

적용 제어에서의 오프셋 영향 제거

○ \* \*\* \*  
최두환 김영철 양홍석

\* 서울대 전기과 \*\* 충북대 전자과

OFFSET ELIMINATION IN ADAPTIVE CONTROL

\* \*\* \*  
DOOWHAN CHOI YEONGCHOL KIM HEUNGSUK YANG

\* DEPT. OF ELECTRICAL ENG. SEOUL NATIONAL UNIV.  
\*\* DEPT. OF ELECTRONICS CHUNGBOOK NATIONAL UNIV.

ABSTRACT

This note considers the class of controllers with integral action which arise directly from appropriate system models. Via internal model principle approach, a corresponding class of self-tuning controller is shown to have both integral action in controller and offset removal in the tuning algorithm. The key idea is to constrain the estimator in each step in order to ensure that dc gain of feedforward and feedback polynomial of adaptive controller are always equal, thus allowing the loop integrator to work properly.

1. Introduction

One of the major specifications in control system design is that the system provides a zero static error, i.e., zero offset.

Offsets themselves are the result of a variety of causes such as load disturbances and the intrinsic steady-state nonlinearity of practical process plant. The rapid regulation against potential offsets due to load disturbance is a significant design consideration.

In some less critical loops this goal can be satisfied by high-gain proportional control. Controllers with integral action are commonly used in industry for this goal. So a number of authors [4, 5, 6] have introduced self-tuning controllers with integral action. The usual approach taken is to force an integrator into controller and so eliminate offsets.

The approach taken here is reverse: the system is modeled to include an offset term, the corresponding controller is found to have integral action, and constrain the feedforward and feedback gain for the loop integrator to work properly.

Self-tuning methods may be divided into two groups [13]: implicit methods where the estimator directly produces controller coefficient and explicit method where the estimator generates system coefficients which are processed to give controller coefficients. This paper concentrates on the implicit method.

When self-tuners were applied [2, 3, 11] however, the need of zero offset became more apparent and a series of 'ad hoc' [11] measures were adopted to achieve it. For the goal, much of this paper is concerned with the derivation and properties of the control algorithm. Although this paper focuses on tuning these control algorithms via a pole-assignment technique [13], other methods of tuning discrete-time

controller coefficients could be used.

## 2. Modified system models

The linearized model of the controlled process is assumed to be

$$A(z^{-1})y(t) = z^k B(z^{-1})u(t) + x(t) \quad \dots (2.1)$$

where  $u(t)$ ,  $y(t)$  are the plant input and output at sample number, and  $A(z^{-1})$ ,  $B(z^{-1})$  are polynomials of degree  $n$ ,  $m$  in backward shift operator  $z^{-1}$ .

The signal  $x(t)$  is a disturbance term. In the literature,  $x(t)$  has been considered to be of moving average form :

$$x(t) = C(q^{-1}) \xi(t)$$

where  $C(q^{-1}) = 1 + c_1 q^{-1} + \dots + c_L q^{-L}$

In this equation,  $\xi(t)$  is an uncorrelated zero mean random sequence. This gives

$$A(z^{-1})y(t) = z^k B(z^{-1})u(t) + C(q^{-1}) \xi(t) \quad \dots (2.2)$$

Though much self-tuning theory is based on this model, it seems to be inappropriate for many industrial application in which disturbances are non-stationary : the disturbance may drift with time. This arises from

- i) Load disturbance.
- ii) The fact that steady-state gain  $\partial y / \partial u$  of a process doesn't equal the state gain.
- iii) nonzero-mean noise.

In practice, two principal disturbances are encountered [14]: random steps occurring at random times ( for example, changes in material quality )

and Brownian motion .

In both these cases, an appropriate model is :

$$x(t) = \frac{C(q^{-1}) \xi(t)}{(1 - q^{-1})}$$

This gives modified system model

$$A(q^{-1}) \Delta y(t) = q^k \Delta B(q^{-1}) u(t) + C(q^{-1}) \xi(t) \quad \dots (2.3)$$

where  $\Delta = 1 - q^{-1}$

This model has been used by [12].

