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AN APPLICATION OF A PERIODICALLY TIME-VARYING DIGITAL FILTER TO A FILTER FOR SCRAMBLE SPECTRUM

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Abstract: This paper proves that a filter for scramble spectrum is realized by a periodically time-varying digital filter. Further, we propose a design method of this filter by using a bi-frequency map. As an example, we implemented this filter with Digital Signal Processor.

1. Introduction

Generally, a periodically time-varying digital filter has a spectrum exchange characteristics in real time. That is, an input signal with a certain frequency is exchanged to A signal with different frequency through a periodically time-varying digital filter. This means that spectrum of an input signal is divided into some frequency regions and those frequency regions are exchanged each other by a periodically time-varying digital filter. A periodically time-varying digital filter can be employed as a filter for scramble spectrum.

This paper proposes a design method of a filter for scramble spectrum. Since the periodically time-varying digital filter is a bi-frequency system, the relationship between spectra of an input signal and of an output signal can be represented by a bi-frequency map. In this paper, the design of a filter for scramble spectrum is derived by using this map.

Finally, we implemented this filter with Digital Signal Processor(MB8764) and confirm this effect.

2. Preliminary

One model of a periodically time-varying digital filter is shown in Fig.1. This model is named polyphase model. This model is implemented by connecting some time-invariant filters in parallel. The number of time-invariant filters is equal to the period of a periodically time-varying digital filter. The output signal is obtained by selecting one of these output signals of these filters. The selection of a output signal corresponds to the sampling time. If the period is P, the number of time-invariant filters is equal to the period 'P'. Let an impulse response of each filter be $h_m(n)$ ($m=0, 1, \cdots, P-1$). The subscript m is assumed to indicate the filter number. The output signal y(n) of a periodically time-varying digital filter is written as follows;

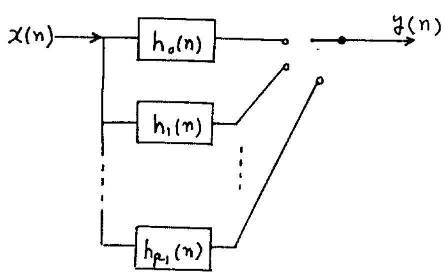

Fig. 1 A polyphase model

$$y(n) = \sum_{k=-\infty}^{\infty} h_{n \bmod P}(n-k)x(k) \qquad (1)$$

where x(k) is an input signal. Since $h_m(n)$ s are periodic with respect to m, they can be transformed by Discrete Fourier Series (DFS) with respect to m. Then a pair of DFS of order P are obtained by following equations.

$$\hat{h}_r(n) = \sum_{m=0}^{P-1} h_m(n)e^{-j2\pi rm/P} \qquad (2)$$

$$h_m(n) = \frac{1}{P}\cdot\sum_{r=0}^{P-1} \hat{h}_r(n)e^{j2\pi rm/P} \qquad (3)$$

where $\hat{h}_r(n)$ is DFS of $h_m(n)$. Substituting Eq.(3) into Eq.(1) yields

$$y(n) = \frac{1}{P}\cdot\sum_{r=0}^{P-1} \hat{h}_r(n-k)x(k)e^{j2\pi rn/P} \qquad (4)$$

From Eq.(4), the z-transform of an output y(n) is

$$Y(z) = \frac{1}{P}\cdot\sum_{r=0}^{P-1} \hat{H}_r(ze^{-j2\pi r/P})X(ze^{-j2\pi r/P}), \qquad (5)$$

where $\hat{H}_r(z)$ and X(z) are z-transforms of $\hat{h}_r(n)$ and x(n), respectively. From Eqs.(4) and (5), the input signal x(n) is filtered by $\hat{h}_r(n)$ and spectra of these obtained signals are shifted by each $\exp(j2\pi rn/P)$. The output signal is obtained by summing P spectrum shifted signals.

