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Receding Horizon Tracking Controller and Its Stability Properties

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ABSTRACT

The receding horizon tracking control for the discrete time invariant systems is presented in this paper. This control law is derived with the receding horizon concept from the standard tracking problems. Stability properties of this control law are analyzed. It is shown that there exists a finite horizon index for which the closed loop systems are always asymptotically stable. The receding horizon tracking control is a kind of predictive control and will add a new clan to many existing predictive controls, with which some comparisons are made.

1. Introduction

We consider a linear time-invariant discrete system described by

$$x(k+1) = Ax(k) + Bu(k) \tag{1a}$$

$$y(k) = cx(k) \tag{1b}$$

where $x(k) \in R^n$ is the state vector, $u(k) \in R^m$ is the control vector, and $y(k) \in R^p$ is the output vector.

The strategy for the receding horizon tracking control (RHTC) can be summarized as follows :

1) it is assumed that the finite future values of the desired trajectory are available at each instant of time. This is an acceptable assumption in many practical control problems.

2) at the present moment, control inputs, u(k), u(k+1), ... , u(k+N-1), are obtained to minimize the finite quadratic cost

$$J = J_1 + J_2 \tag{2a}$$

where the intermediate cost J_1 and the terminal cost J_2 are given by

$$J_1 = \tfrac{1}{2} \sum_{j=0}^{N-1} \{(y(k+j)-y_r(k+j))'Q(y(k+j)-y_r(k+j)) + u'(k+j)Ru(k+j)\} \tag{2b}$$

$$J_2 = \tfrac{1}{2}(y(k+N)-y_r(k+N))'F(y(k+N)-y_r(k+N)) \tag{2c}$$

Here Q and F are p×p real nonnegative definite matrices and R is an m×m real positive definite matrix.

3) only the first control input u(k) is applied at the present moment, and at the next moment the overall procedure is repeated.

We observe that for a given p×n (n>p) matrix C there always exists some n×p matrices L such that $CL=I_{p\times p}$. Therefore the cost (2b) and (2c) can be written as

$$J_1 = \tfrac{1}{2} \sum_{j=0}^{N-1} \{(x(k+j)-Ly_r(k+j))'c'Qc(x(k+j)-Ly_r(k+j)) + u'(k+j)Ru(k+j)\} \tag{3a}$$

$$J_2 = \tfrac{1}{2}(x(k+N)-Ly_r(k+N))'c'Fc(x(k+N)-Ly_r(k+N)) \tag{3b}$$

Since the terminal weighting matrix is a selectable design parameter we can represent the terminal cost as follows

$$J_3 = \tfrac{1}{2}(x(k+N)-Ly_r(k+N))'\tilde{F}(x(k+N)-Ly_r(k+N)) \tag{3c}$$

where $\tilde{F}$ is another design parameter. We denote

$$\tilde{J} = J_1 + J_3 \tag{3d}$$

RHTC may be thought as an extension of the receding horizon regulator [5,6], and is similar to the preview controller [1] in that the finite future values of the desired trajectory are available at the present moment. In the preview controller, however, the global cost interval is fixed while the available future trajectory recedes within the fixed cost interval whereas in RHTC both the cost interval and available future trajectory recede.

Lately, control design methods using the same strategies as in RHTC are found frequently in the literature, for example, generalized predictive control (GPC) [2], dynamic matrix control (DMC) [3], model algorithmic control (MAC) [4], and extended horizon adaptive control (EHAC) [11] etc. These control laws are called predictive controls and thus RHTC is considered as a new clan of predictive controls. Though these design methods have the same strategy as

RHTC, each of them uses a different plant model, a little different form of the quadratic cost, and different constraints on control inputs. All of these assume input/output models such as CARIMA(Controlled AutoRegressive Integrated Moving Average) models, step response models, impulse response models, and ARMAX models. The difference in the quadratic cost, however, could be overcome by proper settings of tuning parameters. Successful applications to real processes by using predictive controls have been reported [12,13].