We'll use the modified model (2.3) for the derivation of algorithm. The next section based on earlier work [8] shows the principle which is sufficient condition for offset elimination.

## 3. THE INTERNAL MODEL PRINCIPLE

If  $W(t)$  is the set point and  $T(z)$ ,  $U(z)$  and  $V(z)$  are general polynomials in  $z$  a general feedback control may be as in fig.1

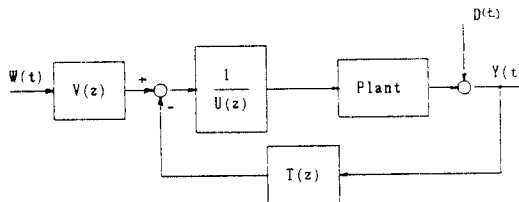


Fig.1 General Feedback Control

Eliminating  $U(t)$  using the plant model  $G(z)$  give closed loop relationship

$$Y(z) = \frac{V(z)G(z)}{U(z) + T(z)G(z)} W(z) + \frac{U(z)}{U(z) + T(z)G(z)} D(z)$$

Thus output will faithfully reproduce the desired output provided that

(A.1)  $U(e^{j\omega})$  is small enough over the range of frequencies dominant in both  $W(e^{j\omega})$  and  $D(e^{j\omega})$

(A.2) Feedforward compensator  $V(e^{j\omega})$  is equal to feedback compensator  $T(e^{j\omega})$  over the frequencies dominant  $\Psi(e^{j\omega})$ .

The method we suggest here is to constrain the estimator in order to satisfy the (A.2) and to construct the control with integral action in order for the (A.1).

#### 4. PREDICTOR

The system considered here is the modified model(2.3). Based on this model the predictor is developed using pole assignment technique. [13] The corresponding controller is found to have integral action.

##### (1) INTEGRAL ACTION

A reference model is given by

$$G_m(q^{-1}) = q^{-k} \frac{B_m(q^{-1})}{A_m(q^{-1})} \quad \text{where } B_m(1) = A_m(1)$$

For simplicity we assume that the plant is minimum phase and time delay  $k$  is known.

Let  $T(q^{-1})$  be the usual observer polynomial.

Then the plant(2.3) is

$$A\Delta Y'(t) = B\Delta U'(t-k) + Z(t)/T$$

$$\text{where } Z(t) = C(q) \xi(t)$$

$$Y'(t) = Y(t)/T$$

$$U'(t) = U(t)/T$$

We consider the control law given a set point  $W(t)$ , that is ,

$$G(q^{-1})U(t) = H(q^{-1})W(t) - F(q^{-1})Y(t)$$

and choose  $G(q^{-1})$ ,  $H(q^{-1})$  in accordance with Internal-

-Model Principle as follows

$$G(q^{-1}) = \Delta B(q^{-1})$$

$$H(1) = F(1)$$

In case of cancelling all zeros of plant

$\beta(q^{-1})$  and  $B_m(q^{-1})$  can be chosen as

$$\beta(q^{-1}) = E(q^{-1})B(q^{-1})$$

$$B_m(q^{-1}) = b_m = A_m(1)$$

where ' $b_m$ ' is constant and  $E(q^{-1})$  is a polynomial in  $q^{-1}$  with the degree of  $k-1$ .

The closed loop model is ,eliminating  $U(t)$ ,

$$(AE\Delta + q^k F)Y'(t) = HW'(t-k) + EZ(t)/T$$

where the diophantine eq. is

$$AE\Delta + q^k F = T A_m \quad (4.1)$$

• degree of  $E(q^{-1}) = k-1$

• degree of  $F(q^{-1}) = \text{degree of } A\Delta - 1$

The corresponding prediction model is

$$\begin{aligned} T A_m Y(t+k | t) &= F Y(t) + E B U(t) \\ &= F Y(t) + \beta \Delta U(t) \end{aligned}$$

If  $T(q^{-1}) = C(q^{-1})$  there should be faster convergence and optimality conditions can be achieved.

Let  $A_m Y(t)$  be  $Y_f(t)$ .

