'88 KACC 1988. 10. 21~22

CONSTRUCTION AND ROLES OF COMPUTER SIMULATOR FOR DIGITAL CONTROLLER DESIGN

Masatoshi **NAKAMURA**

Department of Electrical Engineering, Saga University,
Honjomachi, Saga 840, Japan

Abstract: The structure of a digital controller based on modern control theory is more complex than that of a PID controller. In implementing the digital control of an actual system by using the digital controller, we often encounter gaps between theory and practice e.g. quantization error, sampling error, modeling error, contaminated noise etc. In such cases, simulator plays an important role in detecting difficulties. This paper demonstrates the importance of the computer simulator for designing a digital controller. The controller and the simulator are constructed by different computer respectively, with a link between the blocks by analogue signals through the A/D, D/A converters. Through the simulator test, we can evaluate the digital controller; identify and solve difficulties in the digital control. The controller, which passed the simulator test, is used identically in the actual system. This was a successful procedure for designing the controller. As an example, we successfully constructed the digital controller using the computer simulator for inverted pendulum control. We then compared the control results of simulator and actual equipment. Furthermore we commented on the construction of the computer simulator which exactly expressed the actual system.

1. Introduction

The digital control based on modern control theory has been adopted in various fields. Those examples can be found in mechanical control systems, such as stabilization control of inverted pendulum [1], crane control [2], walking robot control [3], and in thermal flow control systems, such as moisture control of paper machine [4], working fluid flow rate control of the ocean thermal energy control[5]etc. The digital controller based on modern control theory is designed in a way in which the controller precisely grasps the characteristics of the controlled object. The structure of the digital controller then becomes more complex than that of a PID controller. If we express the controller as a mirror image of the actual system, we can clarify the structures of the digital controller and can show the designing procedure systematically [6]. In implementing the digital control of the actual system by using the digital controller, we often encounter gaps between theory and practice. It is usually laborious to find out the causes of difficulties and, for safety and economic reasons, it is not a recommendable way to check the actual system directly. In such cases, a computer simulator plays an important role in detecting difficulties.

Main purpose of this paper is to demonstrate the importance of the computer simulator for designing a digital controller and to show the construction of the computer simulator. The controller and the simulator are constructed by different computer respectively, with a link between the two blocks by analogue signals through the A/D, D/A converters. As an example, we successfully constructed the digital controller using the computer simulator for inverted pendulum control. We then compared the control results of simulation, simulator and actual equipment, and found the coincidence in each case. Furthermore we commented on the construction of the computer simulator which exactly expressed the actual system.

2. Roles of computer simulator for controller design

2.1 Systematical procedure for controller design

The digital controller based on modern control theory aims at an optimal control,therefor, it is required to grasp the characteristics of the controlled object precisely. Unlike the PID controller, the structures of the digital controller become more complex. If we express the controller as a mirror image of the actual system, we can then clarify and classify the structures of the digital controller into three parts: detector part, control theory part and actuator part (see Fig. 1).

The procedure for designing the digital controller is summarized in the following five steps.

1) Setting of the control objective We first clarify the control objective and draw a line between the controlled system and other parts of the system. We classify the variables into control variables u^* and state variables x^*, and confirm the measurement variables y^*.

2) Modeling of the controlled object We construct a mathematical model, equivalent to the controlled object, by using the physical law and identification method based on input-output data.

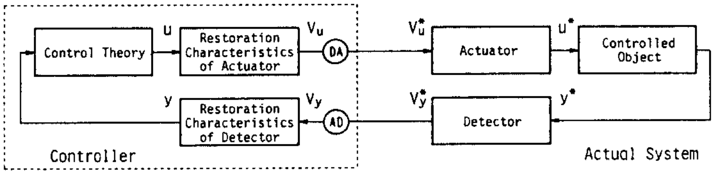

Fig. 1 Scheme of digital control system

3) Application of control theory to the model We select an appropriate control theory which produces a computational algorithm for an optimal control of the current problem.