3.Characteristics of Periodically Time-Varying Digital Filters

Spectrum $Y(e^{j\omega_y})$ of an output signal of a periodically time-varying digital filter can be obtained by inserting $z=e^{j\omega_y}$ into Eq.(5)

$$Y(e^{j\omega_y})=\frac{1}{P}\cdot\sum_{r=0}^{P-1}\hat{H}_r(e^{j\omega_x})X(e^{j\omega_x}) \qquad (6)$$

In Eq.(6), the relationship between ω_x and ω_y is expressed as follows;

$$\omega_x=\omega_y-2\pi r/P+2m\pi \qquad (7)$$

$m=0,\ \pm1,\ \cdots\cdots$

The frequency axis ω_x is changed to ω_y by this filter. ω_x and ω_y are employed as x-axis and y-axis, respectively. This characteristics can be represented with a bi-frequency map. Fig.2 is an example of this map of the period P=4. There are

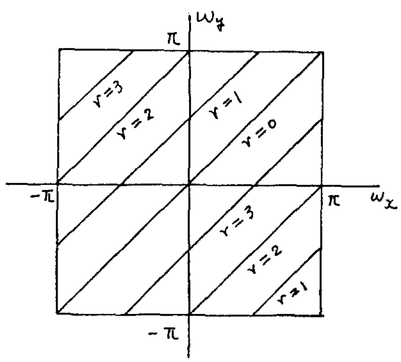

Fig. 2 An example of a bi-frequency map for P=4

four lines (r=0, 1, 2, 3) on the map. The frequency characteristics of $\hat{h}_r(n)$s are only defined as points on line r. Eq.(6) is also realized by the structure of Fig.3 [2]. A filter for scramble spectrum is an application of this structure. If each $\hat{h}_r(n)$ is a bandpass filter, the spectrum of an input signal is divided to some frequency regions. A signal in each frequency region is filtered with $\hat{H}_r(z)$. Each output signal is multiplied by exp(j2πrn/P). This means that

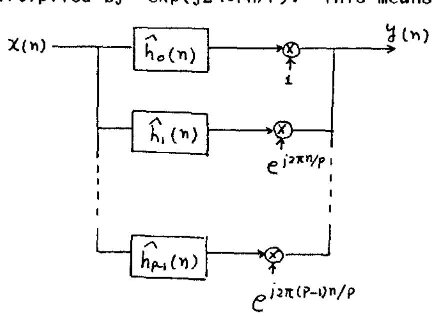

Fig. 3 A filter for scramble spectrum

spectrum of the output signal is shifted. So, we can exchange spectrum of a signal.

4. A Filter for Scramble Spectrum

We show filters for scramble spectrum in the following. This filter is implemented with the periodically time-varying digital filter whose period is even number. In the case of period P=8, an input signal is divided to eight frequency regions as shown in Fig.4 (a). Let the number of frequency regions be q (q=0, 1,⋯,7). Regions which correspond to q=0,⋯, P/2-2 , and P/2-1 represent the negative frequency regions. Regions which correspond to q=p/2,⋯ , P-2, and P-1 represent the positive frequency regions. Then q-th frequency region and (P-1-q)-th frequency region are a complex conjugate pair in frequency

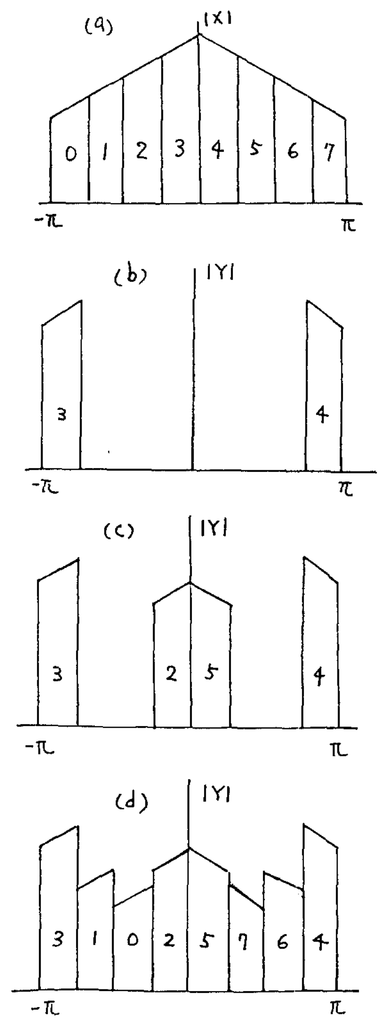

Fig. 4 Spectra of an input signal and an output signal
(a) A spectrum of an input signal
(b),(c) A spectrum of an output signal
(d) A desired spectrum of an output signal