Using Pseudo linear regression model the prediction model can be written as

$$Y_f(t+k | t) = \Phi(t) \Theta_0$$

where

$$\Phi(t) = [ Y(t), \dots, Y(t-n),$$

$$Y_f(t+k-1 | t), \dots, Y_f(t+k-l | t),$$

$$U(t), \dots, U(t-m-k+1) ]^T$$

$$\Theta_0 = [ f_0, \dots, f_m, c_1, \dots, c_l, \beta_0, \dots, \beta_{m-k-1} ]$$

$$= [\theta_1, \dots, \theta_m, \dots, \theta_{m+l+mk}]$$

where  $fn = n + 1$

## B. OFFSET ELIMINATION

It is clear in (4.1) that

$$F(1) = T(1)Am(1)$$

For offset elimination parameter is constrained by

$$F(1) = H(1)$$

Specifying a polynomial  $H(q^{-1})$  with  $H(1) = bmT(1)$  it is clear in (4.1) that  $F(1) = H(1)$  which shows that the system satisfies conditions for offset elimination and also gives the constraint in estimating  $\hat{F}(q^{-1})$ , i.e.,

$$\hat{F}(1) = H(1) \text{ where } H(1) = bm\hat{C}(1)$$

The predictor-based direct self tuning control is given by

$$U(t) = \frac{H(q^{-1})W(q^{-1}) - F(q^{-1})Y(t)}{(1-q^{-1})\beta(q)}$$

which has an integral action and satisfies the condition for offset elimination.

## 5. IMPLEMENTATION

We now propose the following constrained least-square estimation algorithm in accordance with Section 4.

$$\begin{aligned} \bar{\Theta}(t) &= \hat{\Theta}(t-1) + \bar{S}(t-k)\Phi(t-k)e(t) \\ e(t) &= Yf(t) - \Phi^T(t-k)\hat{\Theta}(t-1) \\ S(t) &= S(t-1) + \Phi(t)\Phi^T(t) \\ Yf(t+k|t) &= \Phi(t)\hat{\Theta}(t) \end{aligned}$$

Parameter is constrained by

$$\hat{\Theta}(t) = \bar{\Theta}(t) + \frac{S(t-1)M}{M^T S(t-1)M} [H(1) - F(1)]$$

where  $M = [1, \dots, 1, 0, \dots, 0]$

$\underbrace{\hspace{10em}}_{f_m}$

Remarks:

The constrained estimation algorithm preserves the properties required for system stability analysis [8] and consequently system stability follows.

## 6. Simulation

The simulation study was performed for a stirred-tank heated process [6].

A disturbance was considered as the change in the volume of water : output disturbance.

Assuming a constant water flow rate the dynamic model reduces to the following transfer function

$$\frac{Y(s)}{U(s)} = \frac{4.15e^{-\tau s}}{(119s+1)(71s+1)}$$

where  $\tau$  is time delay.

In case  $\tau = (k-1)h$ , the dynamic model sampled with a sampling time of  $h = 100$  gives the discrete system

$$G(z^{-1}) = z^{-k} \frac{0.2917z^{-1} + 0.1377}{1 - 0.6761z^{-1} + 0.1055z^{-2}}$$

Output disturbance was considered to be random steps.

The simulated system performance is shown in Fig.2 where the zero offset is achieved.

In comparison, the performance of corresponding system with the controller which is cascaded to integrator without constraining the estimate is shown in Fig.3 where some offset exit.

