Design of Reduced-Order Controllers in Two-Degree-of-Freedom Control Systems

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Abstract: In this paper, we propose a new method of designing a reduced-order controller for a linear discrete-time system. Firstly, we study a design problem for a two-degree-of-freedom control system with a feedforward controller. Secondly, in order to obtain a reduced-order controller, frequency-weighted least squares approximation problems are considered. Thirdly, we propose a synthesis procedure of a reduced-order controller. Finally, an example is given to illustrate the effectiveness of this proposed method.

1. Introduction

Controller design procedures, such as a linear quadratic Gaussian (LQG) design or some other standard methods, will often lead to high-order compensators for physical systems described by high-order linear time-invariant state space models. In order to use fewer components or computing resources, or to obtain higher reliability of an implementation, a lower-order controller is required. For this reason, reduction of the order of the controller may be desirable[1-3].

In recent years, a number of new method of designing a two-degree-of-freedom controller has been developed for a model matching problem[4,5]. By using a two-degree-of-freedom controller, two specifications could be independently satisfied; one is closed-loop characteristics such as the disturbance rejection or the sensitivity, and the other is transfer characteristics with respect to the reference command.

In this paper, we consider a design problem for a two-degree-of-control system with a feedforward controller, as shown in Fig.1[6]. First, we design a cascade controller(C_B) in order to obtain a desirable sensitivity and stability margin. Then, we design a feedforward controller(C_F) so that the resulting control system may satisfy a specification with respect to the reference command.

The proposed method is based on minimizing a frequency-weighted error so that the resulting control system approximates the specified reference model. A square error criterion is introduced as a measure of the approximation. By minimizing the criterion, a linear equation is obtained for parameters of a finite impulse response (FIR) controller[3]. This equation can be solved recursively by increasing the order of the controller by a type of Levinson's algorithm[2]. Since the order of the controller can be arbitrarily specified in this method, the lowest order which realizes the desirable accuracy of the approximation can be determined without regard to the order of the plant or the reference model.

2. Two-degree-of-freedom control system

Consider the two-degree-of-freedom control system with a feedforward controller, as shown in Fig.1. Here, $P(z)$, $C_B(z)$ and $C_F(z)$ denote the plant, the cascade controller and the feedforward controller. When $C_F(z)=0$, then the control system has one-degree-of-freedom and is stable if and only if

$$S,\ SP,\ C_BS,\ I-C_BSP \in RH^{\infty}, \tag{2.1a}$$

$$\text{where}\quad S=(I+PC_B)^{-1}, \tag{2.1b}$$

and RH^{∞} denotes the set of all proper stable rational matrices[4].

Lemma. The class of two-degree-of-freedom controllers for the plant $P(z)$, as in Fig.1, is stabilizing if and only if

$$(2.1)\ \text{holds}, \tag{2.2a}$$

$$C_F \in RH^{\infty}. \tag{2.2b}$$

Proof. The condition (2.2a) means that C_B is stabilizing as one-degree-of-freedom controller for the plant. On the other hand, transfer functions from reference command (r) to outputs (y,y_1,y_2) are $(I+PC_B)^{-1}P(C_B+C_F)$, $C_B(I+PC_B)^{-1}(I-$

PC_F) and C_F, respectively. The results obtained by using (2.1) and the well-known identities $(I+PC)^{-1}=I-P(I+CP)^{-1}C$, $C(I+PC)^{-1}=(I+CP)^{-1}C$. □

Here, the attainable transfer characteristics are expressed as

$$G_{yr}=(I+PC_B)^{-1}P(C_B+C_F), \tag{2.3a}$$

$$G_{er}=(I+PC_B)^{-1}(I-PC_F), \tag{2.3b}$$

$$G_{ur}=(I+C_BP)^{-1}(C_B+C_F), \tag{2.3c}$$

$$G_{yd}=(I+PC_B)^{-1}P, \tag{2.3d}$$

$$S=(I+PC_B)^{-1}, \tag{2.3e}$$

$$T=I-S=(I+PC_B)^{-1}PC_B, \tag{2.3f}$$

where G_{yr}, G_{er} and G_{ur} are the transfer functions from r to y, e and u, respectively, and G_{yd} is the transfer function from d to y, S is the sensitivity matrix. Therefore, a cascade controller C_B is designed in order to obtain the desirable feedback properties such as the disturbance rejection or the sensitivity, and then a feedforward controller C_F with the view of responses for the reference command is designed.

3. Frequency-weighted least squares approximation

In this section, in order to obtain a reduced-order controller, we consider frequency-weighted least squares approximation problems.