Since RHTC adopts state space models, it may be thought to be a more general form than predictive controls based on input/output models such as GPC, DMC, MAC, and EHAC. The state space approach, in addition, gives some advantages in studying the internal properties of the closed loop control systems and enables the control design to be easily extended to time-varying systems.

In section 2, the algorithm of the discrete time RHTC is presented and compared with other predictive controls. Section 3 discusses the stability properties of RHTC and shows that the terminal weighting matrix F plays a important role in the stability of the closed loop system. Effects of the horizon depth N are also discussed. Conclusions are given in section 4.

2. Development of the receding horizon tracking control (RHTC)

It will be shown presently that the properties of RHTC depends on the terminal weighting matrix, so we introduce the following three different classes of the terminal weighting matrix.

(a) F (or $\tilde{F}$) is arbitrary but finite.

(b) F is finite but belongs to

$$\mathcal{E} = \{F : F>0,\ -c'Fc+A'[(c'Fc)^{-1}+BR^{-1}B]^{-1}A+c'Qc \le 0\}$$

$$(\text{or } \tilde{F} \in \tilde{\mathcal{E}} = \{\tilde{F} : \tilde{F}>0,\ -\tilde{F}+A'[(\tilde{F})^{-1}+BR^{-1}B]^{-1}A+c'Qc \le 0\})$$

(c) $\tilde{F}$ is infinite.

Following standard optimization procedures, the minimization of the cost (2a) in the cases of (a) and (b) gives the following control law :

$$u(k+j)=-[R+B'K(N-j-1)B]^{-1}B'[K(N-j-1)Ax(k+j) + g(k+j+1)] \quad j = 0,1,\ \dots\ , N-1 \qquad (4)$$

where $K(N-j-1)$ is obtained from the discrete time Riccati equation

$$K(i+1)=A'K(i)A-A'K(i)B[R+B'K(i)B]^{-1}B'K(i)A+c'Qc \qquad (5a)$$

$$=A'K(i)[I+BR\ \ B'K(i)]^{-1}A+c'Qc \qquad (5b)$$

$$,K(0)=c'Fc$$

and $g(k+j+1)$ is obtained from the following equation

$$g(k+j)=A'(I-K(N-j-1)B[R+B'K(N-j-1)B]^{-1}B')g(k+j+1)$$

$$-\ c'Qy_r(k+j) \qquad (6)$$

$$g(k+N)=\ -c'Fy_r(k+N)$$

The receding horizon strategy uses only the first control input $u(k)$, and at the next moment the whole procedure is repeated. Hence actually applied control input $u(k)$ is given by

$$u(k)=-[R+B'K(N-1)B]^{-1}B'[K(N-1)Ax(k)+g(k+1)] \qquad (7)$$

Using the state transition matrix $g(k+1)$ may be written as

$$g(k+1)=-\phi'(N,1)c'Fy_r(k+N)-\sum_{i=1}^{N-1}\phi'(i,1)c'Qy_r(k+i) \qquad (8)$$

where

$$A_c(k)=[I-B(R+B'K(N-k-1)B)^{-1}B'K(N-k-1)]A \qquad (9)$$

and $\quad \phi(k,k_0)=A_c(k-1)A_c(k-2)\ \dots\ A_c(k_0) \qquad (10)$

Now, the closed loop system is represented by

$$x(k+1)=A_c(0)x(k)-B[R+B'K(N-1)B]^{-1}B'[-\phi'(N,1)c'Fy_r(k+N)-\sum_{i=1}^{N-1}\phi'(i,1)c'Qy_r(k+i)] \qquad (11)$$

As we can see in (7), the control input $u(k)$ is composed of two parts : one is the state feedback control term and the other is the feedforward control term. It should be noted that the state feedback control term has fixed gain. If horizon N is large enough for the Riccati equation solution to converge to a steady state value, then RHTC with an appropriate terminal weighting matrix F becomes the same control law as the infinite time preview controller [1]. Fig. 1 shows the structure of the closed loop control system.

