An Algorithm for Ultrasonic 3-Dimensional Reconstruction and Volume Estimation

*Young Min Chin, Sang On Park, Kwang Bang Woo

(Department of Electrical Engineering, Yonsei University)

Abstract

In this paper, an efficient algorithm to estimate the volume and surface area from ultrasonic imaging and a reconstruction algorithm to generate three-dimensional graphics are presented.

The computing efficiency is improved by using the graph theory and the algorithm to determine proper contour points is performed by applying several tolerances. The search for contour points is limited by the change in curvature in order to provide an efficient search of the minimum cost path. These algorithms are applied to a selected mathematical model of ellipsoid. The results show that the measured value of the volume and surface area for the tolerances of 1.0005, 1.001 and 1.002 approximate to the measured values for the tolerance of 1.000 resulting in small errors.

The reconstructed 3-dimensional images are sparse and consist of larger triangular tiles between two cross sections as tolerance is increased.

1. Introduction

In the past few years, ultrasound has been popular because of its advantages of small size, portability, real time image and its merits of harmlessness to men. However, the resolution of ultrasound signal is limited to millimeter by which the diseases may not be characterized exactly.

In the present study, an attempt is made to compensate these weakpoint by developing an efficient algorithm which enables one to estimate volume and surface area of a concerned organ, and 3-dimensional reconstruction algorithm is also presented. Graph theory is used to increase the efficiency. Each algorithm has two parts : one which eliminates unnecessary points in constructing the graph and obtains proper contour points, and another which searches the minimum cost path according to the change in curvature to reduce the searching time. Volume and surface area are measured by using the vector area method based on the polyhedral approximation and 3-dimensional images which consist of triangular tiles are reconstructed from the cross sections.

2. Graph Organization

(1) Data Structure for Graph Organization

The 3-dimensional information for each cross section has to be specified and preserved in record form for an efficient analysis of the object. If information is arranged with array (sequential mapping), data transfer will take longer time and the sufficient memory location is needed to store the data. In this stusy, linked list will be used to compensate such drawback in array. Linked list consists of nodes, with each node consisting of fields.

It is efficient to use the circular list for node search or for node insertion or deletion. Any point in circular list can be connected to any other point in the list. Circular list enables one to delete the node within the fixed time regardless of the number of nodes.

(2) Selection of Contour Points

In the way of selecting the contour points from any cross section, the volume estimated and the image reconstructed are different from those of the real object. The more the contour points, the less the error, but search time or weighting time will increase exponentially and extended memory locations are required to preserve the necessary information.

Therefore, it is necessary that the proper contour points may be selected to eliminate unnecessary points.

Let successive three points in a cross section be P, Q and R as shown in Fig. 2-1. Then

$$T = \frac{PQ + QR}{PR} \geq 1$$

If the three points, P, Q and R are on the same line, the tolerance T has the value of 1. The midpoint Q which makes the tolerance close to 1 can be removed. Proper contour points are selected while keeping the value of the tolerance T between 1.0005 and 1.005.

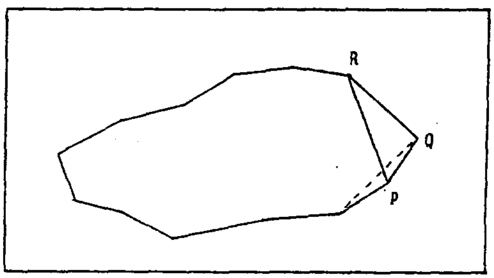


Fig. 2-1 An example of successive three points

(3) Graph Organization

In order to reduce the error, the approximation of two adjacent cross sections is important. This problem of approximation can be described as follows :

Let the upper contour be defined by the sequence of m distinct contour points Po, P1, ---, Pn and let the lower contour be defined by the sequence of n distinct contour points Qo, Q1, ---, Qn. Here, we assume that each points are arranged in counterclockwise direction. Then the problem of approximation can be solved by triangular tiles which are constructed from the contour points in P and Q. The vertices of these tiles are contour points with the vertices of each tile taken two from one sequence and one from tne other. Each tile is defined by a set of three distinct elements, either the form {Pi,Qj,Pk} or {Qi,Pj,Qk}. In other words, the boundary of each tile will consist of a single contour segment and two spans, each connecting one end of the contour segment with a common point on the other contour.

There are many sets of tiles whcih could be defined this way. To select the fitting tiles, restrict is given to those sets of tiles which satisfy the following two conditions :

1) Each contour segment appears in exactly one tile in the set.
2) If a span appears as a left span of some tile, then it also must appear as a right span of at least one tile in the set.

Then the problem can be reduced to one in graph theory. To obtain the fitting tiles, we consturct a following graph.