3.1 Single-term criterion

Let us define a following frequency-weighted error:

$$D(z)=W(z)\{K(z)-C(z)\}$$

$$=M(z)-W(z)C(z), \tag{3.1a}$$

$$\text{where} \quad M(z)=W(z)K(z), \tag{3.1b}$$

K(z) is a high-order controller that realizes a desirable control property, C(z) is a reduced-order controller and W(z) is a weighting function which is frequency dependent. For (3.1), let us consider a following cost function

$$J=\| D(z) \|_2^2$$

$$=tr[(2\pi j)^{-1} \int_C D(z)D^*(z)z^{-1}dz]$$

$$=tr[\sum_{k=0}^{\infty} D_k D_k^T], \tag{3.2a}$$

$$D(z)=\sum_{k=0}^{\infty} D_k z^{-k}, \tag{3.2b}$$

where tr, T and * denote the trace, transpose and conjugate of the matrix, respectively, and C is the unit circle. M(z) and W(z) are assumed to asymptotically stable, are expressed as follow

$$M(z)=\sum_{k=0}^{\infty} M_k z^{-k}, \quad W(z)=\sum_{k=0}^{\infty} W_k z^{-k} \tag{3.3}$$

As a transfer function of the C(z), we consider the FIR type:

$$C(z)=C_0+C_1z^{-1}+\dots+C_mz^{-m}. \tag{3.4}$$

Then equation (3.2) is equivalent to the following

$$J=tr[\Theta_0-2\sum_{i=0}^{m} C_i^T\Psi_i+\sum_{i,j=0}^{m} C_i^T\Phi_{i-j}C_j], \tag{3.5a}$$

$$\Theta_0=\sum_{k=0}^{\infty} M_k^T M_k, \tag{3.5b}$$

$$\Psi_i=\sum_{k=0}^{\infty} W_k^T M_{k+i}, \tag{3.5c}$$

$$\Phi_i=\sum_{k=0}^{\infty} W_k^T W_{k+i}, \tag{3.5d}$$

$$\Phi_{-i}=\Phi_i^T, \tag{3.5e}$$

where $i=0,1,\dots,m$.

Minimizing (3.5a) with respect to $C_i(i=0,\dots,m)$ satisfies

$$\begin{bmatrix} \Phi_0 & \Phi_1^T & \cdots & \Phi_m^T \\ \Phi_1 & \Phi_0 & \cdots & \Phi_{m-1}^T \\ \vdots & \vdots & \ddots & \vdots \\ \Phi_m & \Phi_{m-1} & \cdots & \Phi_0 \end{bmatrix} \begin{bmatrix} C_0 \\ C_1 \\ \vdots \\ C_m \end{bmatrix} = \begin{bmatrix} \Psi_0 \\ \Psi_1 \\ \vdots \\ \Psi_m \end{bmatrix} \tag{3.6}$$

Since the coefficient matrix in (3.6) is positive definite in symmetric Toeplitz form, the equation (3.6) always has a unique solution for C_i[2,3].

3.2 Two-terms criterion

Let us define two frequency-weighted errors as follows,

$$D_j(z)=W_j\{K_j(z)-C(z)\}$$

$$=M_j(z)-W_j(z)C(z), \tag{3.7a}$$

$$M_j(z)=W_j(z)K_j(z), \tag{3.7b}$$

where $j=1,2$.

For (3.7), let us consider the following cost function with two-terms

$$J=\| D_1(z) \|_2^2+\| D_2(z) \|_2^2. \tag{3.8}$$

If all of $M_j(z)$ and $W_j(z)$ for $j=1,2$ are asymptotically stable,

$$M_j(z)=\sum_{k=0}^{\infty} M_{jk} z^{-k}, \quad W_j(z)=\sum_{k=0}^{\infty} W_{jk} z^{-k}, \tag{3.9}$$

where $j=1,2$,

and if $C(z)$ is the FIR type, then (3.8) becomes

$$J=\mathrm{tr}\left[\Theta_0-2\sum_{i=0}^{m} C_i^T \Psi_i+\sum_{i,j=0}^{m} C_i^T \Phi_{i-j} C_j\right], \tag{3.10a}$$

$$\Theta_0=\Theta_{10}+\Theta_{20}, \quad \Psi_i=\Psi_{1i}+\Psi_{2i}, \quad \Phi_i=\Phi_{1i}+\Phi_{2i}, \tag{3.10b}$$

$$\Theta_{j0}=\sum_{k=0}^{\infty} M_{jk}^T M_{jk}, \tag{3.10c}$$

$$\Psi_{ji}=\sum_{k=0}^{\infty} W_{jk}^T M_{jk+i}, \tag{3.10d}$$

$$\Phi_{ji}=\sum_{k=0}^{\infty} W_{jk}^T W_{jk+i}, \tag{3.10e}$$

where $i=0,1,\ldots,m$, $j=1,2$.