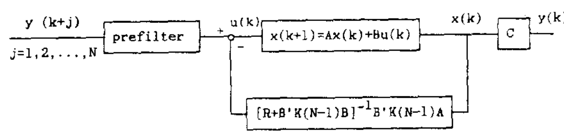

Fig.1 Structure of the closed loop RHTC system

For the cost (3d) the control law can be similarly given by equations (4)-(6) with $K(0)= c'Fc$ and $g(k+N) = -c'Fy_r(k+N)$ replaced by $K(0)= \tilde{F}$ and $g(k+N) = -\tilde{F}Ly_r(k+N)$, respectively.

It should be noted that the case of (c) is equivalent to impose the terminal constraint

$$x(k+N) = Ly_r(k+N) \qquad (12)$$

The optimal solution for this case is obtained by introducing the 2n-dimensional Hamiltonian system

$$\begin{pmatrix} x(k+1) \\ P(k+1) \end{pmatrix} = \begin{pmatrix} A+BR^{-1}B'A^{-T}c'Qc & -BR^{-1}B'A^{-T} \\ -A^{-T}c'Qc & A^{-T} \end{pmatrix} \begin{pmatrix} x(k) \\ P(k) \end{pmatrix} + \begin{pmatrix} 0 \\ A^{-T}c'Qy_r(k) \end{pmatrix} \qquad (13)$$

where $A^{-T}= (A')^{-1}$

Denote by $S(t)$ the 2n 2n state transition matrix of the system with four partitions defined by

$$S(t)=\begin{bmatrix}\Psi(t) & \Omega(t)\\ \chi(t) & \Lambda(t)\end{bmatrix} \qquad (14)$$

If the inverse of $\Omega(t)$ exists, the solution minimizing the cost (3c) is given by

$$u(k)=-R^{-1}B'[\hat{P}(N-1)Ax(k)+\bar{g}(k+1)] \qquad (15)$$

where

$$\hat{P}(k+1)=A^{-1}\hat{P}(k)A-A^{-1}\hat{P}(k)A^{-T}Q^{\frac{1}{2}}[I+Q^{\frac{1}{2}}A^{-1}\hat{P}(k)A^{-T}Q^{\frac{1}{2}}]^{-1}Q^{\frac{1}{2}}A^{-1}\hat{P}(k)A^{-T} \quad k=0,1,2,\ldots\ldots \qquad (16)$$

$$\hat{P}(0)=BR^{-1}B$$

and

$$\bar{g}(k+1)=A^{-T}[\Omega^{-1}(N)Ly_r(k+N)-\sum_{j=0}^{N-1}\Omega^{-1}(j+1)A^{-T}c'Qy_r(j+k)] \qquad (17)$$

We can see in [2,3,4,11] that various predictive control laws have similar forms as (7) and (15). All of them are composed of two parts : one is feedforward control term and the other is feedback control term, of which the receding horizon control laws are composed as well.

Since the parameters in (7) and (15) can be computed offline, the computation efforts for the control law (7) or (15) are comparable to other predictive control laws.

3. Stability of the closed loop system

To study the stability of RHTC we have only to consider the feedback control term in (7) or (15), which gives the closed loop system

$$x(k+1)=(I-B[R+B'K(N-1)B]^{-1}B'K(N-1))Ax(k) \qquad (18)$$

or

$$x(k+1)=(I-BR^{-1}B'\hat{P}(N-1))Ax(k) \qquad (19)$$

which are also obtained from the receding horizon regulation problems. For the continuous time case, it is known that the receding horizon control law with appropriate terminal constraints stabilizes the closed loop system [5,6,16]. We will presently show that in the discrete time RHTC problem the closed loop system is stable with a certain horizon depth N for finite terminal weighting matrices of (a), and for specific classes of the weighting matrices of (b) and (c) a simple horizon depth guarantees the closed loop stability.

For the case of (c) it is shown in [6] that if the pair {A,B} is completely controllable, $Q\geq 0$, and $R>0$, then for any $N\geq n+1$ the system (1) with the RHTC law (15) is asymptotically stable. The RHTC law (15) where terminal weighting matrices are infinite is of greater practical interest since the explicit horizon depth is given by the above result.

we state below a lemma which is prerequisite to the proof of the stability of RHTC for the cases of (a) and (b).

