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다표본 디지탈 탠락 루우프의 가우스 잡음하에서의 성능분석

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Performance Analysis of the Multi-Sampling Digital Tanlock Loop in the Presence of Gaussian Noise

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ABSTRACT

In this paper, we investigate the performance of the first-order multi-sampling digital tanlock loop (MSDTL) with phase and frequency step inputs in the presence of Gaussian noise. The MSDTL yields extended locking range, and reduced steady-state mean and variance of phase error as compared to the conventional DTL. It is shown that as the number of samples taken in one period of the received signal increases, the phase error of the first-order MSDTL decreases sharply.

I. INTRODUCTION

Since the early 1970's, many researchers have studied digital phase locked loops (DPLL's) which are important subsystems used in coherent communication and tracking receivers [1]. DPLL's offer many advantages over analog loops, such as increased reliability, improved stability, and lower cost.

A new type of nonuniform sampling DPLL called the digital tanlock loop (DTL) was introduced by Lee and Un [2]. The phase detector of this loop has a $\tan^{-1}[\cdot]$ function, and therefore, has a linear characteristic in the modulo-2π sense. Consequently, the DTL can be characterized by a linear difference equation. It has many important advantages over the conventional DPLL's with sinusoidal phase characteristics. These include linear characteristics, wider locking range, less steady-state phase error, and insensitivity to the variation of the input signal power.

To improve the performance of the DTL, Cho and Un recently proposed a multi-sampling DTL (MSDTL) which has a structure similar to the original DTL, but has a modified phase detector in which two quadrature input signals are sampled more than once in one period using a reference phase generator and a digital clock [3]. The purpose of this work is to study the performance of the first-order MSDTL in the presence of Gaussian noise, and to compare with that of the original DTL.

II. MATHEMATICAL MODEL OF THE MSDTL

A block diagram of the MSDTL that tracks nonuniformly positive-going zero crossings of a continuous sinusoidal signal is shown in Fig. 1. To detect phase error at a sampling instant, the sampler I takes a sample x(k) from the incoming signal, and at the same time, the sampler II takes another sample y(k) from the 90° phase-shifted incoming signal. Then, the phase detector output z (k) is obtained by getting a function $\tan^{-1}[x(k)/y(k)]$. It is noted that by using the signs of x(k) and y(k) the function of $\tan^{-1}[x(k)/y(k)]$ has no ambiguity in distinguishing phase error in the interval between $-\pi$ and π. Thus, like the DTL, the characteristics curve of the phase detector of the MSDTL is linear with a period of 2π.

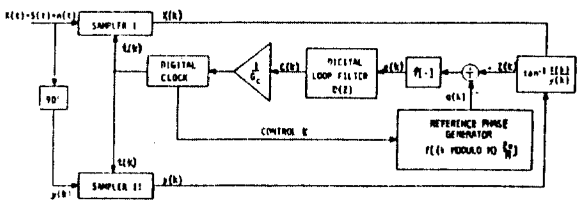

Fig. 1. Block diagram of the MSDTL.
(Samplers I and II take M samples per one signal period)

An important difference between the conventional DTL and the MSDTL is that the former samples only at the positive-going zero crossings, while the later samples more than once in a period T at t=T/M including positive going zero crossings, where T($\triangleq 2\pi/\omega$) is the input signal period, ω is the input radian frequency, and M is the number of samples taken in one input signal period. One can note that, as a result of the multi-sampling operation, the value of z(k) is increased undesirably by the amount of $f[2\pi k/M]$ ($f(x) \triangleq -\pi + \{(x+\pi) \text{ modulo } 2\pi\}$). Hence, as shown in Fig. 1, we cancel it out by inserting a reference phase generator controlled by the output of the digital clock. Consequently, the resulting phase error e(k) is equal in the modulo-2π sense to the phase error between the incoming signal and the reference signal of the digital clock (i,e., the output of the phase error detector of the DTL) [2].

The behavior of the loop depends largely upon the transfer function of the digital loop filter, D(z). The output of the loop filter, c(k), controlled by a gain factor G_c is used as a control signal to the digital clock for the digital clock for the next sampling operation. In our system the digital clock provides the same sampling time to the two samplers. In this way, the MSDTL tracks positive-goihng zero crossings of the incoming signal.