characteristics of filters.

Fig.5 shows the bi-frequency map of this filter. A horizontal frequency line from $-\pi$ to π is divided into P frequency regions. Each frequency region is named with a number. A vertical frequency line is also divided into P frequency regions. Numbers upside of the map indicate input frequency regions. If we assume that the spectrum shown in Fig.4(d) is the desired spectrum of an output signal, numbers of frequency regions in the spectrum of an output signal are written on the left side of this map. Complex conjugate frequency regions in an input signal must be also complex conjugate regions in an output signal. There is a common region on the map which is indicated by a number on the horizontal line and the same number on the vertical line. In the common region, a line expressed with Eq.(7) exits. The line in a common region on this map indicates a passband of each

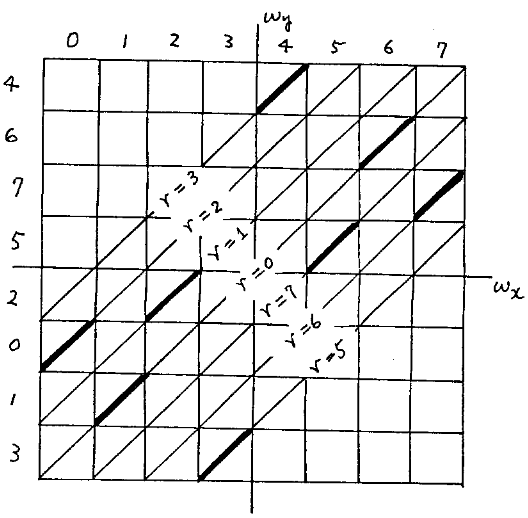

Fig. 5 A bi-frequency map of a filter for scramble spectrum

$\hat{h}_r(n)$. These are represented by deep lines on the map. For an example, the characteristics $H_3(e^{j\omega})$ of $\hat{h}_3(n)$ and $H_5(e^{j\omega})$ of $\hat{h}_5(n)$ are defined respectively by

$$\hat{H}_3(e^{j\omega}) = \begin{cases} P & 0 \le \omega < \pi/4 \\ 0 & \text{otherwise} \end{cases} \qquad (8)$$

$$\hat{H}_5(e^{j\omega}) = \begin{cases} P & -\pi/4 \le \omega \le 0 \\ 0 & \text{otherwise} \end{cases} \qquad (9)$$

By using $\hat{H}_3(e^{j\omega})$ and $H_5(e^{j\omega})$, frequency regions q=3, and q=4 are exchanged to highpass frequency regions in an output signal. It is shown in Fig.4(b). Furthermore, $\hat{H}_1(e^{j\omega})$ and $\hat{H}_7(e^{j\omega})$ are bandpass filters whose passband are expressed in the following.

$$\hat{H}_1(e^{j\omega}) = \begin{cases} P & -\pi/2 \le \omega \le -\pi/4 \\ 0 & \text{otherwise} \end{cases} \qquad (10)$$

$$\hat{H}_7(e^{j\omega}) = \begin{cases} P & \pi/2 \le \omega \le \pi/4 \\ 0 & \text{otherwise} \end{cases} \qquad (11)$$