G = < V, A > : V-Vertex, A-Arc

V = { Vij | i=0,1,---,m-1 ; j=0,1,---,n-1 }

A = {<Vkl,Vst> | (s=k and t=l+1) or (s=k+1 and t=l)}

A vertex in this graph represents a span in a tile and an arc represents a tile which can be constructed between the two contours. The previous conditions are equivalent to the following conditions on G.

1) There is exactly one vertical arc between two adjacent rows and there is exactly one horizontal arc between two adjacent columns.
2) For a vertex Vij of G, **indegree**(Vij)=**outdegree**(Vij)=0 or **indegree**(Vij)>0 and **outdegree**(Vij)>0

(4) Selection of Acceptable Surface

Acceptable surface is defined as a set of selected tiles from the constructed graph, and the optimal acceptable surface must be selected which has the minimum cost. To find efficiently the minimum cost path a directed graph is constructed, and using the characteristics of directed graph the arc cost is restricted.

First, the cost is computed for a distance between the first point P1 in P and each of the points in Q and the point Qj with the minimum cost is found. Then, the first tile <P1, Qj, P2> is selected. The other vertex of tile constructed by the segment P2P3 has to be in Q because from the conditions of section (3) the segment P2P3 construct a tile. The next point in Q is computed by the same way, and the next tiles can be selected. A point in Q according to the change in curvature can be selected to provide efficiency.

(5) Search for Minimum Cost Path

After assigning cost to an arc in graph, the optimal path is determined. The optimal path is defined as follows :

"The tiles constructed between two adjacent contours are represented as the arc in graph and the cost is assigned to arc as proposed in section (4). Namely, the cost is defined as the sum of the length of two spans in a tile. The optimal path is the trail in which the sum of costs of arcs is of minimum cost." The problem of finding an optimal path in a cyclic graph G is reduced to finding a minimum cost path in a planar graph G'. G' is defined by

$G' = \langle V', A' \rangle$

$V' = \{ V_{ij} \mid i=0,1,---,2m \; ; \; j=0,1,---,n \}$

$A' = \{\langle V_{kl}, V_{st}\rangle \mid (s=k \text{ and } t=l+1) \text{ or } (s=k+1 \text{ and } t=l)\}$

$C(\langle V_{kl}, V_{st}\rangle) = C(\text{Cost of } \langle V_{kl}, V_{st}\rangle)$

Let the path start at V_{io}. There exists a one - to - one correspondence between the set of paths from V_{io} to $V_{i+m,n}$ in G' and the set of paths which start and end at V_{io} in G. Therefore, the optimal path can be found by searching the path in which the sum of costs of arcs is of minimum cost.

3. Estimation of the Volume and Surface Area and Reconstruction of the 3-Dimensional Image

(1) Estimation of the Volume and Surface Area

The surface between the two 2 adjacent contours consists of m+n tiles. Let the vertices of the ith triangle be denoted by P(i), Q(i) and R(i), where the vertices P(i) and R(i) lie on the contour line P successively and the vertex Q(i) lies on the contour line Q. Assume that U is a point in the plane P and L a point in the plane Q.

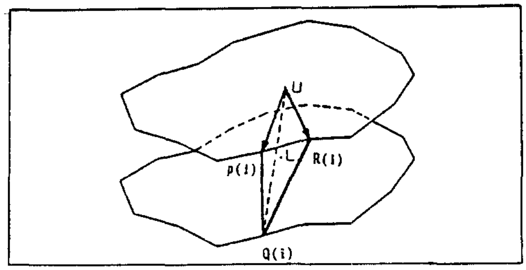


Fig.3-1 Two cross sections and a triangular tile

Form the following vectors as shown in figure 3.1 :

$\overline{A(i)} = \overline{UP(i)}$

$\overline{B(i)} = \overline{UQ(i)}$

$\overline{C(i)} = \overline{UR(i)}$

Rename the points on the contour line Q S(j) j=1,2,---, n) and form the vectors

$\overline{B(j)} = \overline{US(j)}$

$\overline{B(j+1)} = \overline{US(j+1)}$

$\overline{D} = \overline{UL}$

Then the volume between the two cross sections is given by the following formula :

$$V(k-1,k) = \left| \sum_{i}^{n+m} \overline{A(i)}\,\overline{B(i)} \times \overline{C(i)} + \sum_{j}^{m} \overline{B(j+1)}\,\overline{B(j)} \times \overline{D} \right| / 6$$

Here, v(k-1, k) is the volume between the (k-1)th cross section and kth cross section, and the total volume of an object is the sum of these partial volumes.

$$V = \sum_{K}^{n} V(k-1,k)$$

n : (number of cross sections) - 1

Surface area can be obtained by summing the areas of triangle. For the ith triangle, form the vectors

$\overline{P(i)Q(i)} = \overline{E(i)}$ $\quad\quad$ $\overline{P(i)R(i)} = \overline{F(i)}$

Then the surface area S is given by the following formula :

$$S = \left[\sum_{i}^{m} \left|\overline{E(i)} \times \overline{F(i)}\right| + \sum_{i}^{n} \left|\overline{F(i+1)} \times \overline{E(i)}\right| \right] / 2$$

(2) Reconstruction of 3-Dimensional Image

In order to represent an object in a 3-dimensional world, a 3-dimensional coordinate system is needed, rectangular coordinate system or spherical coordinate system. In the present study, the rectangular coordinate system is employed.

