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Delayed State Feedback Controller for the Stabilization of Ordinary Systems

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Abstract: A New type of controller for stabilization of ordinary system in which delayed states are included in feedback loop, is presented. Simple conditions are proposed for the stabilization of ordinary systems with the delayed state feedback controller. Under these conditions, controller gains can be chosen such that desired system performances are satisfied. It is shown that by using this controller the performance and robustness of the resulting closed loop system are much improved compared to the conventional memoryless state feedback controllers.

1. Introduction

It was known that an appropriate time-delay action in feedback control system may improve the performance of the controlled systems. But the excessive use of the time delay action produce severe effects on stability and system performance. Several reports have been published on this matter.[1,2,3,4,5]

It was shown in Tallman and Smith [1] and Rubin [2] that the use of the controller with time delays for a second order lightly damped oscillatory system may produce a deadbeat nonoscillatory response. The problem of designing the Propotional minus delay (PMD) controller for second order system have been studied by Suh and Bien [3,4], where Suh and Bien had suggested a controller utilizing the proportional-minus-delay action and showed that the time-delay action may be used in the design of feedback controller with a fast settling time property since it have the average PD action. However, their work was specifically concerned with second order system and they did not produce general controller design guidelines which both stability and system performance can be satisfied.

In Kim and Chung [5], another type of controller using time delay actions is suggested, which is based on feeding back the measurable state variable and their delayed values. But their work did not show the effects of the time delay for system performance and did not propose the practical design method of the controller for the stabilization of ordinary systems.

In this paper, simple methods are derived to stabilize ordinary systems by the delayed state feedback controller composed of a single or distributed delays, whose design parameters may be chosen to meet the derived sufficient conditions. And the guaranteed stability margins with the parameter uncertainty and the effects of the disturbance rejection are compared with conventional memoryless state feedback control schemes such as Linear Quadratic Regulator or Receding Horizon Control.

And it is shown that the time delays included in the feedback loop may add infinite zeros to the resulting closed loop system, so that ,for an appropriately adjusted h, the transient response of the system may be improved drastically. By these results repersented in this paper, controller design parameters can be chosen easily such that the desired system performance requirements are satisfied. Thereby, in the feedback control system design, the performance and robustness of the system may be more improved than those of the system which uses the memoryless state feedback controller.

This paper is organized as follows. In section 2, for the case when the distributed delays are added to the feedback loop, sufficient conditions for the resulting closed loop system to be memoryless stabilizable are derived. In section 3, robustness against parameter uncertainty is analyzed in terms of the guaranteed bounds of the allowable modeling error. In section 4, the improvements in the transient behavior and the disturbance rejection with the delayed state feedback are discussed. Section 5 presents a conclusion.

2. Construction of the stabilizing delayed state feedback controller

Consider a linear time invariant system.

$$x(t) = A\,x(t) + B\,u(t) \tag{2.1}$$

$$x(t) = \psi(t)\ ,t \in [-\tau ,0] \tag{2.2}$$

,where $x(t)\in R^{n\times 1}$, $u(t)\in R^{m\times 1}$, $y(t)\in R^{m\times 1}$, $t\geq 0$ and $\psi(t)$ is a continuous vector-valued initial function. We assume (A,B) is a controllable pair and (C,A) is a obserbable pair.

In the above system, the control law of the Linear Quadratic Regulator is given by

$$u(t) = -B'\bar{P}\cdot x(t) \tag{2.3}$$

where P is the solution matrix of the Algebraic Riccati equation

$$0 = A'\bar{P} + \bar{P}A - \bar{P}BB'\bar{P} + Q \tag{2.4}$$

We shall attempt to stabilize the above system (2.1) by the following new delayed state feedback

$$u(t) = -K_1x(t) - K_2x(t-h) - \int_0^h K(\tau)x(t-\tau)d\tau \tag{2.5}$$

,where $K_1,K_2,K_3 \in R^{m\times n}$ and K_1 is the control law of the Linear Quadratic regulator.

From (2.1) and (2.5), the closed loop system can be written as

$$x(t) = (A-BK_1)x(t) - BK_2x(t-h) - B\int_0^h K(\tau)x(t-\tau)d\tau \tag{2.6}$$

Conditions are given under which control law (2.5) may be used as a stabilizing control for system (2.1).