The output spectrum is represented by Fig.4(c). Similarly, $H_0(e^{j\omega})$, $H_2(e^{j\omega})$, $H_4(e^{j\omega})$ and $H_6(e^{j\omega})$ are bandpass filters. These characteristics are expressed as follows;

$$\hat{H}_0(e^{j\omega}) = \begin{cases} P & -3\pi/4 \le \omega \le -\pi/2 \\ & \pi/2 \le \omega \le 3\pi/4 \\ 0 & \text{otherwise} \end{cases} \qquad (12)$$

$$\hat{H}_2(e^{j\omega}) = \begin{cases} P & -\pi \le \omega \le -3\pi/4 \\ 0 & \text{otherwise} \end{cases} \qquad (13)$$

$$\hat{H}_4(e^{j}\ ) = 0 \qquad (14)$$

$$\hat{H}_6(e^{j}\ ) = \begin{cases} P & 3\pi/4 \le \omega < \pi \\ 0 & \text{otherwise} \end{cases} \qquad (15)$$

As results, the desired output spectrum is obtained as shown in Fig.4(d). In this case, since all passband of filters are represented in the quadrants 1 and 3 on the map, frequency regions in the positive frequency domain are exchanged to in positive frequency domain. Frequency regions in the negative frequency domain are exchanged to in negative frequency domain.

Fig.6 shows an example map of a filter for scramble spectrum, by which frequency regions in

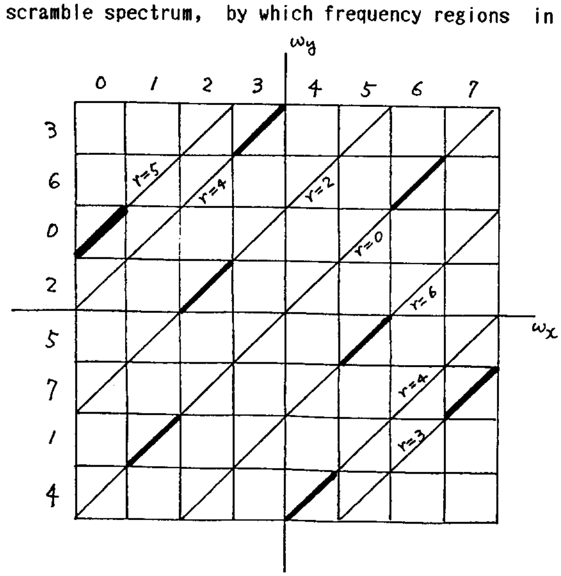

Fig. 6 A bi-frequency map of a filter for scramble spectrum

the positive frequency domain are exchanged to in negative frequency domain and frequency regions in negative frequency domain are exchanged to in positive frequency domain. In this case, passband of filters are represented in the quadrants 2 and 4 on the map. Fig.7 shows the spectrum of the output signal of this filter, when the spectrum of an input signal is shown by Fig.4(a).

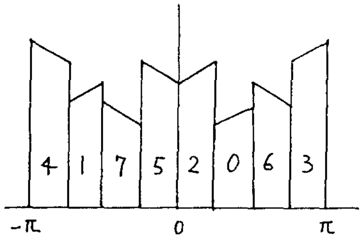

Fig. 7 The spectrum of an output signal of the filter given by Fig. 6

5. A Design of Filter for Scramble Spectrum

At first, we consider a design method of filter with the following frequency characteristic.

$$\hat{H}_1(e^{j\omega}) = \begin{cases} P & -\pi \le \omega \le -\pi/2, \\ & 0 \le \omega \le \pi/2 \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

$$\hat{H}_3(e^{j\omega}) = \begin{cases} P & -\pi/2 \le \omega \le 0, \\ & \pi/2 \le \omega < \pi \\ 0 & \text{otherwise} \end{cases} \tag{17}$$

$$\hat{H}_0(e^{j\omega}) = \hat{H}_2(e^{j\omega}) = 0 \quad \text{for any } \omega \tag{18}$$

The bi-frequency map which corresponds to this spectrum is shown in Fig.8. From the above