The 3-dimensional world coordinate system needs to be transformed into the 2-dimensional screen coordinate system. Let the 3-dimensional world coordinate system be (x, y, z) and the origin be O. Let the eye coordinate system be (xe, ye, ze) and the origin be Oe. And let the viewer's eye be in Oe(D, 0, 0), where D is the distance between O and Oe.

Then the 3-dimensional world coordinate system can be transformed into the eye coordinate system by the following transformation matrix, Tc.

$$Tc = \begin{bmatrix} -\sin\theta & -\cos\theta\cos\phi & -\cos\theta\sin\phi & 0 \\ \cos\theta & -\sin\theta\cos\phi & -\sin\theta\sin\phi & 0 \\ 0 & \sin\phi & -\cos\phi & 0 \\ 0 & 0 & D & 1 \end{bmatrix}$$

A point (xe, ye, ze) in an eye coordinate system can be computed from the point in a world coordinate system.

$$(xe,\ ye,\ ze,\ 1) = (x,\ y,\ z,\ 1)\ Tc$$

$$xe = -\sin\theta + y\cos\theta$$

$$ye = -x\cos\theta\cos\phi - y\sin\theta\cos\phi + z\sin\phi$$

$$ze = -x\cos\theta\sin\phi - y\sin\theta\sin\phi - z\cos\phi + D$$

The screen coordinates (xs, ys) of the projected image of the point P measured in eye coordinates (xe, ye, ze) are easily computed by considering the YeZe plane drawn in Figure 3-2.

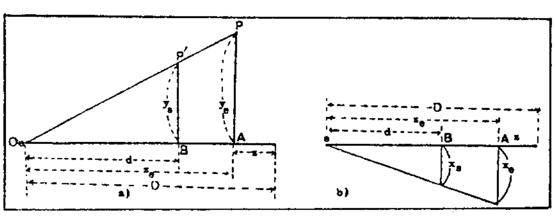


Fig. 3-2 Relation among the projection parameters

Here, d is the parameter representing the distance between the display screen and the view point. From the relationship of similar triangles, xs and ys can be obtained.

$$\frac{ys}{d} = \frac{ye}{ze} \quad \rightarrow \quad ys = d\,\frac{ye}{ze}$$

and

$$\frac{xs}{d} = \frac{xe}{ze} \quad \rightarrow \quad xs = d\,\frac{xe}{ze}$$

In our study, 3-dimensional image from the direction of $\theta = 0$ and $\phi = \sin^{-1}(0.47)$ is drawn.

4. Computer Simulation and Results

For computer simulation, the algorithms are applied to the following ellipsoid.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leqq 1$$

To select the contour points from the primary points, some proper values of T are given to these algorithms. (T=1.000, 1.0005, 1.001, 1.002, 1.003, 1.004, 1.005) For each value of T, the execution time for computing the costs of arcs and the time for searching the optimal path (minimum cost path) are measured. The information on the contour points constructs a data structure using the circular list described previously, and these algorithms are executed on the IBM-PC XT using the C language.

The next tables and figures show the results that the contour points obtained from the 17 cross sections of sphere (a=b=c=2.0) are processed by these algorithms.

Table 4-1 shows the time for weighting the costs of arcs. As the value of T is increased, the time is increased twice at its maximum. This is related to the number of contour points. When the restricted algorithm for searching the optimal path is executed, the time for search is less than 1 second, not affecting the entire execution time.

Table 4-2 shows the error for measured values of volume and surface area. When the tolerance has the value of 1.000, the measured volume and surface area has the least error of 2.88% and 0.352%, respectively. This means that the more the contour points is taken, the more accurate the measured value is. Error for the other values of tolerances is compared with the value of 1.000.

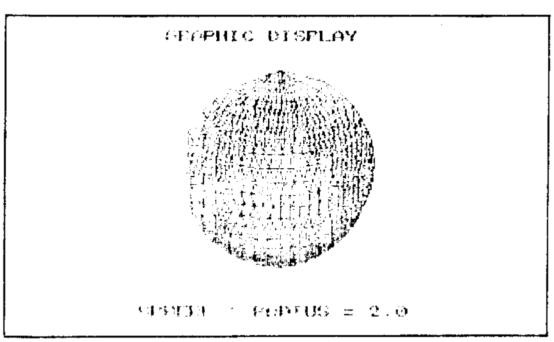
Figure 4-1 shows the reconstruction of 3-dimensional images for the values of tolerances 1.0005, 1.001 and 1.002.

