{ic}\int_0^{\infty}(i_c-i_a)dt$$
 (5-c)

where  $e^{-w}L^{-w}i$  for velocity control, and  $\theta_{L}-\theta_{i}$  for position control.

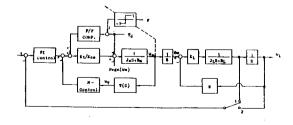


Fig.2 Block-diagram of the DC motor driving system

o A feedforward controller design
Using the current control loop as an

inner loop, a feedforward compensator is designed in this section. Note that a current control method has also the advantages addressing the improvement of the dynamic performances and simplicity in the current limitting(6).

The transfer function of the electrical part is easily derived as follows ;

$$P(s)=N(\varepsilon)K_{t}/(L_{a}s+R_{a}+N(\varepsilon)K_{c}s+K_{t}G(s))$$
 (6)

where the type of the current control is assumed as a hysteresis type. If  $N(\epsilon)$  is chosen as large enough, then P(s) leads to a constant (without dynamics) as follows:

$$P(s) \longrightarrow Kt/Kcs . \tag{7}$$

A similar result can also be derived for the case of PI type current control by using the proper inequalities on PI control gain(6). Therefore, gain of a feedforward compensator can be simply obtained as a constant without phase shift as follows;

$$P(s) = K_{CS}/K_{t}$$
 (8)

The switching function for the realization of the coulomb friction can be implemented using comparator and some passive elements with the speed information.

o Feedback controller design

In the following, motor current is considered as the gain multiplied command of the control law by the effect of the current control loop as explained above. Furthermore, disturbance and coulomb friction

are considered negligible by the effect of a feedforward compensator. With the choice of the state vector as;

$$x(t) = (\theta_m \ w_m \ \theta_L \ w_L \ \theta_T \ w_T)'$$
 (9)

where  $\theta_m$ ,  $w_m$  denote angular position and speed of the motor respectively, and  $\theta_L$ ,  $w_L$  those of load, and  $\theta_T$ ,  $w_T$  those of tachometer respectively. Then, the state equations are developed by ;

$$\dot{x}(t) = A x(t) + B u(t), x(0) = 0$$
 (10-a)

where 
$$u(t) = i_a(t)$$
  
 $y(t)=C x(t), y_{me}(t)=C_{me}x(t)$  (10-b,c)

where prime denotes tanspose, and y(t),  $y_{me}(t)$ , C and  $C_{me}$  denote position of load, tachometer  $output(w_T)$ , output vector and measurable output vector respectively, and

A, B are expressed as follows;

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{-(K_L + NK_T)}{NJ_m} & -B_m/J_m & K_L/J_m & 0 & K_T/J_m & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ K_L/J_L & 0 & \frac{-NK_L}{J_L} & \frac{-B_L}{J_L} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ K_T/J_T & 0 & 0 & 0 & \frac{-K_T}{J_T} & 0 \end{bmatrix}$$

$$B = (0 K_{t}/J_{m} 0 0 0 0)$$

In the above equations, J denotes inertia, B viscous damping coef., and subscript 'm' refers to motor, 'L' load, and 'T' tachometer respectively.

A necessary condition for constant feedback gains being optimal with respect to performance index(2) is given in this section and a computational algorithm is also described. Using the procedures in (7)-(8), a necessary conditions for optimality can be derived as follows;

$$d_{J}/d_{K} = 2(-\overline{B}, \sum_{i=1}^{N+1} (P_{i}L_{i}) + RK\overline{C}L_{N+1})\overline{C},$$

$$-2K_{i}^{-1}(I_{r}, O_{n})P_{N+1}\begin{bmatrix} z_{s}z_{s}, O_{r}, z_{s}x_{s}C_{me} \\ x_{s}z_{s}, O_{r}, x_{s}x_{s}C_{me} \end{bmatrix} = 0 \quad (11)$$

where  $P_i$  and  $L_i$  satisfy the following equations :

$$K=(\mathbf{k}_{i}, \mathbf{k}_{p}, \mathbf{k}_{me}), \overline{C} = \begin{bmatrix} I_{r} & 0 \\ 0 & C \\ 0 & C_{me} \end{bmatrix}$$

$$\overline{A} = \begin{bmatrix} 0 & C \\ 0 & A \end{bmatrix}, \overline{B} = \begin{bmatrix} 0 \\ B \end{bmatrix}, w(0) = \begin{bmatrix} -z_S \\ -x_S \end{bmatrix}.$$

# o A computational algorithm

From the necessary conditions for optimality, it is not possible to find feedback gains directly, however, explicit expressions are avilable for J and dJ/dK. Therefore, a well known gradient technique, for example, the Davidon-Fletcher-Powell algorithm(9), may be used for the determination of optimal gains.

IV. Simulations and discussion The usefulness of the proposed design method is shown by simulations in this section. It is assumed that the normalized parameters with respect to motor inertia( $J_m$ ) are given as follows, and feedback gains are summerized in the following table.

\* Optimal gains used in simulations

velocity control			position control		
<b>k</b> p	kį	k <sub>me</sub>	kp	kį	k <sub>me</sub>
	143.4 WPI)	0.4	-	0.45 TWPI)	1.6
	168.1 entiona				

\* Parameters used;  $J_m=1..J_L=173913$ ,  $J_T=0.022$   $B_m=0.522$ ,  $B_L=52174$ ,  $F_m=6.522$ ,  $F_L=2174$ ,  $K_{\text{CS}}=0.217$ ,  $K_t=15.52$ ,  $K_v=1.739$ , N=150.

With these informations, the output responses are simulated as shown in Fig.3 through 5 by which it is well informed that a ripple component caused by the mechanical resonance is reduced and transint responses are also improved by using the TWPI. In the steady state, disturbance effects are significantly rejected especially in the velocity control and steady state error caused by coulomb friction is also reduced in case of position control using the easily implementable feedforward compensation technique using the current control loop.

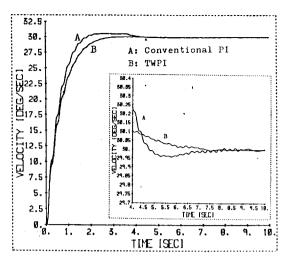


Fig.3 Comparison of output responses in velocity control system

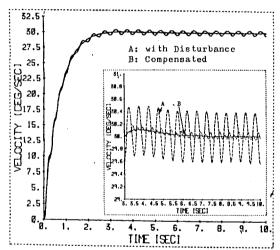


Fig.4 Disturbance rejection in velocity control system

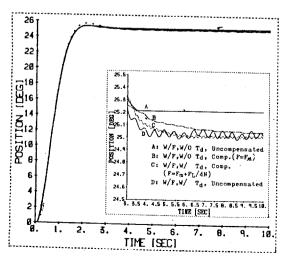


Fig. 5 Rejection of disturbance & coulomb friction in position control system

# V. Conclusion

A design method of the feedforward compensator and feedback controller for the DC motor driving system has been described to improve the control accuracy undergoing large time varying disturbances, coulomb friction, and mechanical resonance. The disturbance and coulomb friction effect has significantly been reduced in a simple manner when a feedforward compensation technique using the current controlled inner loop has been applied. Furthermore, better closed-loop performances have also been obtained by using the optimization technique based on TWPI rather than the conventional performance index. Therefore, it is concluded that the proposed design method is useful for the precision control of the DC motor driving system.

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