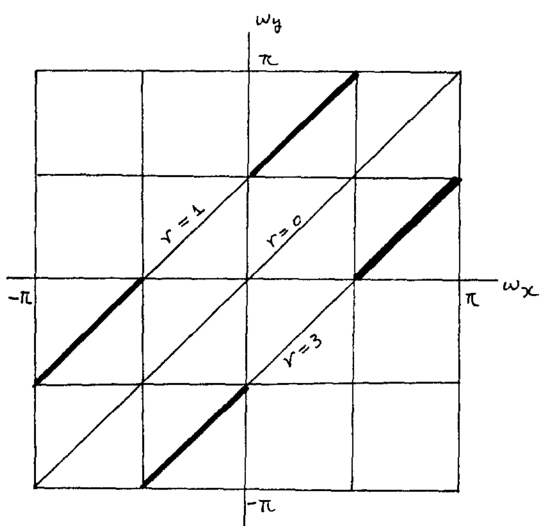

Fig. 8 A bi-frequency map of a filter

spectrum, each passband of $\hat{H}_1(j\omega)$ and $\hat{H}_3(e^{j\omega})$ is not symmetric with respect to $\omega = 0$. Then, these transfer functions have complex coefficients. But, the following relationship between $\hat{H}_1(e^{j\omega})$ and $\hat{H}_3(e^{j\omega})$ exist from the property of passband of these transfer functions.

$$\hat{H}_3(e^{j\omega}) = \hat{H}_1^*(e^{j\omega}) \tag{19}$$

$*$ represents complex conjugate. From Eq.(3), the relationship between $H_m(z)$ and $\hat{H}_r(z)$, which are z-transform of $h_m(n)$ and $\hat{h}_r(n)$, respectively, are obtained as follows;

$$H_m(z) = \frac{1}{P} \cdot \sum_{r=0}^{P-1} \hat{H}_r(z) e^{j2\pi rm/P} \tag{20}$$

So, the $H_m(z)$ is calculated in the case of the frequency characteristic as shown in Fig.8.

$$H_m(z) = \mathrm{Re}[\, H_1(z) e^{j\pi m/2} \,]/2 \tag{21}$$

The above transfer function has real coefficients. Then the transfer function can be implemented with a digital filter.

In general, a filter for scramble spectrum have the following property.

$$\hat{H}_{P-r}(z) = \hat{H}_r^*(z) \tag{22}$$

Therefore, a transfer function of a time-varying digital filter with real coefficients can be obtained. This transfer function can be implemented.

The above transfer function $\hat{H}_1(e^{j\omega})$ can be designed by using a conventional method. At first, we design a low pass filter $H_{LP}(z)$ whose passband is $-\pi/4 \le \omega \le \pi/4$. Inserting $z \cdot \exp(-\pi/4)$ and $z \cdot \exp(3\pi/4)$ into $H_{LP}(z)$, $H_1(z)$ is derived as follows;

$$\hat{H}_1(z) = H_{LP}(z \cdot e^{-j\pi/4}) + H_{LP}(z \cdot e^{j3\pi/4}) \tag{23}$$

From $\hat{H}_1(z)$, a transfer function $H_m(z)$ of a periodically time-varying digital filter is derived.

$$H_m(z) = \mathrm{Re}[\, \{H_{LP}(z \cdot e^{-j\ \ /4}) + H_{LP}(z \cdot e^{j3\ \ /4})\} \cdot e^{j2\ \ m/P}]/2 \tag{24}$$

From the above results, we can summarize a design method of a periodically time-varying digital filter.

(1)Determine a number N of frequency regions between 0 and π.

(2)Obtain a period of a periodically time-varying digital filter; P=2N.

(3)Determine a scramble method of spectrum.

(4)Design a low pass filter $H_{LP}(z)$ with a passband $|\omega| < \pi/2N$.

(5)According to the bi-frequency map, determine a passband of each filter $\hat{H}_r(z)$ and obtain these filters by shifting a center frequency of $H_{LP}(z)$.

(6)Calculate a transfer function from $\hat{H}_r(z)$ by inverse discrete Fourier series.