LEMMA 1 Let $Q>0$ and $R>0$. The solution $K(N-1)$ of the discrete Riccati equation (5) has upper and lower bounds for $N\geq n+1$ if the pair {A,B} is completely controllable, the pair {A,C} is completely observable, and the terminal weighting matrix is finite.

i.e.

$$\beta_1 I \leq K(N-1) \leq \beta_2 I \qquad (20)$$

where β_1 and β_2 are some positive constants.

PROOF : See appendix.

We will show in the next theorem that there exist a finite horizon N over which the closed loop system is asymptotically stable.

THEOREM 1 Let F is finite, $Q>0$, and $R>0$. If the system (1) is completely controllable and observable, then the closed loop system with the receding horizon control law is asymptotically stable.

PROOF : Let $K(\infty)$ be the steady state solution of (5) and let

$$\hat{K}(N-1)=[K^{-1}(N-1)+BR^{-1}B']^{-1}$$

and $\hat{K}_N^e=\hat{K}(\infty)-\hat{K}(N-1)$

Then we have from equation (18)

$$x(i+1)=G(\infty)x(i)+BR^{-1}B'\hat{K}_N^e Ax(i) \qquad (21)$$

where

$$G(\infty)=A-BR^{-1}B'\hat{K}(\infty)A$$

Since the steady state solution $x(i+1)=G(\infty)x(i)$ is known to asymptotically stable [7], for the proof of the asymptotic stability it is sufficient to show that the perturbed term in (21) has the property that $\|BR^{-1}B'\hat{K}_N^e Ax(i)\|/\|x(i)\|$ can be made arbitrary small for some N. The matrix $\hat{K}_N^e$ satisfies

$$\begin{aligned}\hat{K}_N^e&=[K^{-1}(\infty)+BR^{-1}B']^{-1}-[K^{-1}(N-1)+BR^{-1}B']^{-1}\\&=[K^{-1}(N-1)+BR^{-1}B']^{-1}K^{-1}(N-1)[K(\infty)-K(N-1)]\\&\quad K^{-1}(\infty)[K^{-1}(\infty)+BR^{-1}B']^{-1}\end{aligned} \qquad (22)$$

For a positive constant δ, we have

$$\begin{aligned}\|\hat{K}_N^e\|&\leq\|[K^{-1}(N-1)+BR^{-1}B']^{-1}K^{-1}(N-1)\|\,\|K(\infty)-K(N-1)\|\\&\quad\|K^{-1}(\infty)[K^{-1}(\infty)+BR^{-1}B']\|\\&\leq\delta\|K(\infty)-K(N-1)\| \quad \text{for } N\geq n+1\end{aligned} \qquad (23)$$

The second inequality is possible since $K(N-1)$ has upper and lower bounds by lemma 1, and $K(\infty)$ is a positive definite matrix under the controllability condition. B is also bounded by the controllability condition. By a lengthy but elementary calculation we can see that $K^e(i)\triangleq K(\infty)-K(i)$ satisfies

$$\begin{aligned}K^e(i)&=A'K(\infty)A-A'K(\infty)B[R+B'K(\infty)B]^{-1}B'K(\infty)A+c'Qc\\&\quad-A'K(i-1)A+A'K(i-1)B[R+B'K(i-1)B]^{-1}B'K(i-1)A-c'Qc\end{aligned}$$

$$=G'(\infty)K^e(i-1)G(\infty)+G'(\infty)K^e(i-1)B[R+B'K(i-1)B]^{-1}B'K^e(i-1)G(\infty)$$
$$\le G'(\infty)K^e(i-1)G(\infty)+G'(\infty)K^e(i-1)BR^{-1}B'K^e(i-1)G(\infty) \quad (24)$$