Theorem 2.1 : For some positive scalar h, the system (2.1) can be stabilized by means of a delayed state feedback controller of the form (2.5) where K_1 is obtained with any positive definite matrix Q satisfying

$$Q > 2\lambda_{max}[\ K_2K_2' + h^2K_3(\tau)K_3(\tau)'\ ]\cdot I, 0\leq \tau \leq h \tag{2.7}$$

proof: Let

$$V(x_t) = hx'(t)\bar{P}x(t) + h\beta\int_{t-h}^{t} x(s)'x(s)ds + \beta\int_0^h\int_{t-\tau}^{t} x(s)'x(s)dsd\tau \tag{2.8}$$

be a Lyapunov functional, where $x_t \in C([-h,0],R^n)$ is defined by $x_t(s) = x(t+s)$, $-h \le s \le 0$ and β is a positive scalar. P is positive definite, since {A,B} is controllable and Q positive definite.

Taking the derivative of (2.8) along the solution (2.6) yields

$$\dot{V}(x_t) = \int_0^h \begin{bmatrix} x(t) \\ x(t-\tau) \\ x(t-h) \end{bmatrix}' \begin{bmatrix} M \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\tau) \\ x(t-h) \end{bmatrix} d\tau$$

where

$$M = \begin{bmatrix} (A-BK_1)'P+P(A-BK_1)+2\beta I & hPBK_3(\tau) & PBK_2 \\ hK_3(\tau)'B'\bar{P} & -\beta I & 0 \\ K_2'B'\bar{P} & 0 & -\beta I \end{bmatrix}$$

Combining the above equation with (2.4) yields

$$M = -\begin{bmatrix} \bar{P}BB'\bar{P} + Q - 2\beta I & -h\bar{P}BK_3(\tau) & -\bar{P}BK_2 \\ -hK_3(\tau)'B'\bar{P} & \beta I & 0 \\ -K_2'B'\bar{P} & 0 & \beta I \end{bmatrix}$$

Conditions for sign definiteness of partitioned matrices [6] show that the matrix [M] is negative definite if

$$Q-2\beta I+\bar{P}BB'\bar{P}-\beta^{-1}\bar{P}BK_1K_1'B'\bar{P}-\beta^{-1}h_2\bar{P}BK_3(\tau)K_3(\tau)'B\bar{P} > 0$$

Now, by the property of P >0 (because of Q >0), this may be transformed to

$$PB[\beta I-K_2K_2'-h_2K_3(\tau)K_3(\tau)']B'P+\beta(Q-2\beta I) > 0 \qquad (2.10)$$

On the other hand, assume a positive scalar β that satisfies

$$\beta > \lambda_{max}[\,K_2K_2'+h^2K_3(\tau)K_3(\tau)'\,],\ 0\le\tau\le h \qquad (2.11)$$

Take Q satisfies $Q > 2\beta I$, then

$$Q > 2\lambda_{max}[\,K_2K_2'+h^2K_3(\tau)K_3(\tau)\,]\cdot I \quad ,0\le\tau\le h$$

By using (2.11),(2.12), inequality (2.10) is always satisfied. This implies that the system (2.1) with the control law (2.5) is asymptotically stable. This completes the proof.

This result indicates that the system (2.1) with the delayed state feedback control law (2.3) is always asymptotically stable under the condition that controller gains K_1,K_2 and $K_3(\tau)$ satisfy (2.7).

Remark: If only a single delayed state in feedback control loop is used, the control law (2.5) can be written as

$$u(t) = -K_1x(t) - K_2x(t-h) \qquad (2.13)$$

where $K_1 = -B'P$. This control law (2.13) should be easier to implement than (2.5). In this case, sufficient condition for stabilization of the system (2.1) is

$$Q > \lambda_{max}[\,K_2K_2'\,]\cdot I \qquad (2.14)$$

Thus the controller gain K_2 can be chosen easily under this condition (2.14).

From now, we will consider only a single delayed state feedback controller in the following analysis, since the effects of the additional delay may be similar whether distributed delays are added to or a single delay is added to the feedback loop.

3. Robustness against parameter variation

In this section, for the case when there is parameter variations, the conventional LQR(Linear Quadratic Regulator) and the delayed state feedback control system are compared with.

The LQR was considered to be robust against parameter variations because of its guaranteed gain and phase margins. But in spite of its remarkable stability margins, the LQR may have poor stability margins, which have been shown in Soroka and Shaked[7], where the LQR with the cheap optimal control may acquire far off unstable modes due to small plant parameter variations.

