고속순차 최소자승법에 의한 선형위상 유한응답 여파기의 설계

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Fast Sequential Least Squares Design of FIR Filters with Linear Phase

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ABSTRACT

In this paper we propose a fast adaptive least squares algorithm for linear phase FIR filters. The algorithm requires $10\,m$ multiplications per data point where m is the filter order. Both linear phase cases with constant phase delay and constant group delay are examined. Simulation results demonstrate that the proposed algorithm is superior to the LMS gradient algorithm and the averaging scheme used for the modified fast Kalman algorithm.

I. INTRODUCTION

The linear phase digital filters are being used extensively in digital signal processing. Such filters are important for applications (e.g., noise cancelling) where frequency dispersion due to phase nonlinearity is undesirable. Various algorithms for FIR filters with linear phase have been studied under the assumption that the input and desired response are jointly stationary. These algorithms are derived by using a simple relation between the linear phase filter and the unconstrained one. In the general unknown statistics case, there is no simple relation.

The LMS gradient algorithm has been applied to estimate the coefficients of an adaptive linear phase filter [1]. A fast algorithm that produces recursively a linear phase filter for a single block of data has been developed [2]. Other fast algorithms have been studied for the noncausal symmetric filters, motivated by the smoothing problem [3].

In this paper, we present an algorithm that is computationally efficient for sequential least squares estimation. It utilizes the result of the fast adaptive forward backward least squares (FBLS) algorithm studied by Kalouptsidis and Theodoridis [4].

II. FAST SEQUENTIAL ALGORITHM

In the least squares estimation of an FIR linear phase filter, the coefficient vector $\mathbf{a}_m$ is selected to minimize the total squared error

$$\sum_{k=0}^{N} [z(k)-\mathbf{x}_m^t(k)\mathbf{a}_m]^2 \tag{1}$$

with the constraint

$$\mathbf{a}_m = \mathbf{J}\mathbf{a}_m \tag{2}$$

(or $\mathbf{a}_m = -\mathbf{J}\mathbf{a}_m$: constant group delay)

where $z(k)$ and $\mathbf{x}_m(k)$ are the desired response and the reference input samples, respectively, at time k, and the superscript t denotes transpose, and $\mathbf{J}$ is an $m \times m$ reflection (exchange) matrix. The optimum weight vector of a symmetric linear phase filter that satisfies (2) is determined by

$$\mathbf{Q}_m(N)\mathbf{a}_m(N) = \mathbf{r}_m(N) + \mathbf{J}\mathbf{r}_m(N) \tag{3}$$

where

$$\mathbf{Q}_m(N) = \sum_{k=0}^{N} \mathbf{x}_m(k)\mathbf{x}_m^t(k) + \sum_{k=0}^{N} \mathbf{J}\mathbf{x}_m(k)\mathbf{x}_m^t(k)\mathbf{J} \tag{4}$$

$$\mathbf{r}_m(N) = \sum_{k=0}^{N} \mathbf{x}_m(k)z(k). \tag{5}$$

The key to the development of a sequential algorithm is the existence of time update equations. However, the centrosymmetric property of the matrix $\mathbf{Q}_m(N)$ does not render

(3) to a simple recursive reproduction form. Therefore, we use a forward and backward parameter estimation method used in the FBLS algorithm [4]. It can be shown that minimization of the sum of forward and backward error energies with respect to the unknown parameters $\mathbf{c}_m(N)$ results in

$$\mathbf{S}_m(N)\mathbf{c}_m(N) = -\mathbf{s}_m(N) \quad (6)$$

where

$$\mathbf{S}_m(N) = \sum_{k=0}^{N} \mathbf{x}_m(k)\mathbf{x}_m^t(k) + \sum_{k=0}^{N} \mathbf{J}\mathbf{x}_m(k-1)\mathbf{x}_m^t(k-1)\mathbf{J} \quad (7)$$

$$\mathbf{s}_m(N) = \sum_{k=0}^{N} \mathbf{x}_m(k)x(k-m) + \sum_{k=0}^{N} \mathbf{J}\mathbf{x}_m(k-1)x(k). \quad (8)$$