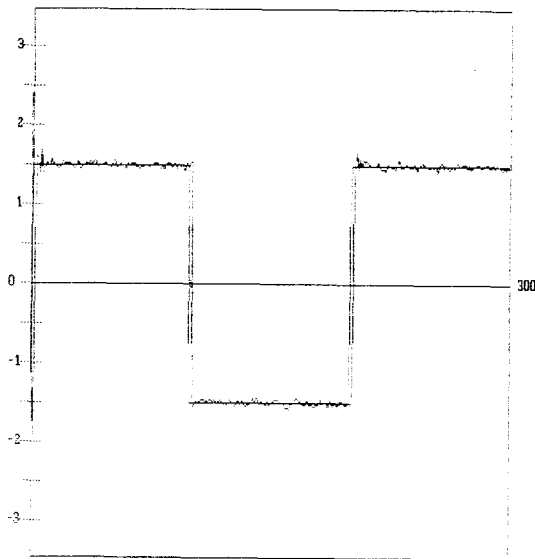


Fig.2 Simulation with estimation constraint and integral action

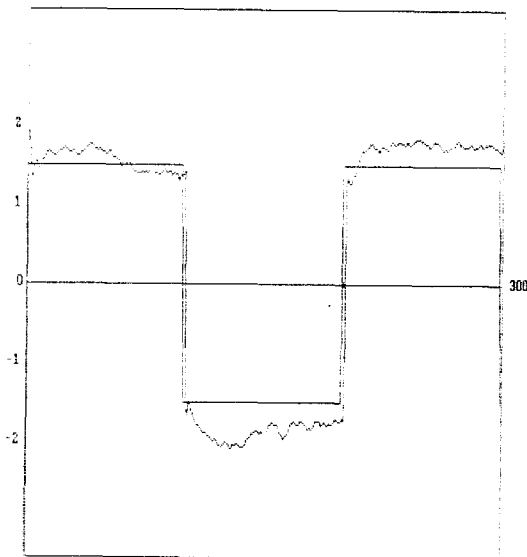


Fig.3 Simulation with cascading integrator

## 7. CONCLUSION

This paper has considered the problem of offset elimination in direct predictive self-tuning algorithm. Using constraint in estimate the corresponding control which has integral action has worked properly. Simulation study is shown that this control eliminate various kinds of offset.

## REFERENCES

- [1] K. J. Astrom and B. Wittenmark, "On self-tuning regulators," *Automatica*, Vol.9, pp. 185-199, 1973.
- [2] D. W. Clark and P. J. Gawthrop, "Self-tuning controller," *Proc. IEE*, Vol.122, pp. 929-934, 1975.
- [3] D. W. Clark and P. J. Gawthrop, "Self-tuning controller," *Proc. IEE*, Vol.126, pp. 633-640, 1979.
- [4] A. Y. Allidina and F. M. Hughes, "Self-tuning controller with integral action," *Opt. Contr. Appl. Methods*, Vol.3, pp. 355-362, 1982.
- [5] D. W. Clark, A. J. F. Hodson, and P. S. Tuffs, "Offset problems and k-incremental predictors an self-tuning control," *Proc. IEEE*, Vol.130, Pt. D, pp. 217-225, 1983.
- [6] F. Cameron and D. E. Seborg, "A self-tuning controller with a PID structure," *Int. J. Contr.*, Vol.38, no.2, pp. 401-417, 1983.
- [7] P. J. Gawthrop, "Self-tuning PID controllers : Algorithms and implementation," *IEEE Trans. Automat. Contr.* Vol.AC-31, pp. 201-209, Mar. 1986.
- [8] G. C. Goodwin and K. Sin, *Adaptive Filtering Prediction and Control*. Englewood Cliffs, NJ: Prentice-Hall, 1984.
- [9] C. Samson, "Stability analysis of adaptive control systems subject to bounded disturbances," *Automatica*, Vol.19, no. 1, pp. 81-86, 1983.
- [10] C. Zhang and R. J. Evans, "Offset elimination in direct self-tuning control," *IEEE Trans. A.C.* Vol.33, pp. 603-607, June, 1988.
- [11] C. J. Harris and S. A. Billings (Eds), "Self-tuning and adaptive control," Peter Peregrinus, 1981.

- [12] P. S. Tuff and D. W. Clark, "Self-tuning control of offset : a unified approach," IEE Proc. Vol.132, pp. 100-110, 1985.
- [13] K. J. Astrom and B. W. Wittenmark, "Self-tuning controllers based on pole-zero placement," IEE Proc. Vol.127, no. 3, pp. 120-130, 1980.
- [14] D. W. Clark, C. Mohtadi and P. S. Tuffs, "Generalized predictive control - Part1. The basic algorithm," Automatica, Vol.23, no. 2, pp. 137-148, 1987.