ON THE COLLISION AVOIDANCE OF TWO MANIPULATORS

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ABSTRACT

This paper presents the recent findings for collision avoidance of two manipulators in addition to the results shown in Lee [4]. The collision situation we assume here is that the prespecified final time $\mathbf{K}_{\mathbf{f}}$ and the prespecified path of one robot can be modified for the purpose of collision avoidance with the other robot.

The collision avoidance problem is resolved into three independent categories for a systematic approach.

INTRODUCTION

Motion planning is composed of path planning and trajectory planning of the robot system. Collision-free motion planning is achieved through collision-free path planning schemes. Most of the path planning schemes concern the problem of avoiding fixed and stationary obstacles in a workspace. Due to the fixed and stationary obstacles, the pathe planning problem is usually converted to a geometric analysis problem for obtaining a collision-free path.

It is also notable that there are several experimental works on obstacle avoidance. Gouzenes [3] discussed collision avoidance of manipulators in a flexible assembly cell, using graph-search techniques and Petri nets. Myers et al. [6] used a fast static collision check for detecting potential collisions with obstacles. They developed a heuristic method to determine a collision-free path in a reasonable amount of time and demonstrated an application on a VAX-11/780 computer and a microcomputer. In particular, the collision avoidance problem associated with two manipulators in a common workspace

can be found in Freund's work [1,2].

In this paper we represent each robot by a single sphere at the wrist. It is assumed that each robot follows a straight line trajectory faithfully which is provided by the algorithmic straight line trajectory planner in Lee[5]. Thus, each robot exactly follows a straight line path at all the servo time periods, which is very important in defining the robot speed in the direction of the straight line path. It is also assumed that no collisions occur at the initial and final locations of the two robots.

PROBLEM FORMULATIONS

The method in obtaining a collision-free path or trajectory may vary depending on various collision situations especially when two robots are moving on the straight line paths simultaneously. When two robots (we call them robot 1 and robot 2) are assumed to move on their planned straight line paths with potential collisions, various collision situations are identified as follows:

- 1. Case 1: The final arrival time $\mathbf{k}_{\hat{\mathbf{f}}}$ of robot 2 can be relaxed but its original path cannot be changed for the purpose of collision avoidance because the modification of path may induce potential collisions with other fixed obstacles.
- 2. Case 2 : The final arrival time $k_{\mbox{f}}$ of robot 2 can be relaxed and its original path can be changed for the purpose of collision avoidance.
- 3. Case 3 : The final arrival time $\mathbf{k}_{\mathbf{f}}$ of robot 2 cannot be relaxed but its original path can be changed for the purpose of collision avoidance.
- 4. Case 4: The final arrival time $\mathbf{k_f}$ of robot 2 cannot be relaxed and its original path cannot be changed for the purpose of collision avoidance. In case 1, we can only change the speed or delay the

motion of robot 2 along the original path to avoid the collision. From now on, the terminology colli-

sion will be used to denote the terminology potential wrist collision. Since the change of robot speed can only be accomplished by modifying the trajectory information, a procedure for a speed change needs to be developed to obtain a collision-free trajectory of robot 2. The delay of robot 2 motion can also be utilized for the purpose of collision avoidance. It then corresponds to the time-coordinated, independent motion of the two robots. In case 2, collision-free motion planning can be considered in the following categories:

- category 1: when speed reduction and/or time delay of the robot 2 motion is applied without any path modification;
- category 2: when only path modification is applied;
- 3. category 3 : when path modification with speed reduction and/or time delay of the robot 2 motion are applied simultaneously.

In category 1, a collision-free path can be found exactly in the same way as in case 1. If a solution from category 1 is not adequate because a fairly large time delay is required or speed reduction is not appropriate for the collision avoidance, then we can consider a solution in categories 2 and 3. In category 2, there are a variety of freedoms in choosing a collision-free path. The choice of the collision-free path depends on the environment of the workspace and various userdesignated performance indices. Sometimes, a solution in category 2 corresponds to a solution in category 3 due to various reasons, for example, a robot speed constraint, a path deviation constraint, etc. Clearly, the unnecessary path deviation in category 2 can be reduced by reducing robot speed along a collision-free path.