Therefor, similarly in section 3.1, minimizing J with respect to C_i $(i=0,\ldots,m)$ yields a linear equation.

4. Synthesis procedure

From the previous arguments, we can give the following synthesis procedure for a cascade controller ($C_B(z)$) and a feedforward controller ($C_F(z)$) in the two-degree-of-freedom control system shown in Fig.1.

4.1 Design procedure for $C_B(z)$

Given a plant $P(z)$ and a high-order controller $K_B(z)$ stabilizing $P(z)$, suppose we replace the controller $K_B(z)$ by a low-order one $C_B(z)$. The one-degree-of-freedom control system with the reduced-order controller $C_B(z)$ can be described as in Fig.2. It is known that if $C_B(z)$ has the same number of unstable of poles as $K_B(z)$, and if

$$\sup_{\omega} \bar{\sigma}[E(e^{j\omega})]<1, \quad -\pi \leq \omega \leq \pi \tag{4.1a}$$

where

$$E(z)=\{I+P(z)K_B(z)\}^{-1}P(z)\{C_B(z)-K_B(z)\}, \tag{4.1b}$$

and $\bar{\sigma}$ denotes the largest singular value of the matrix, then the closed-loop system with the reduced-order controller will be stable also[1]. In this paper, as designing a reduced-order controller $C_B(z)$, we evaluate the error for equation (4.1b) as follows:

$$J_B=\|W(z)E(z)\|_2^2, \tag{4.2}$$

where $W(z)$ is the weighting function. If a cascade controller $C_B(z)$ is the FIR type, $C_B(z)$ is obtained by the method in section 3.

4.2 Design procedure for $C_F(z)$

As designing a feedforward controller $C_F(z)$, we introduce the following cost function:

$$J_F=\|W_1(z)E_1(z)\|_2^2+\|W_2(z)E_2(z)\|_2^2, \tag{4.3a}$$

$$E_1(z)=M_{yr}(z)-G_{yr}(z)$$

$$=M_1(z)-V_1(z)C_F(z), \tag{4.3b}$$

$$M_1(z)=M_{yr}(z)-\{I+P(z)C_B(z)\}^{-1}P(z)C_B(z), \tag{4.3c}$$

$$V_1(z)=\{I+P(z)C_B(z)\}^{-1}P(z), \tag{4.3d}$$

$$E_2(z)=M_{ur}(z)-G_{ur}(z)$$

$$=M_2(z)-V_2(z)C_F(z), \tag{4.3e}$$

$$M_2(z)=M_{ur}(z)-\{I+C_B(z)P(z)\}^{-1}C_B(z), \tag{4.3f}$$

$$V_2(z)=\{I+C_B(z)P(z)\}^{-1}, \tag{4.3g}$$

where $W_1(z)$ and $W_2(z)$ are weighting functions. The desired response and control input are expected by regulating the weighting functions in (4.3a).

5. Example

An example is presented to illustrate the performance of the design method. A transfer function of the plant are given by

$$P(z)=\frac{0.00833z^{-1}+0.00917z^{-2}+0.0025z^{-3}}{1-1.7z^{-1}+0.72z^{-2}}.$$

For a simplification, we consider the two-degree-of-freedom control system which realizes approximately the reference model given by

$$M(z)=\frac{0.28z^{-1}+0.28z^{-2}}{1-0.5z^{-1}+0.06z^{-2}}.$$

The unit step responses and the unit pulse responses of the plant and the reference model are plotted in Fig.3. First, some properties for one-degree-of-freedom control system ($m=1,2$, $C_F(z)=0$) are shown in Fig.4-7, where $W(z)=1$. Since the controller $C_B(z)$ doesn't have a integrator, we observe that the steady state error for a unit step reference input remain(Fig.4). The unit pulse responses are shown in Fig.5. The responses for a unit step disturbance are shown in Fig.6. The effect of the order m of the controller $C_B(z)$ upon performance measure J_B is shown in Fig.7. Next, some properties for two-degree-of-freedom control system ($M_{yr}(z)=M(z)$) are shown in Fig.8-12. It can be seen that the approximation is almost exact when n=6. The effect of the order n of the feedforward controller $C_F(z)$ upon the performance measure J_F is shown in Fig.10. Observe that the response becomes closer to the specified one as n is increased. The effect of weighting function $W_2(z)$ is shown in Fig.11,12, where $M_{ur}(z)$=0. When $W_2(z)$=0.005, the control input becomes small and changes slowly.