The recursive reproduction procedure for $\mathbf{c}_m(N)$ is discussed in [4]. To solve the linear phase filtering problem with the FBLS algorithm, we combine the time update equations with (3) as

$$\mathbf{a}_m(N) = \mathbf{a}_m(N-1) - [\mathbf{u}_m(N) + \mathbf{J}\mathbf{u}_m(N)] \cdot [\hat{L}_m^w(N) - e_m^w(N)]^{-1} e_m(N) \quad (9)$$

$$e_m(N) = z(N) - \mathbf{x}_m^t(N)\mathbf{a}_m(N-1) \quad (10)$$

where

$$\mathbf{u}_m(N) = -\mathbf{Q}_m^{-1}(N-1)\mathbf{x}_m(N). \quad (11)$$

Combining (9) and (10) with the FBLS algorithm results in an recursive algorithm that solves the least squares linear phase filtering problem. It may be shown that the recursion (9) can be replaced by

$$\mathbf{a}_m(N) = \mathbf{a}_m(N-1) - [\mathbf{w}_m^1(N) + \mathbf{J}\mathbf{w}_m^1(N)] \cdot [\hat{L}_m^u(N)]^{-1} e_m(N) \quad (12)$$

where

$$\mathbf{w}_m^1(N) = -\mathbf{S}_m^{-1}(N)\mathbf{J}\mathbf{x}_m(N). \quad (13)$$

In a similar way, the optimum antisymmetric linear phase filter is specified by

$$\mathbf{a}_m(N) = \mathbf{a}_m(N-1) - [\mathbf{u}_m(N) - \mathbf{J}\mathbf{u}_m(N)] \cdot [\hat{L}_m^w(N) + e_m^w(N)]^{-1} e_m(N) \quad (14)$$

or

$$\mathbf{a}_m(N) = \mathbf{a}_m(N-1) + [\mathbf{w}_m^1(N) - \mathbf{J}\mathbf{w}_m^1(N)] \cdot [\hat{L}_m^u(N)]^{-1} e_m(N). \quad (15)$$

Recursions (9), (10), (12), (14), and (15) each require 0.5 m multiplications due to the symmetry property. The number of multiplications in the FBLS algorithm is approximately 9 m per recursion. Hence, we have a total of 10 m multiplications.

III. SIMULATIONS

The performance of the proposed linear phase adaptive filtering algorithm was compared with other linear phase filtering schemes:

1. LMS gradient algorithm - tapped delay line structure [1]
2. LMS gradient algorithm - orthogonalized escalator structure [1]
3. Averaging scheme used for the modified fast Kalman algorithm

In the third scheme, the symmetric constraint is realized by averaging the unconstrained weight updates for corresponding weights on either side of the midsample and using the average to update both weights. The unconstrained weight updates are obtained from the fast Kalman algorithm.

Note that the first algorithm requires 3 m multiplications per data point, the second one requires 0.5 m^2 + 2.5 (or 3.5 : m odd) m multiplications, and the third requires 6 m multiplications. The unity-power white Gaussian input signal is filtered to generate the desired response which is then added by the white Gaussian noise with zero mean. A block diagram of the simulation is shown in Fig. 1. The convergence behaviors of the above mentioned algorithms and

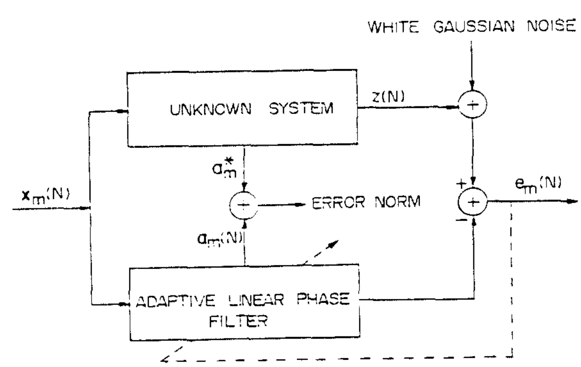

그림 1 유한응답 선형위상 시스템 인식을 위한 모의실험 모델

Fig. 1 Simulation model for FIR linear phase system identification the proposed algorithm for an FIR linear phase system identification for different noise power are shown in Figs. 2 and 3. The learning curves, associated with the tap coefficient error norm, show that the performance of the proposed algorithm is superior to that of the others. The major shortcoming of the LMS algorithm is its slow convergence rate. The escalator algorithm improves the convergence rate, requiring $O(m^2)$ multiplications. The averaging scheme used for the modified fast Kalman algorithm does not efficiently track the desired response. However, the proposed algorithm overcomes this disadvantage without a significant increase of the computational complexity.