In case 3, due to the fixed final arrival time $\mathbf{k_f}$, there is no guarantee that there exists a collision-free path for robot 2 satisfying the final arrival time $\mathbf{k_f}$. In case 4, the collision avoidance must be realized by changing the path and/or trajectory of the other moving robot.

As mentioned earlier, the results on the case 1 situation can be found in Lee [4], and the discussions on the case 3 and case 4 are not practically applicable in reality. Here, we investigate the case 2 collision situation through three independent categories.

COLLISION AVOIDANCE FOR CASE 2

A collision-free path of robot 2 is considered in the following categories: (1) when speed reduction or time delay is applied without any path modification, (2) when only path modification is applied without considering the trajectory information of two robots, and (3) when path modification and speed reduction or time delay are applied simultaneously.

Time Delay

As indicated in $\{4\}$, time delay yields the smaller final arrival time than speed reduction for avoiding a potential collision. Thus, time delay is considered in this section. If we denote the final arrival time of robot 2 as $\mathbf{k}_{\mathbf{f}}$ for the original trajectory, then the total traveling time after a time delay will be :

$$k_{TS} = k_f + \Delta k_f \tag{1}$$

where Δk_f is a required time delay for avoiding a potential collision. If Δk_f is very small compared to k_f , then time delay of the original trajectory provides us a good solution for the purpose of collision avoidance.

When Δk_f is fairly large, we can consider a solution in categories (2) and (3). Also, since two robots are working simultaneously in a common workspace, and their movements are coordinated in a time sequence, there is likely to be a constraint on the time delay of the robot 2 motion.

Path Modification

First, we find a collision-free path which deviates from the robot 1 path for at least a distance of $\mathbf{r}_1 + \mathbf{r}_2$ through a geometric analysis, where \mathbf{r}_1 and \mathbf{r}_2 denote the radius of the sphere model for each robot. The path is then guaranteed for the collision avoidance with robot 1. A collision situation is shown in Figure 1, where robot 1 moves from \mathbf{A}_1 to \mathbf{B}_1 , while robot 2 moves from \mathbf{A}_2 to \mathbf{B}_2 . We assume that a potential collision exists between the two moving robots. The following analysis indicates how to choose a point \mathbf{C}_2 such that the path from \mathbf{A}_2 to \mathbf{B}_2 via \mathbf{C}_2 deviates from the robot 1 path for at least a distance of $\mathbf{r}_1 + \mathbf{r}_2$. We denote the nearest two points on the robot 1 and robot 2 path as $\mathbf{K}_1(\mathbf{x}_{11}, \mathbf{y}_{11}, \mathbf{z}_{11})$ and $\mathbf{K}_2(\mathbf{x}_{12}, \mathbf{y}_{12}, \mathbf{z}_{11})$

robot 2 path as $K_1(x_{K1}, y_{K1}, z_{K1})$ and $K_2(x_{K2}, y_{K2}, z_{K2})$, respectively, as shown in Figure 1. It is notable that points A_2 , B_2 , K_1 , and K_2 are on the

same plane which can be constructed by $\overline{A_2B_2}$ and $\overline{K_1K_2}$.

It is notable that the swept volume of a sphere in the direction of a straight line forms a cylinder capped with two semi-spheres at both ends. Thus, for considering the distance of $r_1 + r_2$ between the two robots, we view the collision situation between the cylinder surface of radius $r_1 + r_2$ in the direction of A_1B_1 and the robot 2 path. There are two intersection points on the cylinder surface, we call them A and B, at which the cylinder surface is perforated by the robot 2 path. To maintain the distance of $r_1 + r_2$ between the robots, we consider the intersecting curve between the cylinder surface and the plane which is constructed by $\overline{A_2B_2}$ and K_1K_2 . We call the intersecting curve on the cylinder surface a collision-free locus. The collisionfree locus connecting A and B forms a part of an ellipse, which is centered at K_1 .

