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유한한 두께를 가지는 평행 평판 도파관에 있는 슬릿에 대한 해석

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An analysis of the slitted parallel plate waveguide with finite thickness

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ABSTRACT

A slit in a parallel plate waveguide with finite thickness is investigated. Two coupled integral equations whose unknowns are magnetic currents are formulated exactly and solved by use of the conventional collocation method.

From knowledge of the magnetic current, the quantities such as reflection coefficient, transmission coefficient and far field pattern are computed.

I. INTRODUCTION

The analyses of the slit in the parallel plate waveguide have been carried out by many authors but the results are valid only for infinitely thin and negligible plate thickness.

In practice, the effects of the plate thickness are not negligible and should be taken into account carefully.

Here, the solution of the parallel plate waveguide with the slit in its upper plate of finite thickness is considered.

II. DEVELOPEMENT OF FORMULATION

The geometrical structure of the problem is shown in Fig.1.

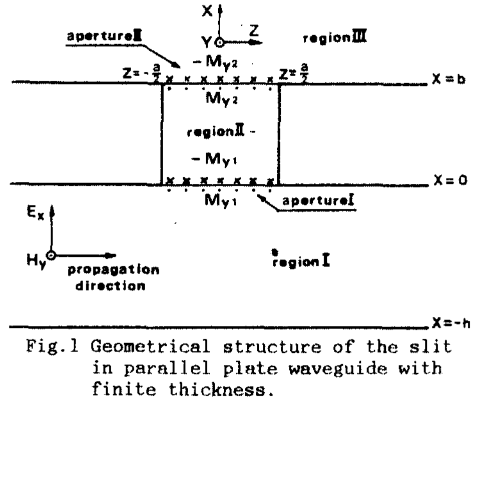

Fig.1 Geometrical structure of the slit in parallel plate waveguide with finite thickness.

Each plates are assumed to be perfect conductors and the media filled in each region is lossless and homongeneous as free space.

When a TEM wave whose electric field amplitude is assumed to be unity propagates along the waveguide shown in Fig.1, electromagnetic fields scattered from the slit may be calculated from an equivalent magnetic currents.

The time dependence factor $e^{j\omega t}$ is suppressed throughout and $h< \lambda_0/2$ such that the only TEM wave can propagate along the waveguide.

Choosing the appropriate Neumann Green function in each region I, II and III, two coupled integral equations are obtained from the continuity of tangential magnetic field on aperture I and II and shown as follow.

$$\int_{-\frac{a}{2}}^{\frac{a}{2}} M_{y1}(z')G_I(0,z;0,z')dz' + H_{y,inc} = -\int_{-\frac{a}{2}}^{\frac{a}{2}} M_{y1}(z')G_{II}(0,z;0,z')dz' + \int_{-\frac{a}{2}}^{\frac{a}{2}} M_{y2}(z')G_{II}(0,z;b,z')dz' \quad (1)$$

on aperture I

$$\int_{-\frac{a}{2}}^{\frac{a}{2}} M_{y1}(z')G_{II}(b,z;0,z')dz' + \int_{-\frac{a}{2}}^{\frac{a}{2}} M_{y2}(z')G_{II}(b,z;b,z')dz' = -\int_{-\frac{a}{2}}^{\frac{a}{2}} M_{y2}(z')G_{III}(b,z;b,z')dz' \quad (2)$$

on aperture II

In Eqs.(1) and (2) G_I, G_{II} and G_{III} represent Neumann Green function in region I, II and III respectively, and satisfy the conditions as follow.

$$(\nabla^2 + k_0^2)G_{I,II,III}(x,z;x',z') = j\omega\epsilon_0\delta(x-x')\delta(z-z') \quad (3)$$

$$\left.\frac{\partial G_I}{\partial x}\right|_{x=0,-h} = 0 \quad (4) \qquad \left.\frac{\partial G_{II}}{\partial z}\right|_{z=\pm\frac{a}{2}} = 0 \quad (5)$$

$$\left.\frac{\partial G_{I}}{\partial x}\right|_{x=0,b} = 0 \qquad (6) \qquad \left.\frac{\partial G_{II}}{\partial x}\right|_{x=b} = 0 \qquad (7)$$

The unknown $My_1(z')$ and $My_2(z')$ are approximated by pulses, and by partitioning the interval $(-a/2,+a/2)$ into N segments of length $\Delta = a/N$ and selecting the match point locations z_m at pulse center according to $z_m = -a/2+(m-1/2)\Delta$, Eqs.(1) and (2) are reduced to the following algebraic equations by use of conventional moment method.

$$\sum_{n=1}^{N}[My_{1,n}\, Y_{mn}^{11} + My_{2,n}\, Y_{mn}^{12}] = Hy,inc(z_m) \qquad (8)$$

$$\sum_{n=1}^{N}[My_{1,n}\, Y_{mn}^{21} + My_{2,n}\, Y_{mn}^{22}] = 0 \qquad (9)$$

$m = 1,\ 2,\ 3,\ \ldots\ N$

$n = 1,\ 2,\ 3,\ \ldots\ N$

z_m; observation point

z_n; source point

in which $My_{1,n}$ and $My_{2,n}$ are the unknown coefficients of the n-th pulse located at z_n.