Let x(t) be the incoming signal to the sampler I consisting of the input signal s(t) and additive Gaussian noise n(t), and y(t) be the input to the sampler II which is a 90° phase-shifted signal of x(t). In this study we assume that the signal s(t) has a phase offset θ_o, and an actual frequency ω with a frequency offset $\Delta\omega(\underline{\Delta}\omega-\omega_o)$ from the nominal frequency ω_o of the digital clock. When x(t) and y(t) are sampled at the kth sampling instant, the sampled values, x(k) and y(k), are given respectively, by

$$x(k) = (2P)^{1/2} \sin[\omega_o t(k) + \theta(k)] + n(k) \qquad (1)$$

and

$$y(k) = (2P)^{1/2} \cos[\omega_o t(k) + \theta(k)] + n'(k) \qquad (2)$$

$$\theta(k) \triangleq \theta[t(k)], \quad n(t) \triangleq n[t(k)], \text{ and}$$

where

$$n'(k) \triangleq n'[t(k)].$$

Where P is the power of the signal s(t), $\theta(t)$ ($\triangleq \omega.t+\theta_o$) is the phase process of the signal s(t) and n'(t) is the 90° phase-shifted noise process of n(t).

The sampling interval of the digital clock between t(k) and t(k-1) is dependent on the signal c(k-1), the gain contnrol factor G_c, and the number of samples taken in one signal period, M. That is,

$$T(k) = T_o/M - c(k-1)/G_c \qquad (3)$$

Where T_o is $2\pi/\omega_o$ representing the nominal period of the digital clock. The total time t(k) up to the kth sampling instant is

$$t(k) = t(o) + \sum_{i=1}^{k} T(i)$$
$$= k\frac{T_o}{M} - \frac{1}{G_c}\sum_{i=0}^{k-1} c(i) \qquad (4)$$

Where t(0) has been assumed to be zero. As will see later, M and G_c play important roles in reducing the steady-state mean and variance of phase error and in extending the locking range.

In the absence of noise, the output of the $\tan^{-1}$ function (see Fig. 1), z(k), is given by

$$z(k) = f[\omega_o t(k) + \theta(k)]$$
$$= f[2\pi k/M + \phi(k)] \qquad (5)$$

where

$$\phi(k) \triangleq \theta(k) - \frac{\omega_o}{G_c}\sum_{i=0}^{k-1} c(i) \qquad (6)$$

and

$$f[x] \triangleq -\pi + \{(z+\pi) \text{ modulo } 2\pi\}. \qquad (7)$$

To eliminate the undesirable term, $2\pi k/M$, in (5) that results from the multi-sampling operation, we subtract the output of the reference phase generator q(k) from z(k), q(k) being represented by

$$q(k) \triangleq f[\frac{2\pi k}{M}]$$
$$= f[(k \text{ modulo } M)\frac{2\pi}{M}]. \qquad (8)$$

Consequently, in the noiseless case, e(k) is equal to $f[\phi(k)]$. On the other hand, in the presence of noise, e(k) consists of the phase error $f[\phi(k)]$ due to the noise-free incoming signal and the random phase error disturbance $\eta(k)$ due to additive noise. That is,

$$e(k) = f[\phi(k)] + \eta(k). \qquad (9)$$

Since $f[\eta(k)]=\eta(k)$ in the interval $(-\pi, \pi)$, $c(k)=D(z)e(k)$, and $\theta(k)=\Delta\omega.t(k) + \theta_o$, we obtain from (4) and (5) a system difference equation describing the phase error process of the MSDTL as

$$\phi(k+1) = [1 - \frac{\omega}{G_c}D(z)]\phi(k) + \frac{\Lambda_o}{M} - \frac{\omega}{G_c}D(z)\eta(k) \qquad (10)$$

where $\Lambda_o \triangleq 2\pi.\Delta\omega/\omega_o$. Note that, when $M=G_c=1$, the equation of (10) becomes that for the conventional DTL.