Since $K(\infty)$ is known to be bounded and the given F is also bounded, the terminal condition $K^e(0)=K(\infty)-F$ is bounded. Since $G(\infty)$ is known to be stable [7], $K^e(i)$ in (24) is asymptotically stable for a bounded terminal condition $K^e(0)$. Hence for every $\epsilon_1>0$ there exists a finite $N'=N'(\epsilon_1)$ such that $\|K^e(i)\|\le\epsilon_1$ for $i\ge N'$. From (23),

$$\|\hat{K}_N^e\|\le\delta\|K^e(N-1)\|\le\delta\,\epsilon_1 \qquad \text{for} \quad N\ge N'+1\ge n+1$$

Now we have

$$\|BR^{-1}B'\hat{K}_N^e Ax(i)\|/\|x(i)\|\le\|BR^{-1}B'\|\|\hat{K}_N^e\|\|A\|$$
$$\le\|BR^{-1}B'\|\|A\|\,\delta\,\epsilon_1 \qquad (25)$$

$\|A\|$ is bounded from the controllability condition. Given an $\epsilon_2>0$, choose $\epsilon_1=\epsilon_2/\|BR^{-1}B'\|\,\|A\|\,\delta$. Then it is clear that there exists a positive constant N such that

$$\|BR^{-1}B'\hat{K}_N^e Ax(i)\|/\|x(i)\| < \epsilon_2$$

This ends the proof

Q.E.D

Determining a suitable N over which the asymptotic stability is guaranteed is an important problem. Generally, small terminal weighting matrices lead to large horizons and large terminal weighting matrices to small horizons as can be seen in the next example.

EXAMPLE 1. We consider a SISO, time-invariant system and a modified quadratic cost

$$x(i+1)=ax(i) + bu(i)$$
$$J_m=\sum_{k=0}^{N-1}[qx^2(i+k)+ru^2(i+k)]+fx^2(i+N)$$

where $a>0$ and $b\neq 0$. By an elementary algebraic computation, we can show that the system can be stabilized by the receding horizon control law with the finite horizon depth N as follows :

$$N>1 \qquad \text{for } f=\infty$$
$$N>1+\log_\alpha\{b^2K[(a-1)K+q)][b^2K-(a-1)r]^{-1}/q\} \quad \text{for } f=0$$

where

$$K=\{ra^2+qb^2-r+[(ra^2+qb^2-r)^2+4rqb^2]^{\frac{1}{2}}\}/(2b^2)$$

and

$$\alpha=(Kb^2+r)/(ra^2+qb^2-kb^2)$$

Besides the determination of a proper N to guarantee the asymptotic stability, system performance analysis with different values of N is another important problem to be studied [8,9,14].

Related to the above arguments, we want to seek a somewhat general classes of terminal weighting matrices which render an explicit horizon depth. We have seen that the case of (c) gives an explicit horizon depth. Here we consider the class of matrices defined as (b). First, we present the following lemma.

LEMMA 2 If the matrix F belongs to the class defined as in (b), then the solution matrix K(j) of the Riccati equation (5) satisfies the following inequality :

$$K(j)\ge K(i) \quad \text{for} \quad j\le i \qquad (26)$$

PROOF This result can be derived by some monotone properties of discrete Riccati equations. Consider two matrix equations as follows :

$$M_{i+1}=A'[M_i^{-1}+BR^{-1}B']^{-1}A+Q_1, \quad M_0=F_1 \qquad (27a)$$
$$N_{i+1}=A'[N_i^{-1}+BR^{-1}B']^{-1}A+Q_2, \quad N_0=F_2 \qquad (27b)$$

It is well known that

(A) if $F_1=F_2$ and $Q_1\ge Q_2$, then $M_i\ge N_i$ for all $i\ge 0$

(B) if $Q_1=Q_2$ and $F_1\ge F_2$, then $M_i\ge N_i$ for all $i\ge 0$

These inequality immediately provide (26) since (A) and $F=K(j)$ imply $F_i\ge K(j-i)$ for $j\ge i\ge 0$ and thus $K(j-i)\ge K(k-i)$ for $k\ge j\ge i$ by the result of (B) and $K(j)=F\ge K(k-j)$.

Q.E.D

It is clear that in the proof of LEMMA 2, F is no less than the steady state solution $K(\infty)$. Now, we are in a position to prove the following result which gives an explicit horizon depth for a class of terminal weighting matrices.