The matrix elements in Eqs.(8) and (9) are defined as

$$Y_{mn}^{11} = -\int_{z_n-\Delta/2}^{z_n+\Delta/2} [G_I(0,z_m;0,z') + G_{II}(0,z_m;0,z')]\, dz' \qquad (10)$$

$$Y_{mn}^{12} = \int_{z_n-\Delta/2}^{z_n+\Delta/2} G_{II}(0,z_m;b,z')dz'$$

$$= \int_{z_n-\Delta/2}^{z_n+\Delta/2} G_{II}(b,z_m;0,z')dz' = Y_{mn}^{21} \qquad (11)$$

$$Y_{mn}^{22} = \int_{z_n-\Delta/2}^{z_n+\Delta/2} \left[\frac{k_0}{2\eta_0}H_0^{(2)}(k_0|z_m-z'|) - G_{II}(b,z_m;b,z')\right] dz' \qquad (12)$$

where $k_0 = w\sqrt{\mu_0\varepsilon_0}$, $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$, w, μ_0 and ε_0 denote angular frequency, permeability and permittivity repectively and $H_0^{(2)}$ means the Hanhel function of the second kind of index zero.

Substituting the solutions of Green functions to Eqs.(10), (11) and (12), the matrix elements are given as follow.

$$Y_{mn}^{11} = \frac{1-e^{-jk_0\Delta/2}}{j\eta_0 k_0 b} - \frac{jk_0\Delta}{\eta_0\pi}\{\ln(\frac{\Delta\pi}{2eb}) - \sum_0 (\frac{k_0 b}{\pi})\}$$
$$- \frac{j}{\eta_0 N \tan k b} + \frac{j2}{\eta_0\pi}\sum_{i=1}^{\infty}[\frac{1}{\alpha\tanh[k_0 b\alpha]}\frac{1}{i}$$
$$\sin(\frac{i\pi}{2N})\{\cos[\frac{i\pi}{N}(2m-1)] + 1\}]$$

in case of m = n (13)

$$= \frac{1}{\eta_0 k_0 h}\sin(k_0\Delta/2)\, e^{-jk_0|m-n|\Delta}$$
$$-j\frac{k_0 h}{\pi^2\eta_0}[f(\pi[|m-n|+1/2]\Delta/h) - f(\pi[|m-n|-1/2]\Delta/h)$$
$$+\sum_{i=1}^{\infty}\{(\frac{1}{i^2-(\frac{k_0 h}{\pi})^2}\, e^{-\pi\sqrt{i^2-(\frac{k_0 h}{\pi})^2}[|m-n|+1/2]\Delta/h}$$
$$-\frac{1}{i^2-1/4}\, e^{-i\pi[|m-n|+1/2]\Delta/h})$$
$$-(\frac{1}{i^2-(\frac{k_0 h}{\pi})^2}\, e^{-\pi\sqrt{i^2-(\frac{k_0 h}{\pi})^2}[|m-n|-1/2]\Delta/h}$$
$$-\frac{1}{i^2-1/4}\, e^{-i\pi[|m-n|-1/2]\Delta/h})\}]$$
$$-\frac{j}{\eta_0 N\tan[k_0 b]} + \frac{j4}{\eta_0\pi}\sum_{i=1}^{\infty}\{\frac{1}{\alpha\tanh[k_0 b\alpha]}\frac{1}{i}$$
$$\cos[\frac{i\pi}{2N}(2m-1)]\cos[\frac{i\pi}{2N}(2n-1)]\sin(\frac{i\pi}{2N})\} \qquad (14)$$

in case of $m \neq n$

$$Y_{mn}^{12} = \frac{j}{\eta_0 N\sin[k_0 b]} - \frac{j4}{\eta_0\pi}\sum_{i=1}^{\infty}\{\frac{1}{\alpha\sinh[k_0 b\alpha]}\frac{1}{i}$$
$$\cos[\frac{i\pi}{2N}(2m-1)]\cos[\frac{i}{2N}(2n-1)]\sin(\frac{i\pi}{2N})\}$$

$$= Y_{mn}^{21} \qquad (15)$$

$$Y_{mn}^{22} = \frac{k_0\Delta}{2\eta_0}[1-j\frac{2}{\pi}\ln(\frac{\gamma k_0\Delta}{4e})] - \frac{j}{\eta_0 N\tan[k_0 b]}$$
$$+\frac{j2}{\eta_0\pi}\sum_{i=1}^{\infty}[\frac{1}{\alpha\tanh[k_0 b\alpha]}\frac{1}{i}\sin(\frac{i\pi}{2N})\{\cos[\frac{i\pi}{N}(2m-1)]+1\}]$$

