An Adaptation Algorithm for Parallel Model Reference Bilinear Systems

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An Adaptation algorithm is presented and a convergence criterion is derived for parallel model reference adaptive bilinear systems. The output error converges asymptotically to zero, and the parameter estimates are bounded for stable reference models. The convergence criterion depends only upon the input sequence and a priori estimates of the maximum parameter values.

1. System

The system to beidentified is described by a single-input single-output discrete time bilinear ARMA equation of the form

$$y(k)= \sum_{i=1}^{n} [a_i y(k-i) + b_i y(k-i)u(k-i) + c_i u(k-i)] = p^T x(k-1) \qquad (1)$$

The model to be adjusted is represented by

$$\hat{y}(k) = \sum_{i=1}^{n} [\hat{a}_i(k)\hat{y}(k-1) + \hat{b}_i(k)\hat{y}(k-i)u(k-i) + \hat{c}_i(k)u(k-1)] = \hat{p}^T(k)w(k-1) \qquad (2)$$

Where

$$x(k-1) = [y(k-1),\ldots, y(k-n), y(k-1)u(k-1),\ldots, y(k-n)u(k-n), u(k-1), \ldots , u(k-n)]^T$$

and

$$w(k-1) = [\hat{y}(k-1), \ldots , \hat{y}(k-n), \hat{y}(k-1)u(k-1), \ldots , \hat{y}(k-n)u(k-n), u(k-1), \ldots , u(k-n)]^T$$

The objective is to obtain the parameter estimate vector p(k) such that the output tracking error

$$e(k) = y(k) - \hat{y}(k) \qquad (3)$$

converges asymptotically to zero, i.e.,

$$\lim_{k\to\infty} e(k) = 0$$

2. Identification algorithm

The identification algorithm is given by

$$\hat{p}(k) = \hat{p}(k-1) + \xi(k-1)w(k-1)e^*(k) \qquad (4)$$

Where

$$e^*(k) = Y(k) - \hat{p}^T(k-1)w(k-1) \qquad (5)$$

and $\xi(k-1)$ denotes the gain of the algorithm to be specified.

Theorem : For the stable identification system defined by (1) and (2) and the identification algorithm defined by (4) and (5), if the algorithm satisfies

$$\xi(i-1) = \frac{\zeta}{1 + \zeta\|w(i-1)\|^2} \qquad (6)$$

then it follows that

$$\lim_{k\to\infty} e(k) = 0$$

and $\|\hat{p}(k)\|^2$ is bounded for all k, where

$$\zeta \geq \frac{h(i-1)}{\|\bar{u}(i-1)\|^2} \qquad (7)$$

$$h(i-1) = 2n|a|_{max} + n|b|_{max}|u(i)| + b_{max}\sum_{j=1}^{n}|u(i-j)| \qquad (8)$$

$$\|\bar{u}(i-1)\|^2 = \bar{u}^T(i-1)\bar{u}(i-1)$$

$$\|w(i)\|^2 = w^T(i)w(i)$$

$$\bar{u}(i-1) = [u(i-1), \ldots, u(i-n)]^T$$

and $|a|_{max}$ ans $|b|_{max}$ are the maximum values of $|a_i|$ and $|b_i|$, respectively.

Proof : Subtracting (5) from (3) and substituting (2) and (4), we obtain

$$e(k) = [1-\xi(k-1)\|w(k-1)\|^2]e^*(k) \qquad (9)$$

Substitution of (9) into (4) yields

$$\hat{p}(k) = \hat{p}(k-1) + \frac{\xi(k-1)w(k-1)e(k)}{[1-\xi(k-1)\|w(k-1)\|^2]} \qquad (10)$$

Using (1) and (2), we can write the output tracking error e(k) as

$$e(k) = y(k) - y(\hat{k})$$

$$= a^T\bar{e}(k-1) + b^T\bar{z}(k-1) - \bar{p}^T(k)w(k-1) \qquad (11)$$

Where

$$a = [a_1, \ldots, a_n]^T$$

$$b = [b_1, \ldots, b_n]^T$$

$$\bar{e}(i-1) = [e(i-1), \ldots, e(i-n)]^T$$

$$\bar{z}(i-1) = [e(i-1)u(i-1), \ldots, e(i-n)u(i-n)]^T$$

and

$$\bar{p}(k) = \hat{p}(k) - p$$

Substitution of (10) into (11) yields

$$e(k) = a^T\bar{e}(k-1) + b^T\bar{z}(k-1) - \bar{p}^T(k-1)w(k-1) - \frac{\xi(k-1)\|w(k-1)\|^2 e(k)}{[1-\xi(k-1)\|w(k-1)\|^2]} \qquad (12)$$

From (10) we obtain

$$\bar{p}^T(k-1)w(k-1)e(k) = \frac{[1-\xi(k-1)\|w(k-1)\|^2]}{2\xi(k-1)}$$

$$[\|\bar{p}(k)\|^2 - \|\bar{p}(k-1)\|^2] - \frac{\xi(k-1)\|w(k-1)\|^2e^2(k)}{2[1-\xi(k-1)\|w(k-1)\|^2]} \qquad (13)$$