6. Conclusion

This paper has proposed a method of designing a reduced-order controller in two-degree-of-freedom control system. Using the method, we can design a control system such that two specifications with respect to responses for reference commands and closed-loop characteristics could be satisfied. Because we use the FIR controller, it is easy to calculate the parameters of the controller. Since the order of the controller can be arbitrarily specified in this method, the lowest order which realizes the desirable accuracy of the approximation can be determined without regard to the orders of the plant or the reference model.

It is possible to extend this technique to an integrated finite impulse response (IFIR) controller or an infinite impulse response (IIR) controller, namely, a rational controller[2]. Because of the simplicity of the proposed algorithm, this design method is suitable for a computer aided design (CAD) system.

The further problem to be done is to investigate the servomechanism problem and the effect of the weighting function which is frequency dependent.

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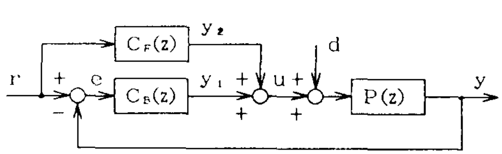

Fig.1 Two-degree-of-freedom control system (Feedforward type).

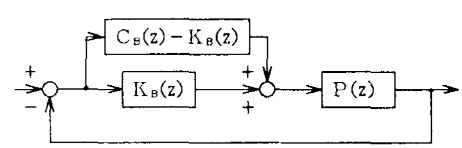

Fig.2 Closed-loop system with reduced-order controller.

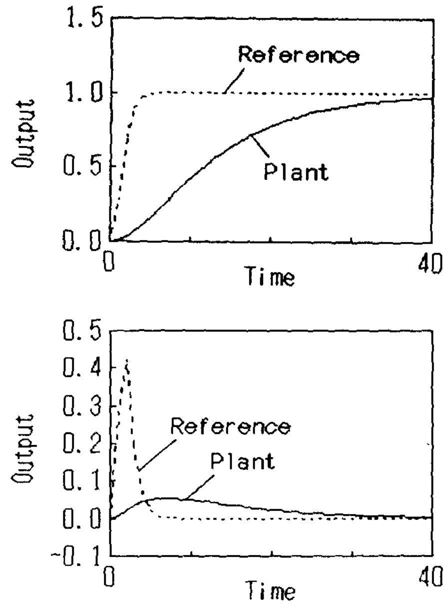

Fig.3 Unit step responses and unit pulse responses of the plant and the reference model.

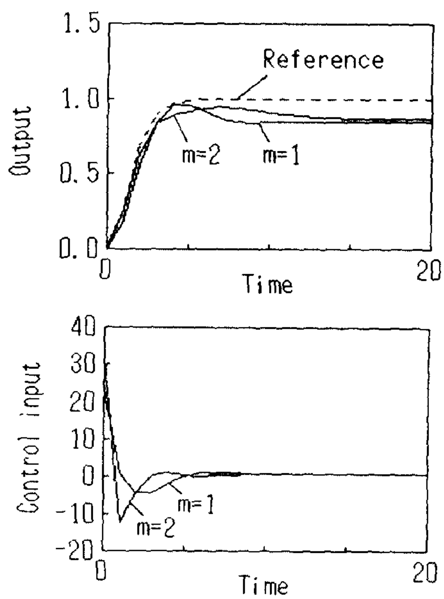

Fig.4 Outputs and control inputs for a unit step reference input ($W(z)=1$, $C_F(z)=0$).

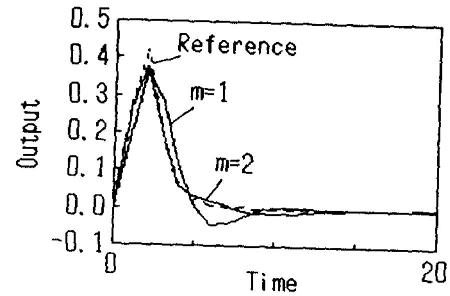

Fig.5 Unit pulse responses ($W(z)=1$, $C_F(z)=0$).

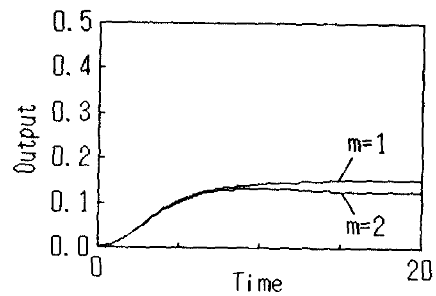

Fig.6 Outputs for a unit step disturbance ($W(z)=1$, $C_F(z)=0$).