in case of m = n (16)

$$\simeq \frac{k}{2\eta_0}\Delta H_0^{(2)}(k_0|m-n|\Delta) - \frac{j}{\eta_0 N\tan[k_0 b]}$$
$$+\frac{j4}{\eta_0\pi}\sum_{i=1}^{\infty}\{\frac{1}{\alpha\tanh[k_0 b\alpha]}\frac{1}{i}\cos[\frac{i\pi}{2N}(2m-1)]$$
$$\cos[\frac{i\pi}{2N}(2n-1)]\sin(\frac{i\pi}{2N})\} \qquad (17)$$

in case of $m \neq n$

where e = 2.718$\cdots$, γ = 1.781$\cdots$.

The above equations are formulated under the condition that the slit width is smaller than half wavelength in free space.

Solving the linear simultaneous equations whose coefficient elements Y_{mn}^{11}, Y_{mn}^{12}, Y_{mn}^{21} and Y_{mn}^{22} are given above, one obtains the unknown coefficients $My_{1,n}$ and $My_{2,n}$.

From knowledge of $My_{1,n}$, the magnetic field reflection and transmission coefficient are found to be

$$\Gamma_H = -\frac{\sin(k_0\Delta/2)}{k_0 h}\sum_{n=1}^{N} My_{1,n}\, e^{-jk_0 z_n} \qquad (18)$$

$$T_H = 1 - \frac{\sin(k_0\Delta/2)}{k_0 h}\sum_{n=1}^{N} My_{1,n} e^{jk_0 z_n} \qquad (19)$$

Γ_H ; Reflection coefficient
T_H ; Transmission coefficient

The magnetic far field pattern is calculated from knowledge of $My_{2,n}$.

$$H_\phi(\theta) = \frac{k_0\Delta}{\eta_0\sqrt{2\pi}} e^{-j(\pi/4)} \sum_{n=1}^{N} My_{2,n} e^{jk_0 z_n \cos\theta} \qquad (20)$$

III. SUMMARY OF RESULTS

The values of magnetic currents computed for relatively thin slit are shown in Fig.2 and compared with that of infinitely thin case.

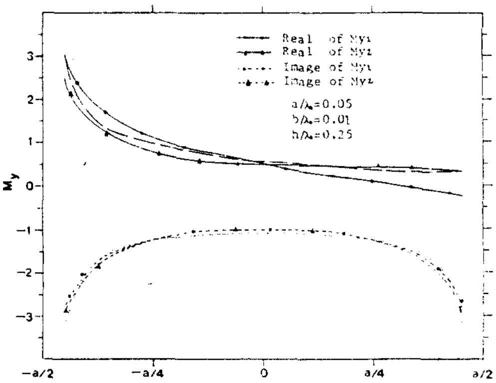

Fig.2 Values of magnetic currents for thin slit.
(— — —), (··········) represent real and image magnetic current for infinitely thin case.

And Fig.3 shows the magnetic currents values for relatively thick case.

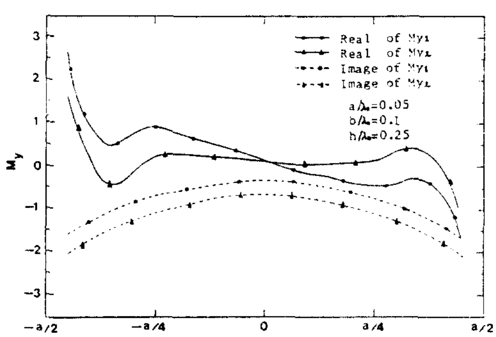

Fig.3 Values of magnetic currents for relatively thick slit.

Magnetic field reflection and transmission coefficients are readily calculated from knowledge of My_1 .

Far field patterns of relatively wide and narrow slits obtained from knowledge of My_2 are shown in Fig.4.

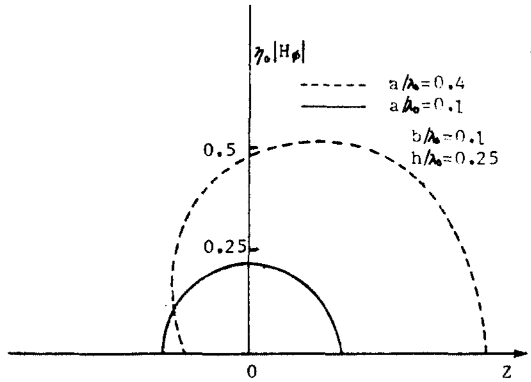

Fig.4 Far field patterns of narrow and wide slits.

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