IV. CONCLUSION

In this paper, a fast least squares linear phase adaptive filtering algorithm has been presented. The algorithm requires $10\,m$ multiplications for time-sequential recursive filtering. This reduced computation stems from the fast adaptive forward backward least squares method. Computer simulations demonstrate that the proposed filter performs better than other existing filters.

REFERENCES

[1] D. H. Youn and S. Prakash, "On realization and related algorithms for adaptive linear phase filtering," in Proc. ICASSP 1984, pp. 3.11.1 -3.11.4.

[2] N. Kalouptsidis and G. D. Koyas, "Efficient block LS design of FIR filters with linear phase," IEEE Trans. Acoust., Speech, Signal Processing, vol. ASSP-33, pp. 1435-1444, Dec. 1985.

[3] S. L. Marple, Jr., "Fast algorithms for linear prediction and system identification filters with linear phase," IEEE Trans. Acoust., Speech, Signal Processing, vol. ASSP-30, pp. 942-953, Dec. 1982.

[4] N. Kalouptsidis and S. Theodoridis, "Fast adaptive least squares algorithms for power spectral estimation," IEEE Trans. Acoust., Speech, Signal Processing, vol. ASSP-35, pp. 661-670, May 1987.

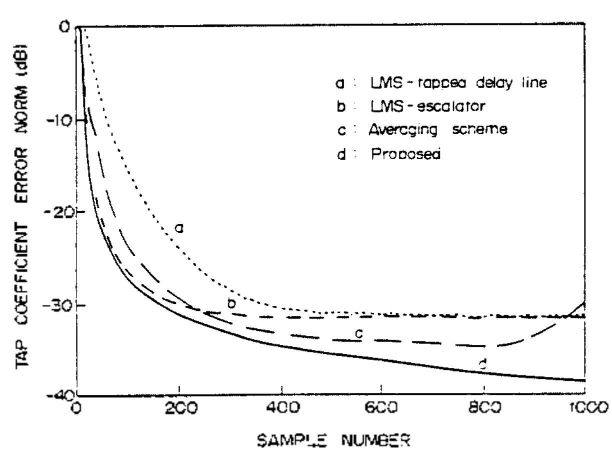

그림 2 잡음전력이 -20dB일 때 차수가 29인 선형위상 유한응답 여파기의 수렴동작 (앙상블 평균을 위하여 독립적으로 100번 반복하였음)

Fig. 2 Convergence behaviors of linear phase FIR filter with order of 29 when the noise power is -20dB (100 independent iterations were done for ensemble average)

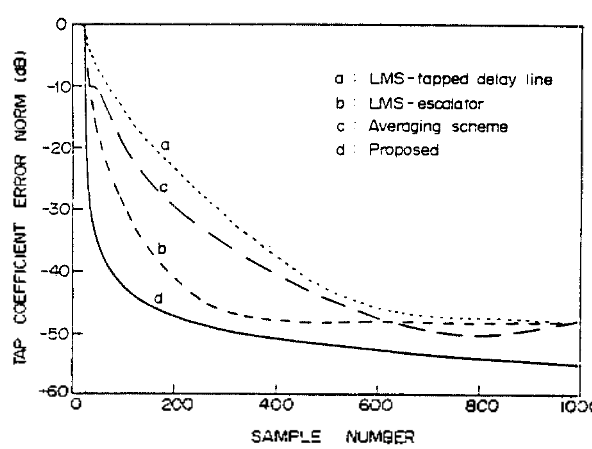

그림 3 잡음전력이 -40dB일 때 차수가 50인 선형위상 유한응답 여파기의 수렴동작 (앙상블 평균을 위하여 독립적으로 100번 반복하였음)

Fig. 3 Convergence behaviors of linear phase FIR filter with order of 50 when the noise power is -40dB (100 independent iterations were done for ensemble average)