4) Characterization of the detector part and actuator part The information between the actual system and the digital controller is mutually exchanged by analogue voltage signals. Through those analogue voltage signals, we can know the values of physical variables of the input and output of the controlled object. By means of signal transmission, the output of actuator u^* and the input of detector y^* should be equal to the output and input of the control theory part u and y, respectively. The restoration characteristics of the actuator $(u - V_u)$ are determined as an inverse function of the characteristics of the actuator $(V_u^* - u^*)$, which were obtained earlier from the measured quantities of the actuator. In the same way, the restoration characteristics of the detector $(V_y - y)$ are determined as an inverse function of the characteristics of the detector $(y^* - V_y^*)$.

5) Construction of the controller The digital controller is constructed by an digital computer, in which the restoration characteristics of the detector $(V_y - y)$, control theory part $(y - u)$ and the restoration characteristics of the actuator $(u - V_u)$ are programmed and A/D, D/A converters are equipped.

2.2 Roles of a computer simulator

In implementing the control of an actual system by using the digital controller thus designed, we often encounter gaps between theory and practice e.g. quantization error, sampling error, modeling error, contaminated noise etc. It is usually laborious to find out the causes of difficulties and, for safety and economic reasons, it is not a advisable to check the actual system directly. In such cases, the digital simulator plays an important role in detecting difficulties one after another. A successful completion of the controller design requires three stages: simulation, conventional simulator and complete simulator. Fig. 2 shows a scheme of simulation, conventional simulator and complete simulator.

(A) Simulation

Figure 2(A) illustrates a widely-used simulation, where a model of the controlled object and a control theory part are programmed in the same computer. By means of the simulation, we can check the following items: selection of control theory, computational algorithm, selection of the control parameter, sampling error etc. The program of the model part and the control theory part are identically used afterwards as a part of the simulator and the controller, respectively.

(B) Conventional simulator

The conventional simulator is constructed by a computer in which the model of the controlled object is programmed and A/D, D/A converters are equipped. The conventional simulator is controlled by a conventional controller which is programmed in another computer with the control theory part and equipped with A/D, D/A converters. These two computers are linked by analogue signals through the A/D, D/A converters. By using the conventional simulator, we can detect the following items: quantization error of the A/D, D/A converters, effect of the asynchronized computation and noise contamination.

(C) Simulator

Figure 2(C) shows the complete controller and the complete simulator. The simulator is constructed by adding the characteristics of the actuator and the detector to the conventional simulator. The simulator is controlled by the complete controller, which consists of the conventional controller and the

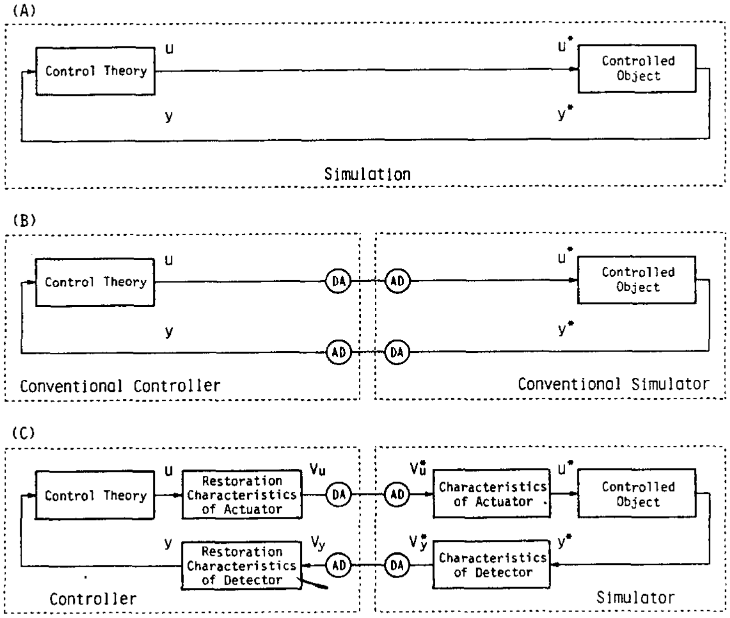

Fig. 2 Digital control system: simulation, conventional simulator and simulator

restoration characteristics of the actuator and the detector. Through simulator test, we can evaluate the modeling of actuator and detector, and resolving power of the A/D, D/A converters in the complete controller. The likely problems in the controller may be solved at this stage.