THEOREM 2 Suppose F belongs to the class $\mathcal{E}$, $Q>0$, and $R>0$. If the pairs {A,B} and {A,C} are completely controllable and observable, respectively, then for any $N\ge n+1$ the closed loop system is asymptotically stable.

PROOF We consider the adjoint system of (18) :

$$\hat{x}(k+1)=\{A-B[R+B'K(N-1)B]^{-1}B'K(N-1)A\}^{-T}\hat{x}(k) \quad (28a)$$
$$=\{A-BR^{-1}B'\hat{K}(N-1)A\}^{-T}\hat{x}(k) \qquad (28b)$$

By lemma 1 and the boundedness of B, we obtain the following inequality

$$\beta_2^{-1}I\le\hat{K}^{-1}(N-1)\le(\beta_1+\alpha_2^{-1}\beta^2)I \qquad (29)$$

where α_4 is a positive constant such that $\|B\|\le\alpha_4$. If we define a scalar valued function

$$v(\hat{x},k)=\hat{x}(k)'A^{-1}\hat{K}^{-1}(N-1)A^{-T}\hat{x}(k) \qquad (30)$$

then we have for some positive constants α_5 and α_6

$$\alpha_5\|x\|^2\le v(\hat{x},k)\le\alpha_6\|\hat{x}\|^2. \qquad (31)$$

That is, $v(\hat{x},k)$ is a positive definite function of $\hat{x}$. Now, we obtain the difference of $v(\hat{x},k)$ along the solution of the adjoint system (28) as follows :

$$v(\hat{x}(k),k)-v(\hat{x}(k+1),k+1)$$
$$=\hat{x}'(k)A^{-1}\hat{K}^{-1}(N-1)A^{-T}\hat{x}(k)-\hat{x}'(k+1)A^{-1}\hat{K}^{-1}(N-1)A^{-T}\hat{x}(k+1)$$
$$=\hat{x}'(k+1)\{I-BR^{-1}B'\hat{K}(N-1)]\hat{K}^{-1}(N-1)[I-\hat{K}(N-1)BR^{-1}B']\hat{x}(k+1)-\hat{x}(k+1)A^{-1}K^{-1}(N-1)A^{-T}\hat{x}(k+1)$$
$$=\hat{x}'(k+1)\{\hat{K}^{-1}(N-1)-2BR^{-1}B'+BR^{-1}B'\hat{K}(N-1)BR^{-1}B'-A^{-1}\hat{K}^{-1}(N-1)A^{-T}\}\hat{x}(k+1) \qquad (32)$$

It can be easily checked that the matrix $\hat{K}^{-1}(N-1)$ satisfies the following equation

$$\hat{K}^{-1}(N-1) = A^{-1}\hat{K}^{-1}(N-2)A^{-T}+BR^{-1}B'-z(N-2) \tag{33}$$

where z(N-2) is some nonnegative definite matrix. Hence

$$\begin{aligned} & v(\hat{x}(k),k)-v(\hat{x}(k+1),k+1) \\ & = \hat{x}'(k+1)\{A^{-1}[\hat{K}^{-1}(N-2)-\hat{K}^{-1}(N-1)]A^{-T}-BR^{-1}B'-z(N-2) \\ & \quad + BR^{-1}B'\hat{K}(N-1)BR^{-1}B'\}\ \hat{x}(k+1) \\ & = -\hat{x}'(k+1)\{A^{-1}[\hat{K}^{-1}(N-1)-\hat{K}^{-1}(N-2)]A^{-T}+z(N-2) \\ & \quad + BR^{-\frac{1}{2}}[I-R^{-\frac{1}{2}}B'\hat{K}(N-1)BR^{-\frac{1}{2}}]R^{-\frac{1}{2}}B'\}\ \hat{x}(k+1) \\ & \le -\hat{x}'(k+1)BR^{-\frac{1}{2}}s(N-1)R^{-\frac{1}{2}}B'\hat{x}(k+1) \end{aligned} \tag{34}$$

where

$$\begin{aligned} s(N-1) & \triangleq I-R^{-\frac{1}{2}}B'\hat{K}(N-1)BR^{-\frac{1}{2}} \\ & = I-R^{-\frac{1}{2}}B'[K^{-1}(N-1)+BR^{-1}B']^{-1}BR^{-\frac{1}{2}} \\ & = [I+R^{-\frac{1}{2}}B'K(N-1)BR^{-\frac{1}{2}}]^{-1} \end{aligned} \tag{35}$$