With model (2.1), we consider the modeling error $\triangle A$,$\triangle B$. Then the real plant is represented by

$$\dot{x}(t) = (A + \triangle A)\,x(t) + (B + \triangle B)\,u(t) \qquad (3.1)$$

The guaranteed bound of allowable modeling errors in the LQR is as following. These results are represented in Patel, Toda and Sridhar[8]. In the LQR, the resulting closed loop system is asymptotically stable if the following inequalities holds

$$\bar{\sigma}(\triangle A) < 1 / \bar{\sigma}(P_L) = M1 \ (\triangle B=0) \qquad (3.2)$$

$$\bar{\sigma}(\triangle B) < 1 /\{\bar{\sigma}(P_L)\bar{\sigma}(K_1)\} = M2 \ (\triangle A=0) \qquad (3.3)$$

where $\bar{\sigma}(\cdot)$ and $\underline{\sigma}(\cdot)$ denote the maximum and the minimum singular values and P_L is the solution of a Lyapunov equation,

$$(A-BK_1)'P_L + P_L(A-BK_1) = -2I_n \qquad (3.4)$$

Without matching conditions, as K_1 approaches infinity(,which implies $Q\to\infty$), the bound of $\bar{\sigma}(\triangle A)$ tends to finite but does not grow infinite. On the other hand, in this case, the bound of $\bar{\sigma}(\triangle B)$ tends to zero since P_L does not tends to zero. Thus the closed loop system in LQR may be unstable due to an arbitrary small modeling error in B ($\triangle B$).

Now in this case, consider the delayed state feedback control system. The following theorem represents the guaranteed bounds due to modeling errors in B when the delayed state feedback control law (2.13) is applied to the real plant (3.1).

Theorem 3.1 : Assume that $\triangle A=0$ and control (2.13) is applied to (3.1). Then the resulting closed loop system is asymptotically stable if the following inequalitie hold.

$$\bar{\sigma}(\triangle B) < M3 \qquad (3.5)$$

where

$$M3 = \frac{-a(b+c) + (a-2c^2)\,D^{1/2}}{a(a-c^2)} \qquad (3.6)$$

$$D = a(1-d) + b^2+2bc + c^2d \qquad (3.7)$$

$$a = \bar{\sigma}^2(P_L)\cdot\bar{\sigma}^2(K_2) \qquad (3.8)$$

$$b = \bar{\sigma}(P_L)\cdot\bar{\sigma}^2(K_2)\cdot\bar{\sigma}(P_LB) \qquad (3.9)$$

$$c = \bar{\sigma}(P_L)\cdot\bar{\sigma}(K_1) \qquad (3.10)$$

$$d = \bar{\sigma}^2(P_LB)\cdot\bar{\sigma}^2(K_2) \qquad (3.11)$$

$$\bar{\sigma}(K_1) \ne \bar{\sigma}(K_2) \ne 0 \qquad (3.12)$$

proof : From (2.13) and (3.1), the resulting closed loop system can be written as

$$\dot{x}(t) = (A-BK_1)x(t)-\triangle BK_1x(t)+BK_1x(t-h)+\triangle BK_2x(t-h) \qquad (3.13)$$

We choose as a Lyapunov function

$$V(x_t) = x(t)'P_Lx(t)+\beta\int_{t-h}^{t} x(s)'x(s)\,ds \qquad (3.14)$$

Its derivative along the solution of (3.13) is

$$\dot{V}(x_t) = -\begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}' \begin{bmatrix} N \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix} \tag{3.15}$$

where, [N]=

$$\begin{bmatrix} (2-\beta)I+(\triangle BK_1)'P_L+P_L(\triangle BK_1)' & , & -P_L(B+\triangle B)K_2 \\ -K_2'(B+\triangle B)'P_L & , & \beta I \end{bmatrix}$$

Block matrix [N] is negative definite[13] if

$$\begin{aligned} &2\beta I + \beta[\ K_1'\triangle B'P_L + P_L\triangle BK_1\ ] \\ &> \beta^2 I + P_LBK_2K_2'B'P_L + P_L\triangle BK_2K_2'\triangle B'P_L \\ &\quad + P_L\triangle BK_2K_2'B'P_L + P_LBK_2K_2'\triangle B'P_L \end{aligned} \tag{3.16}$$