In order to obtain straight line segments which approximate the collision-free locus, we construct lines \mathbf{l}_1 and \mathbf{l}_2 which lead separately from \mathbf{A}_2 and \mathbf{B}_2 and are tangent to the ellipse. Consider two points \mathbf{Q}_1 and \mathbf{Q}_2 in the direction of $\overline{\mathbf{K}_1\mathbf{K}_2}$. We denote their deviations from \mathbf{K}_1 as \mathbf{d}^1 and \mathbf{d}^2 , respectively. First, we obtain the location of \mathbf{Q}_1 and \mathbf{Q}_2 in terms of \mathbf{d}^1 and \mathbf{d}^2 , respectively. Then, we find constraints on \mathbf{d}^1 and \mathbf{d}^2 to guarantee that the distances between two straight line paths $\mathbf{A}_1\mathbf{K}_1$ and $\mathbf{A}_2\mathbf{Q}_1$, and $\mathbf{B}_1\mathbf{K}_1$ and $\mathbf{B}_2\mathbf{Q}_2$ are equal to $\mathbf{r}_1+\mathbf{r}_2$. The location of $\mathbf{Q}_1(\mathbf{x}_{\mathbf{Q}1},\mathbf{y}_{\mathbf{Q}1},\mathbf{z}_{\mathbf{Q}1})$ is obtained in terms of \mathbf{d}^1 as:

$$\overrightarrow{oQ}_1 = \overrightarrow{oK}_1 + \frac{\overrightarrow{oK}_2 - \overrightarrow{oK}_1}{\|\overrightarrow{oK}_2 - \overrightarrow{oK}_1\|} \cdot d^1$$
 (2)

Since the value of \mathbf{d}_1 specifies the location of \mathbf{Q}_1 , we can represent the distance between two straight line paths $\mathbf{A}_1\mathbf{K}_1$ and $\mathbf{A}_2\mathbf{Q}_1$ in terms of the deviation \mathbf{d}^1 . If we denote this distance as DS₂, then :

$$DS_2 = f_1(d^1) \tag{3}$$

where \mathbf{f}_1 relates the distance between the paths of $\mathbf{A}_1\mathbf{K}_1$ and $\mathbf{A}_2\mathbf{Q}_1$ with the deviation \mathbf{d}^1 . To maintain the distance between the paths $\mathbf{A}_1\mathbf{K}_1$ and $\mathbf{A}_2\mathbf{Q}_1$ for at least \mathbf{r}_1 + \mathbf{r}_2 , we must have :

$$DS_{2} = f_{1}(d^{1}) \ge r_{1} + r_{2}$$
 (4)

Here, we will consider the equality only to avoid an unnecessary large deviation from \mathbf{K}_1 . Then, we can obtain a constraint on the deviation \mathbf{d}^1 as :

$$d^{1} = f_{1}^{-1}(r_{1} + r_{2}) \tag{5}$$

Similarly, for the paths of ${\rm B_1K_1}$ and ${\rm B_2Q_2}$, we have a constraint on the deviation ${\rm d}^2$ as :

$$d^2 = f_2^{-1}(r_1 + r_2) \tag{6}$$

where f_2 relates the distance between the paths of B_1K_1 and B_2Q_2 with the deviation d^2 . The location of Q_1 can be obtained from Eq. (2) by using d^1 of Eq. (5). Also, the location of Q_2 can be obtained from Eq. (2) by substituting d^2 of Eq. (6) for d^1 .

Using the locations of Q_1 and Q_2 , the two straight lines l_1 and l_2 in the directions of $\overline{A_2Q_1}$ and $\overline{B_2Q_2}$ can be identified easily. It is notable that l_1 and l_2 are on the same plane which is constructed by $\overline{A_2B_2}$ and $\overline{K_1K_2}$. Now, we consider the intersection point of these two straight lines l_1 and l_2 . Apparently, the path from A_2 to B_2 via the intersection point, which is denoted as C_2 , is a collision-free path in category (2). To find the deviation of this path from the original robot 2 path, the nearest point on $\overline{A_2B_2}$ from C_2 , which we call $\overline{C_2}$, is found from a vector projection and addition as:

$$\overrightarrow{OC}_{2} = \overrightarrow{OB}_{2} + (\overrightarrow{OA}_{2} - \overrightarrow{OB}_{2}) \cdot \frac{\overrightarrow{B_{2}C_{2}} \cdot (\overrightarrow{OA}_{2} - \overrightarrow{OB}_{2})}{\|\overrightarrow{OA}_{2} - \overrightarrow{OB}_{2}\|^{2}}$$
(7)