Multiplying both sides of (12) by e(k) and substituting (13), we have

$$e^2(k) = a^T\bar{e}(k-1)e(k) + b^T\bar{z}(k-1)e(k) - \frac{[1-\xi(k-1)\|w(k-1)\|^2}{2\xi(k-1)}[\|\bar{p}(k)\|^2 - \|\bar{p}(k-1)\|^2] - \frac{\xi(k-1)\|w(k-1)\|^2e^2(k)}{2[1-\xi(k-1)\|w(k-1)\|^2]} \qquad (14)$$

Therefore,

$$\sum_{i=0}^{k} e^2(i) = e^2(0) + \sum_{i=1}^{k} a^T\bar{e}(i-1)e(i) + \sum_{i=1}^{k} b^T\cdot \bar{z}(i-1)e(i) + \sum_{i=1}^{k}\frac{1}{2\xi(i-1)}[1-\xi(i-1)\|w(i-1)\|^2] [\|\bar{p}(i-1)\|^2 - \|\bar{p}(i)\|^2] - \sum_{i=1}^{k}\frac{\xi(i-1)\|w(i-1)\|^2e^2(i)}{2[1-\xi(i-1)\|w(i-1\|^2]} \qquad (15)$$

We examine the first summation on the right hand side of (15).

From the triangle inequality and the definition of $|a|_{max}$, it follows that

$$\sum_{i=1}^{k} a^T\bar{e}(i-1)e(i) \leq \frac{n}{2}|a|_{max}e^2(0) + n|a|_{max}\sum_{i=1}^{k}e^2(i) + \frac{M}{2}|a|_{max} \qquad (16)$$

Where

$$M = \sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} e^2(i-j)$$

We examine the second summaton on the right hand side of (15). From the definition of $|b|_{max}$ and $\bar{z}$, we obtain

$$\sum_{i=1}^{k} b^T\bar{z}\ (i-1)e(i) \leq \frac{|b|_{max}}{2} \{ne^2(0)|u(0)| +$$

$$N + \sum_{i=1}^{k} e^2(i)[n|u(i)| + \sum_{j=1}^{n} |u(i-j)|]\} \qquad (17)$$

Where

$$N = \sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} e^2(i-j)|u(i-j)|$$

We examine the third summation on the right hand side of (15). From the assumption that the system is stable and from (6), it follows that

$$\frac{1 - \xi(i-1)\|w(i-1)\|^2}{2\xi\ (i-1)} = \frac{1}{2\zeta} \qquad (18)$$

With (18), the third summation on the right hand sode of (15) becomes

$$\sum_{i=1}^{k} \frac{1}{2\xi(i-1)} [1 - \xi(i-1)\|w(i-1)\|^2] \cdot$$

$$[\|\bar{p}\ (i-1)\|^2 - \|\bar{p}(i)\|^2] =$$

$$\frac{1}{2\zeta} [\|\bar{p}\ (0)\|^2 - \|\bar{p}(k)\|^2] \qquad (19)$$

Now we examine the fourth summation on the right hand side of (15). From (6), we have

$$\sum_{i=1}^{k} \frac{\xi(i-1)\|w(i-1)\|^2 e^2(i)}{2\{1 - \xi(i-1)\|w(i-1)\|^2\}} =$$

$$\sum_{i=1}^{k} \frac{1}{2}\zeta\|w(i-1)\|^2 e^2(i) \qquad (20)$$

Substituting (8), (16), (17), (19), and (20) into (15) and after rearranging, we obtain

$$\sum_{i=0}^{k} e^2(i) < L + \frac{1}{2\zeta} [\|\bar{p}(0)\|^2 - \|\bar{p}(k)\|^2] +$$

$$\frac{1}{2} \sum_{i=1}^{k} e^2(i)F(i) \qquad (21)$$

Where

$$F(i) = h(i-1) - \zeta\|w(i-1)\|^2 \qquad (22)$$

and

$$L = e^2(0) + \frac{n}{2} |a|_{max}e^2(0) + \frac{M}{2} |a|_{max} +$$

$$\frac{n}{2} |b|_{max}e^2(0)|u(0)| + \frac{N}{2} |b|_{max}$$

L is a bounded, positive constant containing only past values of the output tracking error and the input.
From (7), (22) yields

$$F(i) \leq h(i-1) \{1 - \frac{\|w(i-1)\|^2}{\|u(i-1)\|^2}\} \leq 0 \qquad (23)$$

Therefore,

$$\sum_{i=0}^{k} e^2(i) < L + \frac{1}{2\zeta} [\|\bar{p}\ (0)\|^2 - \|\bar{p}(k)\|^2] \qquad (24)$$

From (24), we have

$$\lim_{k \to \infty} e(k) = 0$$

Furthermore, from the rearrangement of (24),

$$\|\bar{p}(k)\|^2 < 2\zeta L + \|\bar{p}(0)\|^2$$

Which guarantees the boundedness of $\hat{p}(k)$ and completes the proof.

References

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(2) B.K. Altay, "Elimination of real positivity condition and error filtering in parallel MRAS," IEEE Trans. Auto. Contr., Vol. AC-29, pp. 1017-1019, 1984.