(D) Actual system

The controller, which passed the simulator test, is used identically in the actual system. If the digital control for the actual system is successful, the design of the controller is complete. Even in case of any failure, we can easily detect the cause of the failure beyond those occurring in the already checked items. These checks are rather easier because the causes of the failure exist in a localized part e.g. the modeling errors in the simulator, contaminated noise in the actual system etc.

3. Controller construction using the computer simulator

3.1 Analysis of inverted pendulum control system

Using the stabilizing control problem of an inverted pendulum, as an typical example, we try to construct the digital controller through the computer simulator. The control objective is to stabilize the pendulum inverted by an appropriate position control of the cart. The controlled object, the pendulum, is fixed at a end to a rotary axis of a potentiometer and rotates freely. The cart with the potentiometer and the pendulum can be actuated by an AC servo-motor with position loop controller. The angle of the inverted pendulum and the position of the cart are measured by the potentiometers. All equations for the inverted pendulum control are summarized in the following sections.

(1) Mathematical model of the equipment

The dynamic equation is derived by the Newton law. The linear dynamic equation and the measurement equation is obtained by using the approximation sin $\theta^* \fallingdotseq \theta^*$, cos $\theta^* \fallingdotseq 1$ and neglecting friction as

$$\dot{x}^* = Ax^* + Bu^*, \qquad (1)$$

$$y^* = Cx^* \qquad (2)$$

where the state variables $x^* = [\theta^* \; r^* \; \dot{\theta}^* \; \dot{r}^*]^T$, θ^* is the angle of the inverted pendulum and r^* is the position of the cart, the manipulated variable is the force u^*, the measurement variables $y^* = [\theta^* \; r^*]^T$ and T denotes transposition. The matrices in (1) and (2) are expressed as

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ (M+m)mhg/Q & 0 & 0 & 0 \\ -m^2h^2g/Q & 0 & 0 & 0 \end{bmatrix},$$

$$B = [\, b_1 \; b_2 \; b_3 \; b_4 \,]^T = [\, 0 \; 0 \; -mh/Q \; J/Q \,]^T,$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

where M is the mass of the cart, m is the mass of the pendulum, h is the distance between the fixed point and the center of gravity of the pendulum, g is the acceleration of gravity, J is the inertia moment of the pendulum, and $Q=(M+m)J-m^2h^2$.

Concerning the characteristics of the actuator, that is, the AC servo-motor, the coefficient (1/G) of the relation $r^*=(1/G)V_u^*$ is determined by the least squares method based on the available data (r^*, V_u^*). The position of the cart r^* and the input voltage of the AC servo-motor V_u^* are recorded with the pendulum detached. The mathematical equation for the characteristics of the actuator can be expressed by the multiplication of the mass by the acceleration of the cart:

$$u^* = (M/G)\ddot{V}_u^*. \qquad (3)$$

Concerning the characteristics of the detector, that is, the potentiometers, the coefficients of the relations are also determined by the least squares method. The relation between the pendulum angle θ^* and the output voltage of the potentiometer V_θ^* is

$$V_\theta^* = (1/W_1)\theta^* \qquad (4.1)$$

and the relation for the cart position is expressed as

$$V_r^* = (1/W_2)r^*. \qquad (4.2)$$

(2) Equations for controller side

We adopt the regulator and observer of pole placement [7]which stabilizes the inverted pendulum in a center position of the cart. The control variable is obtained as

$$u = K\hat{x}. \qquad (5)$$

The feedback gain K is calculated by the regulator of pole placement as

$$K = [\, \lambda^4 b_4/b_3{}^2 g - 6\lambda^2/b_3 - a_{31}/b_3, \; -\lambda^4/b_3 g,$$

$$-4\lambda^3 b_4/b_3{}^2 g + 4\lambda/b_3, \; 4\lambda^3/b_3 g \,]$$

where λ is the 4th multiple pole which the designer chooses appropriately in negative region. The state estimation $\hat{x}$ is obtained by the observer of pole placement as

$$\hat{x} = Dz + Hy, \qquad (6.1)$$

$$\dot{z} = Az + K_c y + \tilde{B}u \qquad (6.2)$$

where $L=-\rho I$, I is an identity matrix, ρ is the duplicated pole which the designer chooses appropriated in a negative region. It is required that the control variable u transmits correctly to the input of the controlled object through the actuator. The restoration characteristics of the actuator are then determined as an inverse function of the characteristics of the actuator:

$$V_u = G \iint u dt^2/M. \qquad (7)$$

For the same reason, the restoration characteristics are chosen as inverse functions of the characteristics of the detectors as

$$\theta = W_1 V_\theta, \qquad (8.1)$$

$$r = W_2 V_r. \qquad (8.2)$$

3.2 Controller construction through the computer simulator

(A) Simulation

The computer simulation consisted of the mathematical model of the controlled object (1), (2) and the control theory part (5), (6) in a computer. The numerical calculation of the differential equations (1) and (6) were implemented by the use of Euler method as

$$x(k) = x(k-1) + \Delta f(x(k-1)) \tag{9}$$

which is the simplest algorithm and gives the shortest calculation time. The notation Δ denotes the sampling interval. In the stage of the simulation, the poles λ and ρ were determined by taking into account the maximum power of the actuator, the overshoot of the response and the sampling interval. The smaller the values of λ and ρ, the faster, the response becomes and accordingly, the peak of the required power of the actuator increases. For this equipment, the values λ and ρ were set at -3 and -8, respectively, and the sampling interval was chosen as 20 msec.

(B) Conventional simulator

The conventional simulator consisted of the mathematical model of the controlled object (1), (2) and A/D, D/A converters. The conventional controller consisted of the control theory (5), (6) and the A/D, D/A converters. The two blocks were linked by analogue signals through the A/D, D/A converters, and the control was implemented in real time asynchronizedly. By means of the conventional simulator, we ensured that the resolving power of the 12 bits A/D converters of 0.000077 rad and 0.212 mm, and the resolving power of the 12 bits D/A converter of 0.125 mm, were respectively enough for this control.

(C) Simulator

The complete simulator consisted of the conventional simulator and the characteristics of the actuator (3) and the detector (4). Equation (4) is linear and then does not cause any problem in numerical calculation, while equation (3) is the second order derivative function of which the numerical derivative often causes numerical errors. We adopted here a simple difference approximation:

$$u^*(k') = (M/G\Delta^2)\,\{V_u^*(k') - 2V_u^*(k'-1) + V_u^*(k'-2)\}\,. \tag{10}$$

The complete controller consisted of the conventional controller and the restoration characteristics of the actuator (7) and the detector (8). The integral equation (7) was calculated numerically by an exact second order difference equation as

$$V_u(k) = V_u(k-1) + \Delta w(k-1) + (\Delta^2/2M)u(k-1), \tag{11.1}$$

$$w(k) = w(k-1) + (\Delta/M)u(k-1). \tag{11.2}$$

where Δ denotes the sampling interval. The time chart of the simulator and the controller is illustrated in Fig. 3. Through simulation test, the numerical calculation of (10), (11) for the characteristics of the actuator proved to bring a good result for this study.

(D) Actual system

The controller thus designed, was used identically in the actual system. The output of the D/A converter of the controller was connected to the input terminal of the AC servo-motor and the inputs of the A/D converters were connected to the output terminal of the potentiometers. Through the