6. Implementation of a Filter for Scramble Spectrum

We implemented a filter for scramble spectrum with Digital signal processor (DSP MB8764). it is known that a polyphase model in Fig.1 can be implemented by a coefficient varying digital filter shown in Fig.9 . When each filter in a polyphase model is realized by a FIR digital

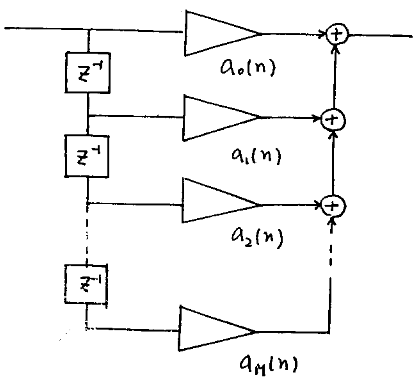

Fig. 9 A coefficient varying model

filter, it is easy to prove that coefficients $a_n(m)$s equal to $h_m(n)$s. This type of the structure is implemented easily with Digital Signal Processor.

We show a bi-frequency map of a designed filter in Fig.10 . A filter $H_{LP}(z)$ is designed by Remerz algorithm. We assume that a passband is $0 \leq \omega \leq 0.1\pi$ and a stopband is $0.122\pi \leq \omega \leq \pi$. The characteristic of this filter is shown in Fig.11 . The order of this filter is 96. The sampling frequency is 8 KHz. Fig.12 shows a relationship between a spectrum of an input signal and a spectrum of an output signal. From Fig.12 , we can see that a signal of 3.25 KHz is exchanged to a signal of 1.75 KHz.

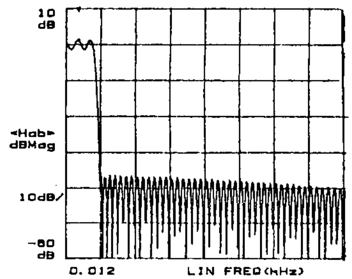

Fig. 11 The characteristic of a low pass filter

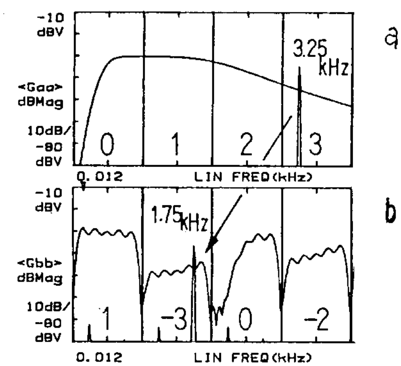

Fig. 12 Frequency characteristics of an input signal and an output signal of an example

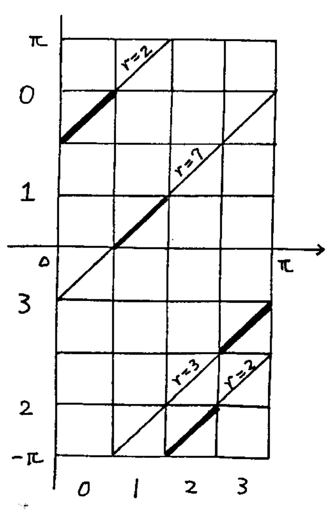

Fig. 10 A bi-frequency map of a filter

7. Conclusion

This paper made it clear that a filter for scramble spectrum is realized with a periodically time-varying digital filter. Moreover, we proposed a design method of this filter and implemented this filter by using Digital Signal Processor.

References

[1] C. M. Loeffler: Optimal Design of Periodically Time-Varying and Multirate Digital Filter, IEEE Trans. A.S.S.P., vol.ASSP-32, pp.991-997 (1984)

[2] E. R. Ferrara, Jr.: Frequency Domain Implementation of Periodically Time-varying Filters, IEEE Trans. A.S.S.P., vol.ASSP-33, pp.993-892 (1985)

[3] R. Ishii and M. Kakishita: Analysis of a Periodically Time-varying Digital filters, Procedings of 1986 ICASSP, pp.2607-2610 (1986)