Taking the singular value to (3.16), we get

$$\begin{aligned} &\bar{\sigma}^2(\triangle B)\cdot[\ \bar{\sigma}^2(P_L)\bar{\sigma}^2(K_2)\ ] \\ &+2\bar{\sigma}(\triangle B)\cdot[\ \bar{\sigma}(P_L)\bar{\sigma}^2(K_2)\bar{\sigma}(P_LB) + \beta\bar{\sigma}(P_L)\bar{\sigma}(K_1)\ ] \\ &+[\ \beta^2 - 2\beta + \bar{\sigma}^2(P_LB)\bar{\sigma}^2(K_2)\ ] < 0 \end{aligned} \tag{3.17}$$

This implies

$$\bar{\sigma}(\triangle B) < g(\beta) \tag{3.18}$$

where

$$g(\beta) = \frac{-b-\beta c+[\ (b+\beta c)^2 - a(\beta^2-2\beta+d)\ ]^{1/2}}{a} \tag{3.19}$$

In order to maximize the guaranteed bound $g(\beta)$ of $\triangle B$, we choose β^* such that $g(\beta^*)=0$, where get $g(\beta^*)$, that is, the maximum guaranteed bound of $\triangle B$, as follows.

$$g(\beta^*) = M3 = \frac{-a(b+c) + (a-2c^2)\ D^{1/2}}{a(a-c^2)}$$

where

$$D = a(1-d) + b^2 + 2bc + c^2 d$$

$$a = \bar{\sigma}^2(P_L)\cdot\bar{\sigma}^2(K_2)$$

$$b = \bar{\sigma}(P_L)\cdot\bar{\sigma}^2(K_2)\cdot\bar{\sigma}(P_LB)$$

$$c = \bar{\sigma}(P_L)\cdot\bar{\sigma}(K_1)$$

$$d = \bar{\sigma}^2(P_LB)\cdot\bar{\sigma}^2(K_2)$$

$$\bar{\sigma}(K_1) \neq \bar{\sigma}(K_2) \neq 0$$

This completes the proof.

By comparing M2 of (3.3) with M3, we can see that M2 is less than M3. Moreover, without matching condition, the bound of $\bar{\sigma}(\triangle B)$ in LQR tends to zero as $Q\rightarrow\infty$ (cheap control), but, on the other hand, the bound of $\bar{\sigma}(\triangle B)$ in the delayed state feedback control system converses to a finite value for infinite Q.

From the above fact, we can conclude that, if delayed states are added to the feedback loop of the LQR, the resulting closed loop system may be robust than the conventional LQR against the modeling error ($\triangle B$).

4. Transient behavior and disturbance rejection

In this section, we consider the improvement in the transient behavior of the system and the effect of the disturbance rejection with delayed state feedback control law.

It is well-known that by state feedback controller pole locations can be moved to any region in left half plane but zero locations can not be affected. However the proposed delayed state feedback conroller can affect zero locations as well as pole locations in the closed loop transfer function, since the existence of delays included in the feedback loop generates infinite number of poles and zeros[9].

The location of these zeros which are added to open loop zeros can have a drastic effect on the shape of the transients [10], and therefore the appropriate use of the delayed state may produce improvements in the transient response of the system. These new transmission zeros, the location of which can be controlled to selecting h, can be approximated by lower order Padé approximation[9]. For example, to approximate the delay operator exp(-hs), let us use 2nd order Pade approximation, as follows,

$$\exp(-hs) = \frac{1 - h/2\cdot s + h^2/12\cdot s^2}{1 + h/2\cdot s + h^2/12\cdot s^2} = \frac{Q(s)}{P(s)} \tag{4.1}$$

Using this approximation, the resulting closed loop system transfer function, where control law is used (2.13), is given by

$$\begin{aligned} H_c(s) &= C[\ sI-A+BK_1+BK_2\exp(-hs)\ ]^{-1}B \\ &= P(s)C[\ sI-A+BK_1+BK_2Q(s)]^{-1}B \end{aligned} \tag{4.2}$$

Thus

$$\begin{aligned} \det[H_c(s)] &= \frac{P(s)\det[\ C(sI-A)^{-1}B\ ]\det(sI-A)}{\det[\ (sI-A+BK_1)P(s) + BK_2Q(s)\ ]} \\ &= \frac{\psi(s)}{\phi(s)} \end{aligned} \tag{4.3}$$

From denominator $\psi(s)$, the new transmission zeros added to open loop zero are given by the roots of P(s)=0, that is,

$$s = -\frac{3}{h} \pm \frac{3^{1/2}}{h} \quad ,h\neq 0 \tag{4.4}$$

Since the additional zeros in (4.4) is minimum phase zero, the resulting closed loop has a fast settling time property[10]. Therefore we can conclude that the system employing the delayed state feedback control law may be better than the conventional LQR from the view point of the transient response behavior.