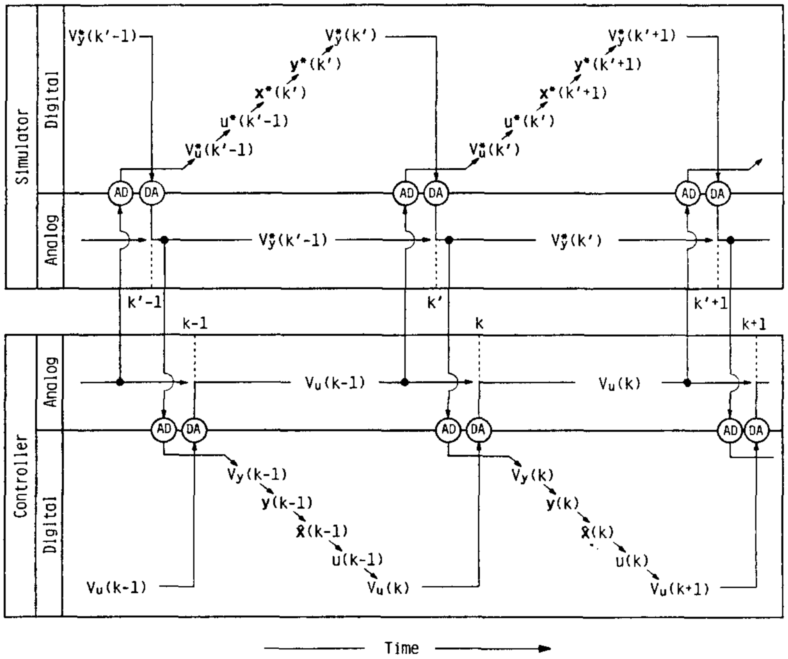

Fig. 3 Time chart of simulator and controller

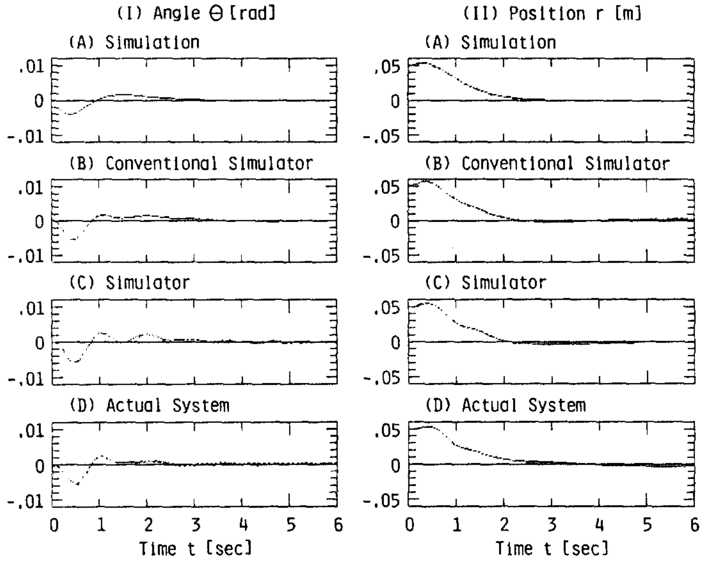

Fig. 4 Transient response of inverted pendulum control

implementation test, low pass filters (cut off frequency 170 Hz) were proved to be necessary at the detector part in order to eliminate high frequency noise contamination. In the next section, we shall discuss the results of inverted pendulum control for simulation, conventional simulator, simulator and actual system.

4. Experiment results

4.1 Transient response

Figure 4 shows the transient response of inverted pendulum control for simulation, conventional simulator, simulator and actual system. It illustrates the curves for the angle of the pendulum and the position of the cart. We can see a great similarity in the four cases. The controller design using the computer simulator was successful. The conditions for the controller were as follows: resolving power was 0.000077 rad, 0.212 mm; sampling interval was 20 msec; the multiple poles of the regulator λ were -3; the duplicated poles of the observer ρ were -8. The length of the pendulum l m was and the weight of the pendulum was 0.08 kg.