Table 4-1 Time for weighting the cost of arc

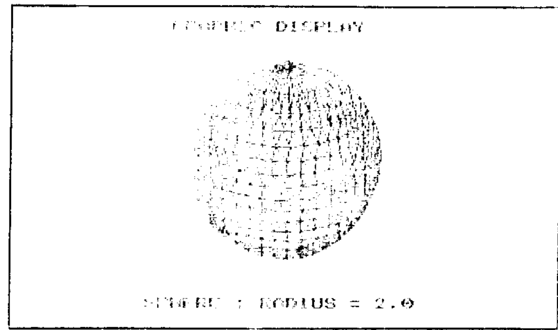
Tolerance	Time for Weighting the Cost of Vertex	Time for Weighting the Cost of Arc	Entire Execution Time	Number of Points
1.0000	129 Sec	21 Sec	150 Sec	993
1.0005	83 Sec	18 Sec	101 Sec	792
1.0010	41 Sec	16 Sec	57 Sec	558
1.0020	20 Sec	9 Sec	29 Sec	360
1.0030	12 Sec	7 Sec	19 Sec	280
1.0040	10 Sec	5 Sec	15 Sec	232
1.0050	7 Sec	4 Sec	11 Sec	205

Table 4-2 Measurement of volume and surface area

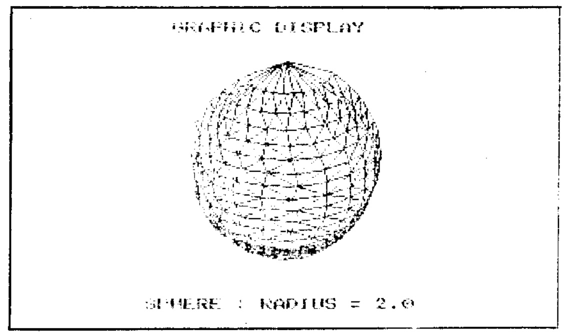
Tolerance	Estimated Volume	% Difference with *P	Estimated Surface Area	% Difference with *S	Number of Points
1.0000	*P 32.545	0.00	*S 50.083	0.000	993
1.0005	32.535	0.03	50.150	0.134	792
1.0010	32.394	0.46	50.074	0.017	558
1.0020	32.004	1.66	50.042	0.082	360
1.0030	31.645	2.76	53.450	6.720	280
1.0040	31.256	3.96	54.480	8.780	232
1.0050	30.840	17.14	55.390	10.600	205
Real Volume V=33.510			% Difference with *P (V-*P)/V 100=2.88		
Real Surface Area S=50.265			% Difference with *S (S-*S)/S 100=0.36		



(a) T = 1.0005



(b) T = 1.001



(c) T = 1.002

Fig.4-1 Reconstruction of 3-dimensional image

5. Conclusion

The results are summarized as follows : for the tolerance values of 1.0005, 1.001, and 1.002, the execution time for the algorithms is reduced to 66% - 80% and the error for the measured value is less than 3%. Thus, it is demonstrated that the algorithms proposed here is efficient to reduce the execution time and error for measurement.

When these algorithms are applied to the data obtained from human organ by the ultrasound image, the volume and surface area can be measured with less error by selecting the proper values of tolerances. These quantitative factors and the reconstruction of 3-dimensional image will be able to enhance ultrasound imaging.

References

1) H.Fuchs, Z.M.Kedem, and S.P.Uselton, "Optimal surface reconstruction from planar contours", Comm. ACM, vol.10, pp 693-702, 1977

2) P.N.Cook, L.T.Cook, K.R.Lee, et al., "An algorithm for volume estimation based on polyhedral approximation", IEEE Trans. Biomed. Eng., vol. BME-27, pp 493-500, 1980

3) Y.Watanabe, Y.Nose, S.Sanefuji, M.Yokota, and M.Nakamura, "A method to estimate volume and surface area of organ two dimensional echocardiography", IEEE Trans. Biomed. Eng., vol. BME-28, pp 294-297, 1981

4) Y.Watanabe, "A method for volume estimation by using vector areas and centroids of serial cross sections", IEEE Trans. Biomed. Eng., vol. BME-29, pp 202-205, 1982

5) F.B.James, K.M.Saundra, L.P.Richard, et al., "In vitro evaluation of an ultrasonic 3-dimensional imaging and volume system", Utrasonic Imaging, vol. 4, pp 126-139, 1982