III. PERFORMANCE ANALYSIS OF THE MSDTL IN THE PRESENCE OF GAUSSIAN NOISE

In the presence of Gaussian noise, the system equation of the first-order MSDTL is obtained from (10) by

$$\phi(k+1) = (1 - \frac{K_1'}{G_c})\phi(k) + \frac{\Lambda_o}{M} - \frac{K_1'}{G_c}n(k)$$

where $K_1' = G_1\omega$. Here one can regard the phase error process $\{\phi(k)\}$ as a first-order, discrete time, continuously variable Markov process. Consequently, the pdf of $\phi(k)$, conditioned on an initial phase error, satisfies the following Chapman-Kolmogorov (C-K) equation [2]:

$$p_{k+1}(\phi|\phi_o) = \int_{-\infty}^{\infty} q_k(\phi|u)p_k(u|\phi_o)du \qquad (12)$$

where $\phi_o \Delta \phi(o)$ is initial phase error, $p_k(\phi|\phi_o)$ is the pdf of $\phi(k)$ given ϕ_o, and $q_k(\phi|u)$ is the transition pdf of $\phi(K+1)$ given $\phi(k)=u$. Notice that the range of the range of the noise term $\eta(k)$ is in the interval $(-\pi\text{-}f[\phi(k)],\ \pi\text{-}f[\phi(k)])$. Therefore, one can see from (11) that the range of $q_k(\phi|u)$ is in the interval $(u+\Lambda_o/M\text{-}K_1'\pi/G_c,\ u+\Lambda_o/M+K_1'\pi/G_c)$. Accordingly, $_k(\phi|u)$ is represented from (11) and (12) as

$$q_k(\phi|u) = \frac{G_c}{K_1'}\frac{\exp(-\alpha)}{2\pi} + \frac{G_c}{K_1'}\frac{\sqrt{\alpha}}{\sqrt{\pi}}\cdot\cos\left(\frac{\phi-\phi_p}{K_1'/G_c}\right)$$

$$\cdot\exp\left[-\alpha\cdot\sin^2\left(\frac{\phi-\phi_p}{K_1'/G_c}\right)\right] \qquad (13)$$

$$\cdot\left\{\frac{1}{2} + \mathrm{erf}\left[\sqrt{2\alpha}\cdot\cos\left(\frac{\phi-\phi_p}{K_1'/G_c}\right)\right]\right\}$$

where $\phi_p \Delta u\text{-}K_1'.f[u]/G_c+\Lambda_o/M$ and $\alpha \Delta P/\sigma_n^2$. It should be noted that, although ϕ and u in (12) are in the interval $(-\omega, \omega)$, $q_k(\phi|u)$ given by (13) is zero outside the interval $(u+\Lambda_o/M\text{-}K_1'.\pi/G_c,\ u+\Lambda_o/M+K_1'.\pi/G_c)$.

Considering the modulo-2π phase error process $\{\psi(k)\}$ defined as

$$\psi(k) \ \underline{\Delta}\ f[\phi(k)]\ \underline{\Delta}\ -\pi + [\{\phi(k) + \pi\} \text{ modulo } 2\pi], \qquad (14)$$

we now obtain the steady-state pdf $p(\psi)$ represented by an integral equation [4] given as

$$p(\psi) = \int_{-\pi}^{\pi} r(\psi|z)\ p(z)dz \qquad (15)$$

where

$$r(\psi|z)\ \underline{\Delta} \sum_{n=-\infty}^{\infty} q(\psi+2n\pi|z). \qquad (16)$$

The steady-state pdf $p(\psi)$ may be obtained by solving (16) numerically, as was done by Weinberg and Liu [5]. Fig. 2 shows the calculated results together with simulation results for various parameter values. One can see that as M increases, the mean of phase error is shifted to $\psi=0$; and as G_c increases, the variance of phase error is reduced sharply.