Inequality comes from that $K^{-1}(N-1) \ge K^{-1}(N-2)$ by lemma 2. Let $\alpha_2 I \le R \le \alpha_3 I$

Since $\| s(i)\| \ge (1+\beta_2\alpha_2^{-1}\alpha_4^{2}) \triangleq \alpha_7$

$$v(\hat{x}(k+1),k+1)-v(\hat{x}(k),k) \ge \alpha_6^{-1}\alpha_8\ \hat{x}'(k+1)BB'\hat{x}(k+1) \tag{36}$$

Thus we have for $k \ge k_0+n-1$

$$\begin{aligned} & v(\hat{x}(k+1\ ;\hat{x}(k_0),\ k_0),\ k+1)-v(\hat{x}(k_0),\ k_0) \\ & \ge \alpha_3^{-1}\alpha_7\ \hat{x}'(k_0)\ [\sum_{i=k_0}^{k}\phi_p(k_0-i-1)BB'\phi_p(k_0-i-1)]\hat{x}(k_0) \\ & \ge \alpha_3^{-1}\alpha_7\alpha_8\,|\ \hat{x}(k_0)\ |^2 \end{aligned} \tag{37}$$

where $\phi_p(i)$ is the state transition matrix of the closed loop system (18) and α_8 is a positive constant such that

$$\| \sum_{i=k_0}^{k}\phi_p(k_0-i-1)BB'\phi_p'(k_0-i-1)\| \ge \alpha_8 \tag{38}$$

It is known that the pair {A-BG,B} is completely controllable if {A,B} is completely controllable and G is bounded [10]. Hence $\{A-BR^{-1}B'\hat{K}(N-1)A,B\}$ is completely controllable, from which the inequality (38) follows. Therefore, the solution of the adjoint system (28) is exponentially increasing, which implies that the original system (18) is exponentially decreasing. This shows that the closed loop system (18) is asymptotically stable.

Q.E.D

The above theorem indicates that there exists a class of terminal weighting matrices which gives stabilizing control laws with a horizon slightly greater than the system order. From the above results it is conjectured that large horizons are necessary if terminal weighting matrices are small, whereas for large terminal weighting matrices horizons can be made small.

Up to now, the stability properties of predictive control laws based on input/output models have not fully investigated. It should be noted, however, that input/output models can be easily transformed to state space models, and it is possible to study stability properties of one control law in the transformed models. In this context, the stability properties of RHTC proven here can be used for the stability proof of other predictive control laws which is based on input/output models. The stability result of RHTC for the case (c) can be directly used for EHAC since two controls become equal problems if Q=0 in RHTC. The result for the case (a) can be exploited in GPC since GPC with proper parameter settings is equal to RHTC. In addition, it is believed that the results of this paper can give some insights for the stability properties of MAC and DMC .

4. Conclusions

This paper has described the receding horizon tracking control algorithm which is based on the minimization of the quadratic cost with receding horizon concept. The obtained control law is the simple fixed gain state feedback controller with feedforward control term that provides the preview actions. Indeed, due to the comparatively simple form of the control algorithm, this could be mounted in a microcomputer.

It has been shown that the control law has a finite horizon N over which the closed loop system is asymptotically stable. It is an interesting result that for specific classes of the terminal weighting matrix F the horizon depth N greater than the system order provides the asymptotic stability. The stability properties proven in this paper will give insights into the proof of the similar predictive control design methods.

RHTC can be formulated as well for time-varying systems, and similar stability results can be obtained by the same procedures.