4.2 Steady state response

To see the experimental results in greater

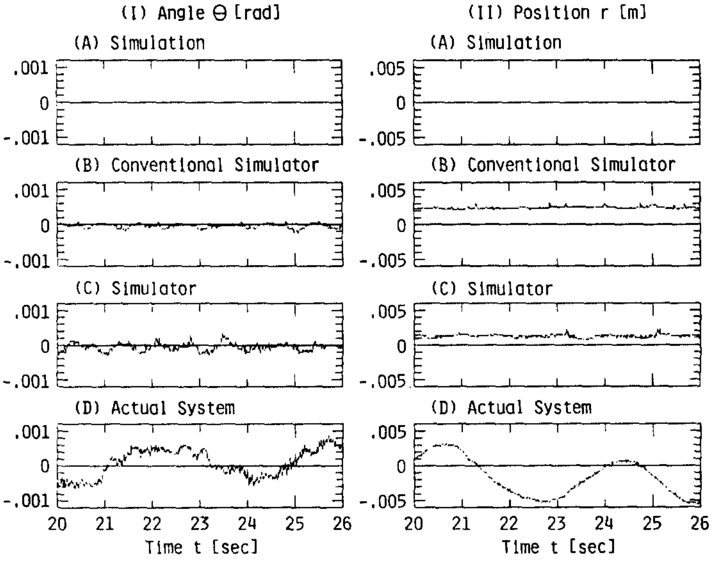

Fig. 5 Steady state response of inverted pendulum control

detail, we examined the steady state response of the inverted pendulum control. The conditions for the experiment were the same as the previous ones. The response after 20 sec is shown in Fig. 5. The scale in Fig. 5 is magnified ten times of the scale in Fig. 4. We found some differences in the simulation, conventional simulator, simulator and actual system. In the simulation of linear system control, both angle and position converge to zero. In the control of the actual system, we can see the limit cycles around equilibrium point in Fig. 5(D). In the simulator results, we find small fluctuations in Fig. 5(B) and (C). In order to detect causes of the differences, we examined the signal in the controller and the simulator.

4.3 Exact simulator construction

To see the causes of the differences between the theoretical and practical aspects, we examined the signal in greater detail at the following four points: u, V_u, V_u^*, u* (see Fig. 6). Theoretically, the signal u is the same as the signal u*, because the restoration characteristic of the actuator is the inverse function of the characteristic of the actuator. But the signal u* was different from u because of calculation error in numerical derivative. However, we may say that the more accurate the numerical derivative we uses, the closer the result will be. Furthermore, nonlinear terms which cause the limit cycle should be taken into the simulator. These are problems to be dealt with in future.

5. Conclusion

This paper demonstrates the importance of the computer simulator for designing a digital controller. The computer simulator plays an important role in detecting difficulties in the design of the digital controller. Through the simulator detection, we can evaluate the modeling of actuator, detector, and resolving power of the A/D, D/A converters and noise contamination etc. As an example, we constructed the digital controller and the computer simulator for inverted pendulum control. We were successful in designing the digital controller using the computer simulator and we commented on the construction of the computer simulator which exactly expressed the actual system.

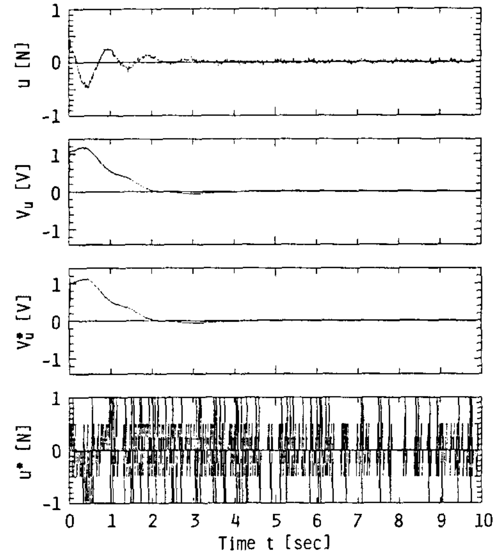

Fig. 6 Control signals in the controller and the simulator

References

[1] H. Kano, Adaptive Control of Inverted Pendulum, Computrol, Vol.2, pp.69-75 (1983) (in Japanese)

[2] T. Mita and H. Kanai, Optimal Control for a Crane System Using the Maximum Speed of the Trolley, Transactions of the SICE, Vol.15, No.6, pp.833-838 (1979) (in Japanese)

[3] T. Mita, Digital Control of High Speed Walking Robot, Computrol, Vol.2, pp.76-87 (1983) (in Japanese)

[4] K.J. Åström, U. Borisson, L. Ljung and B. Wittenmark, Theory and Applications of Self-Tuning Regulators, Automatica, Vol.13, pp.457-476 (1977)

[5] M. Nakamura, N. Egashira and H. Uehara, Digital Control of Working Fluid Flow Rate for an OTEC Plant, ASME journal of Solar Energy Engineering, Vol.108, pp.111-116, May (1986)

[6] M. Nakamura and S. Tahara, Design of a Hybrid Adaptive Controller for an Inverted Pendulum Hindered by a Moving Weight — Causes of and Compensation for Limit Cycles —, Transaction of IEE Japan, Vol.107-D, No.8, pp.1026-1032 (1987) (in Japanese)

[7] H. Ogo and T. Mita, Introduction to System Control Theory, Jikkyo (1979) (in Japanese)