Then the actual deviation of the collision-free path from the original robot 2 path is found as :

$$d^{act} = \| \overrightarrow{oc}_2 - \overrightarrow{oc}_2 \|$$
 (8)

Note that no time delay of the robot 2 motion is required for the path from \mathbf{A}_2 to \mathbf{B}_2 via \mathbf{C}_2 for the purpose of collision avoidance, while $\Delta\mathbf{k}_{\mathbf{f}}$ in Eq. (1) is required for the original robot 2 path. Of course the time of travel from \mathbf{A}_2 to \mathbf{B}_2 via \mathbf{C}_2 will exceed $\mathbf{K}_{\mathbf{f}}$.

Time Delay with Path Modification

We now consider a collision-free path by both path modification and time delay. A path, which deviates from \overline{c}_2 for a distance of Δd , is considered. A point c_2 can be found for the deviation of Δd from \overline{c}_2 as :

$$\overrightarrow{oc_2^1} = \overrightarrow{oc}_2 + \frac{\overrightarrow{oc}_2 - \overrightarrow{oc}_2}{\|\overrightarrow{oc}_2 - \overrightarrow{oc}_2\|} \cdot \Delta d$$
 (9)

The point C_2^1 specifies the lengths of $\overline{A_2C_2^1}$ and $\overline{C_2^1B_2}$ as \overline{I}_1^1 and \overline{I}_2^1 , respectively.

As mentioned earlier, the two robots are moving simultaneously in a time-coordinated sequence. Other objects may need to move close to the initial location of robot 2. Thus, there is likely to be a constraint on the time delay at the initial location for avoiding the potential collision. If the allowable time delay at the initial location is denoted as Δk_{allow} , we want to obtain a collisionfree path which does not violate this constraint. It is notable that the path from A_2 to B_2 via C_2 does not need any time delay for the purpose of collision avoidance. Thus, this path always meets the constraint on the time delay at the expense of the deviation d^{act} . If Δk_f in Eq. (1) is smaller than or equal to Δk_{allow} , then the solution in category (1) will be enough for the purpose of collision avoidance. However, if Δk_f is bigger than Δk_{allow} , then time delay on the original robot 2 path cannot be used for the purpose of collision avoidance. In this aspect, we want to find a collision-free path by both path modification and time delay of the robot 2 motion. Note, however, that although the time delay we determine will be less than Δk_{allow} , the total traveling time may be larger than k_{TS} .

If the required time delay for a path between $\overline{A_2B_2}$ and the path from A_2 to B_2 via C_2^1 exceeds Δk_{allow} then we can increase the path deviation from the original robot 2 path. Otherwise, we can decrease the path deviation from the original robot 2 path. A bisection method between \overline{C}_2 and C_2 can be used to increase or to decrease the path deviation depending on whether the required time delay exceeds. Note that, since the path from ${\bf A_2}$ to ${\bf B_2}$ via ${\bf C_2}$ does not need any time delay for the purpose of collision avoidance, the existence of a collision-free path from the bisection process is always quaranteed.

SUMMARY

The collision avoidance problem between two manipulators was considered for the case 2 collision situation, where the original path and trajectory of robot 2 can be modified for the purpose of collision avoidance.

The problem was investigated through three different phases; time delay, path modification, and time delay with path modification.

Together with the results in Lee [4], this paper will constitute a Keystone in collision avoidance 924

planning of two manipulators.

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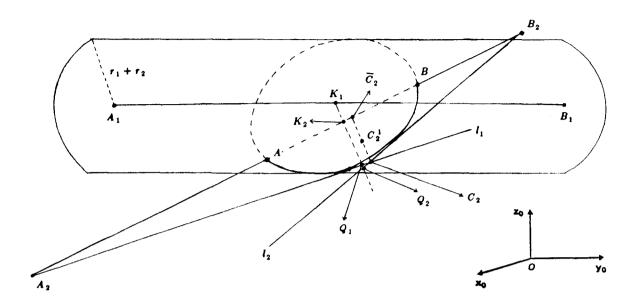


Figure 1. Collision Avoidance by Path Modification