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Appendix : Proof of lemma 1

A similar result is found in [15] where continuous time systems are dealt with. Thus we only sketch the proof. The Riccati equation (5) is obtained as well from the regulator problem for the system (1), where the performance index is given by

$$J_1 = y'(N)Fy(N) + \sum_{k=0}^{N-1} [y'(k)Qy(k)+u'(k)Ru(k)]$$

Vector functions of time, $u(k)$ and $y(k)$, can be considered as elements of the Hilbert spaces $H1= R^m \times R^m \times \ldots \times R^m$ (N times) and $H2= R^p \times R^p \times \ldots \times R^p$ (N+1 times), respectively, with corresponding inner products defined as

$$\langle u_1, u_2 \rangle_R = \sum_{k=0}^{N-1} u_1'(k)Ru_2(k)$$

and

$$\langle y_1, y_2 \rangle_Q = \sum_{k=0}^{N} y_1'(k)Q(k)y_2(k)$$

where $Q(k) = \begin{cases} F & \text{if } k=N \\ Q & \text{otherwise} \end{cases}$

The linear operator G, mapping H1 into H2, is defined as

$$(Gu)(i) = \sum_{k=0}^{i-1} CA^{i-k-1}Bu(k)$$

Let $y_h(i)=CA^i x(0)$ and $J_h= \| y_h \|_Q^2$. If $\hat{u}$ is an arbitrary control and $\hat{y}$ is the corresponding output, then it holds that $\hat{y} = y_h + G\hat{u}$

and $J_1(\hat{u},x(0)) = \|\hat{u}\|_R^2 + \| G\hat{u} + y \|_Q^2$

The vector pair $(\hat{u},\hat{y})$ of the Hilbert space H3=H1×H2 with inner product defined as

$$\langle (\hat{u}_1,\hat{y}_1),\ (\hat{u}_2,\hat{y}_2) \rangle = \langle \hat{u}_1,\hat{u}_2 \rangle_R + \langle \hat{y}_1,\hat{y}_2 \rangle_Q$$

belongs to the linear variety V of H3. Therefore, by projection theorem

$$\langle (u^*,y^*),\ (u^*-\hat{u},\ y^*-\hat{y}) \rangle = 0$$

from which

$$J_1(u^*,x(0)) \triangleq J_1^* = \langle (u^*,u^*),\ (y^*,y^*) \rangle = \langle u^*,\hat{u} \rangle_R + \langle y^*,\hat{y} \rangle_Q.$$

From the fact that $J_1^* = \|u^*\|_R^2 + \|y_h + Gu^*\|_Q^2$ it follows that

$$J_1^* = J_h - \|Gu^*\|_Q^2 - \|u^*\|_R^2 = J_h + \langle Gu^*,\ y_h \rangle_Q .$$

Using this equation it can be obtained that

$$\|Gu^*\|_Q^2 \geq (J_h - J_1^*)^2/J_h$$

and

$$J_1^*/J_h \leq 1 - (\|Gu^*\|^2/J_h)(1+1/\|G\|^2).$$

Thus it is obtained that

$$1 \geq J_1^*/J_h \geq 1/(1+\|G\|^2) \qquad \text{(A.1)}$$

It is known that

$$J_1^* = x(0)'K(N)x(0) \qquad \text{(A.2)}$$

and

$$J_h = x(0)'K_h(N)x(0) \qquad \text{(A.3)}$$

where $K_h(N)$ is the solution of the matrix Lyapunov equation

$$K_h(i+1) = A'K_h(i)A + cQc$$

$$K_h(0) = c'Fc$$

Since {A,C} is completely observable, $K_h(N)$ is a positive definite matrix for $N \geq n$. This fact with (A.1), (A.2), and (A.3) says that

$$K(N) \geq \frac{1}{1+\|G\|^2} K_h(N) \triangleq \beta_1 I \text{ for } N \geq n$$

where β_1 is a positive constant. The upper bound of K(N) for $N \geq n$ is guaranteed by the complete controllability condition, viz.

$$K(N) \leq \beta_2 I \quad \text{for} \quad N \geq n$$

where β_2 is a positive constant.