The above approach will be extended to the delayed state feedback control system used distributed delays in the same manner of a single delay. In this case, additional zeros added to open loop zeros are as follows

$$\lim_{\delta\rightarrow 0}\ -\frac{3}{\delta} \pm \frac{3^{1/2}}{\delta}\ ,\ \cdots\cdots\ ,\ -\frac{3}{h} \pm \frac{3^{1/2}}{h} \tag{4.5}$$

Now consider the effect of the disturbance rejection with delayed state feedback controller. For the case when there is a sustained input disturbance, the conventional LQR cannot attain and maintain the desired equilibrium conditions. It can be done by the inclusion of a feedback path containing a single integration to reject a constant disturbance, where the form of the control law is as follows[15]

$$u(t) = -F_1x(t) - F_2\int_{t0}^{t} x(\tau)d\tau + f(x(t0)) \tag{4.6}$$

On the other hand, the proposed controller (2.5) may be expressed as follows

$$u(t)= -K_1 x(t) - K_3 \int_{t-h}^{t} x(\tau)d\tau + K_2 x(t-h) \qquad (4.7)$$

If h tends to a sufficient large value, so that approaches to initial time (t_0), one can find the similarity (4.6) to (4.7), which implies that by appropriatly selected parameters ($h, K_1, K_2, K_3(\cdot)$) the property of the disturbance rejection may be achived.

Consequently, for the case when controller design parameters are chosen in a suitable manner, the asymptotic stability of the closed loop system is always guaranteed and additional delayed states in feedback loop are to increase the robustness against parameter variation($\triangle B$), improve the transient response of the system and considerably accommodate input disturbances. The performance of the delayed state feedback control system compared with the conventional LQR is shown in Fig.1

As a simple example, consider the problem of stabilizing a scalar system

$$x(t) = 2\ x(t) + 1.5\ x(t) \qquad (4.8)$$

by the delayed state feedback control

$$u(t) = -\ 1.5\ P\ x(t) - 1.2\ x(t-h) \qquad (4.9)$$

where Q is chosen so that the condition (2.14) is satisfied, as Q=3. For differents values of h, the response of x(t) is represented Fig 1 (a),(b).

5. Conclusion

It was known that intentional time delays included in the feedback path may improve the transient response of the system but ,due to time delays, the overall system may be unstable.

In this paper, by the Lyapunov functional approach, the delay state feedback controller is proposed ,which always guarantes the stability of the overall system. Several characteristics, such as the robustness, the transient behavior and disturbance rejection, are investigated.

By the use of delayed states the better performance can be achived in comparison with the conventional memoryless state feedback controllers when controller gains are determined according to the proposed method in this paper.
These result can be extended to the state observation problem, where the delayed observation may be used for the better performance of the state observation.

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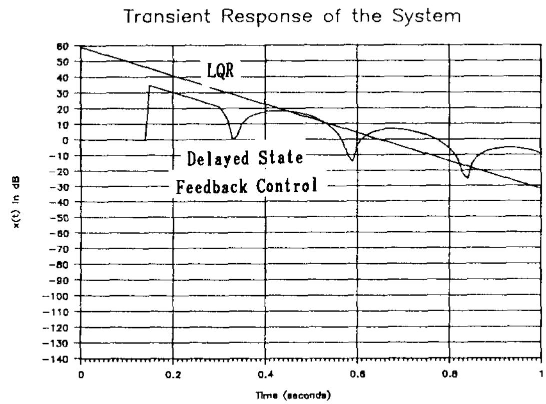

Fig 1 (a). h = 0.15 second

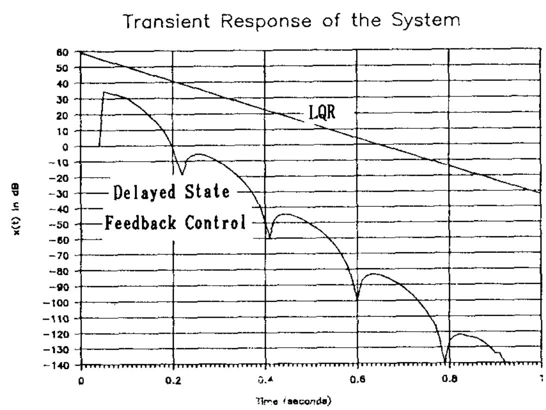

Fig 1 (b). h = 0.05 second