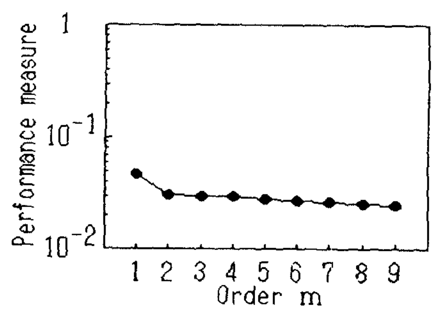

Fig.7 Effect of the order m upon performance measure J_B ($W(z)=1$, $C_F(z)=0$).

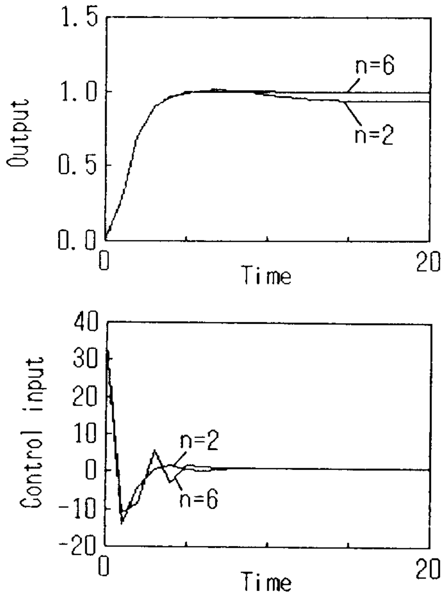

Fig.8 Outputs and control inputs for a unit step reference input (m=2, W_1(z)=1, W_2(z)=0).

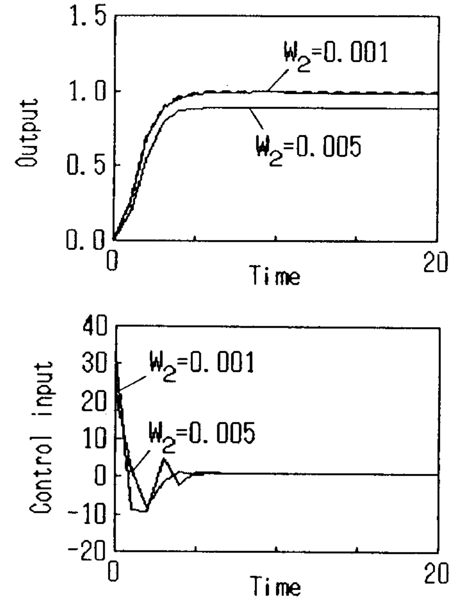

Fig.11 Outputs and control inputs for a unit step reference input (m=2, n=6, W_1(z)=1, M_{ur}(z)=0).

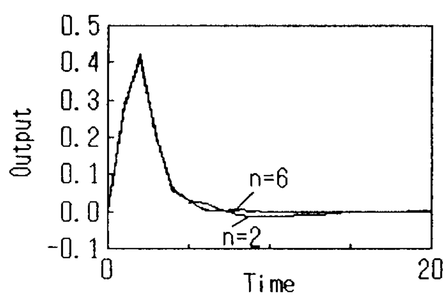

Fig.9 Unit pulse responses (m=2, W_1(z)=1, W_2(z)=0).

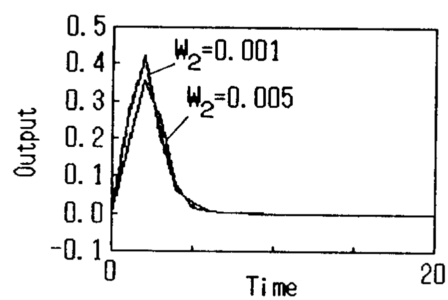

Fig.12 Unit pulse responses (m=2, n=6, W_1(z)=1, M_{ur}(z)=0).

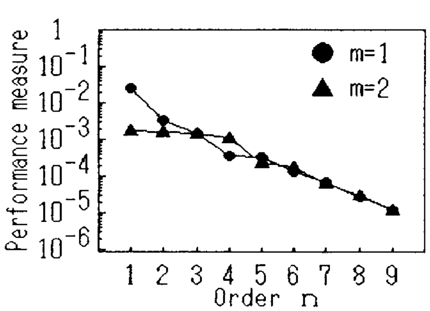

Fig.10 Effect of the order n upon performance measure J_F (m=1,2, W_1(z)=1, W_2(z)=0).