And from (11) and (14), we obtain the following

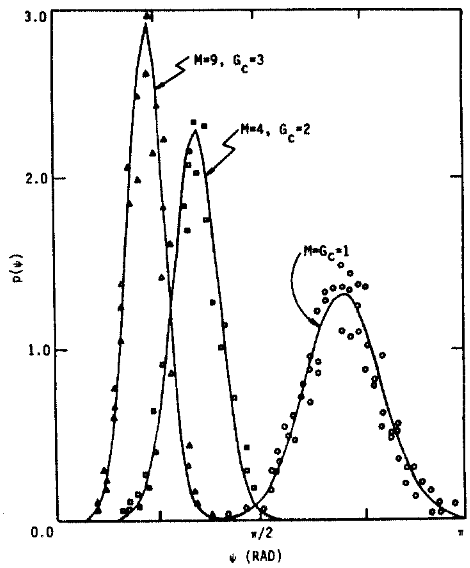

Fig. 2 Steady-state pdf's of phase error ψ of the first-order MSDTL ($M=G_c=1$) and the first-order MSDTL with various values of M and G_c for SNR=7 dB, $\Lambda_o=\pi/2$, and $K_1'=$ 0.7. (Solid lines are numerical results, and points are simulation results.)

system equation for the modulo-2π phase error process $\{\psi(k)\}$ in the presence of Gaussian noise:

$$\psi(k+1) = \left(1 - \frac{K_1'}{G_c}\right)\psi(k) + \frac{\Lambda_o}{M} - \frac{K_1'}{G_c}\eta(k). \qquad (17)$$

By taking expectation of both sides of (17) in the steady state, we have the steady-state mean of phase error, $E[\psi_{ss}]$, given by

$$E[\psi_{ss}] = \frac{G_c}{M}\frac{\Lambda_o}{K_1'}, \qquad (18)$$

since $E[\eta]$ is approximately equal to zero for the SNR in the region of interest. Similarly, squaring both sides of (17) and then taking expectation of each term, we have the steady-state variance of phase error, $\mathrm{var}[\psi_{ss}]$, represented by

$$\mathrm{var}[\psi_{ss}] = \frac{K_1'}{2\cdot G_c - K_1'}\cdot E[\eta^2] \qquad (19)$$

where $0<K_1'<2.G_c$ since $\mathrm{var}[\psi_{ss}]$ must be nonnegative. It is noted from (19) that, unlike the conventional sinusoidal DPLL, $\mathrm{var}[\psi_{ss}]$ is an explicit function of the parameter K'_1. It decreases as the parameter K'_1 decreases or as the input SNR α increases since $E[\eta^2]$ is a decreasing function of α. In Figs. 3, variances of the modulo-2π phase error ψ together with simulation results of the sinusoidal DPLL and

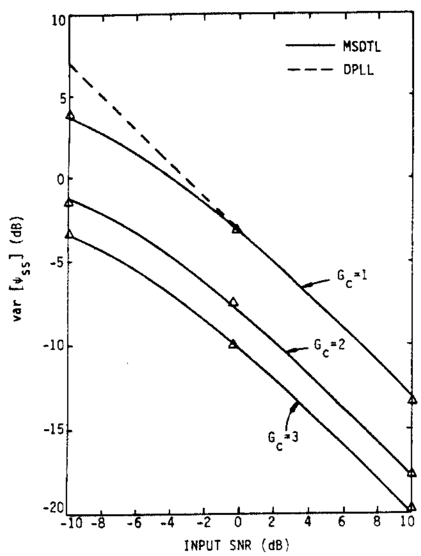

Fig. 3 Variances of steady-state phase error ψ_{ss} versus input SNR of the first-order MSDTL for various values of G_c ($K_1'=1.0$, $\omega/\omega_o=1$).

the MSDTL are shown for various parameter values. And, it is seen that as in the conventional DTL, for high SNR's the variances of the two loops do not reveal appreciable difference for the same parameter values, but for low SNR's the performance of the MSDTL is superior to that of the sinusoidal DPLL. Also, one can note that var[ψ_{ss}] decreases as G_c increases.

IV. CONCLUSIONS

We have studied the performance analysis of the multi-sampling digital tanlock loop in the presence of Gaussian noise, which employes a new types of phase error detector with multi-sampling operation. And, we have derived the mean and variance of steady-state phase error of the first-order MSDTL, and compared with those of the sinusoidal DPLL. It has been found that, as the number of samples taken in one period, M or G_c, increases, the mean or variance of steady-state phase error dereases abruptly, respectively. The MSDTL studied should be useful in synchronous communication and control systems, because it has many advantages over other conventional DPLL's.

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