

## DYNAMIC CHARACTERISTICS OF AN IDEALLY DESIGNED ROBOT

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**ABSTRACT:** A conventional robotic manipulator is usually a very complicated system whose dynamics is too computationally time consuming for dynamic analysis and real time control. The authors have proposed the general design criteria of the robot links which greatly simplify the robot dynamic characteristics. In this paper these design guidelines are applied to a 6 degree of freedom PUMA 560 robot in order to realize actual implementation of the design concept. Based upon the design concept, the dynamic equations of the redesigned robot were derived. Dynamic characteristics of two systems, the ideally designed and conventional robot, are compared with respect to the joint input torque characteristics and degree of the coupling between the robot joints.

## 1. INTRODUCTION

A conventional robot arm is usually a very complicated system whose dynamics is too computationally time consuming for dynamic analysis and real time control. One approach towards rendering the dynamic equations computationally feasible in real time is the simplification of the equations based on the significance analysis of the terms. Bejczy[1] simplified the dynamic equations of a robot by geometric and numerical analysis. Armstrong et al.[2] obtained a simplified model of the PUMA 560 robot with a derivation procedure comprised of several heuristic rules for simplification. The simplified model, abbreviated from the full explicit model with a 1 % significance criterion, can be evaluated with 305 calculations, one fifth the number required by the recursive Newton-Euler method. However, this approach does not change the nonlinear characteristics itself and there is a problem of modeling error.

Another approach is the simplification of the dynamic equations through proper design of the robot links. Yang and Tzeng [3] simplified dynamic equations of robots by inertia distribution design, and suggested ten design conditions for a six degrees of freedom robot under which the dynamic equations become simplified. The design conditions cancel some coefficients of the regrouped total energy equation of the robot. Yousef-Toumi and Asada [4] derived necessary conditions for the kinematic structure and mass distribution of a robot to possess the decoupled and/or configuration-invariant inertia. Chung and Cho [5-10] proposed the balancing concept in which a counter-balancing mass installed in each link achieves static balancing in such a way that the mass center of each link always remains exactly at each joint. They showed that the dynamic characteristics can be greatly simplified due to this balancing. The authors[11] have

proposed the general design criteria to use in designing an ideal robot whose dynamics is very much simplified than that of the conventional one. This approach is expected to enhance dynamic characteristics of robot, in addition to simplifying the complex robot dynamics.

In this paper the dynamic characteristics of an ideally designed robot is investigated. A 6 degree of freedom PUMA 560 robot is used as a robot model. Dynamic characteristics of two systems, the ideally designed and conventional robot, are compared with respect to the joint input torque characteristics and degree of the coupling between the robot joints.

## 2. REDESIGN OF THE PUMA 560 ROBOT

A schematic diagram of the PUMA 560 robot is shown in Fig.1. Link parameters are the mass( $m$ ), mass center location( $r_x, r_y, r_z$ ), and moments of inertia of each link( $I_{xx}, I_{yy}, I_{zz}$ ), which are summarized in Table 1. The data were measured by Armstrong et al [2]. The link parameters of the conventional PUMA 560 robot do not coincide with the ideal conditions, which results in very complex dynamics. We will reduce the dynamic complexity by redesigning the links according to the ideal robot design conditions [11]. The design procedure is summarized here in the followings.

The design procedure consists of the conditions about the mass balancing and inertia distribution. Mass balancing condition with respect to the link  $j$  can be achieved by locating mass center position of the link  $j$  such that the mass center of the links  $j$  through 6 locates on the rotation axis of the joint  $j$ . Due to the structural characteristics of the PUMA 560 robot, we need not to redesign all the links. The rotation axis of the joint 1 is parallel to the gravitational acceleration vector and thus the position of the joint 1 does not effect the dynamic coefficients of the PUMA robot [12]. This implies that the link 1 need not to be redesigned. Since the rotation axes of the links 4 and 6 are parallel to the axial direction of the corresponding link, the links 4 and 6 satisfy the balancing condition. Consequently, only the links 2, 3, and 5 should be redesigned. The link 5 is redesigned such that the mass center of the links 5 and 6 locates on the rotation axis of the joint 5. In a similar manner the mass center locations of the links 3 and 2 are determined. The redesigned mass center locations of the links are shown in Table 2. In actual application, to achieve the balancing condition, the magnitude of the link mass may be changed. However, in this paper we assume that the balancing condition can be met with the change of the mass center location only.

After the links are designed according to the mass

balancing condition, the inertia distribution condition should be applied to the links such that each link has three identical moments of inertia with respect to the principal axes of the link. In this case the link 1 is also excluded for the same reason as in the mass balancing condition. The links 2 and 3 are excluded, too, because the application of the above inertia distribution condition is not suitable for the links 2 and 3. Therefore, we should redesign only the links 4, 5, and 6. It is also assumed that these can be achieved without change of the other link parameters. The resultant moments of inertia of the links are shown in Table 2.

When the robot links are redesigned as shown in Table 2, we get the following very simplified dynamic equations:

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} D_{11} & 0 & 0 & D_{14} & D_{15} & D_{16} \\ 0 & D_{22} & D_{23} & 0 & D_{25} & D_{26} \\ 0 & D_{23} & D_{23} & 0 & D_{25} & D_{26} \\ D_{14} & 0 & 0 & D_{44} & 0 & D_{46} \\ D_{15} & D_{25} & D_{25} & 0 & D_{55} & 0 \\ D_{16} & D_{26} & D_{26} & D_{46} & 0 & D_{66} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \\ \ddot{q}_4 \\ \ddot{q}_5 \\ \ddot{q}_6 \end{bmatrix} +$$

$$\begin{bmatrix} H_{122}\dot{q}_1\dot{q}_2 + H_{123}\dot{q}_1\dot{q}_3 + H_{124}(\dot{q}_2 + \dot{q}_3)\dot{q}_4 + H_{125}(\dot{q}_2 + \dot{q}_3)\dot{q}_5 + H_{126}(\dot{q}_2 + \dot{q}_3)\dot{q}_6 \\ + H_{145}\dot{q}_4\dot{q}_5 + H_{146}\dot{q}_4\dot{q}_6 + H_{156}\dot{q}_5\dot{q}_6 \\ - H_{124}\dot{q}_1\dot{q}_4 - H_{125}\dot{q}_1\dot{q}_5 - H_{126}\dot{q}_1\dot{q}_6 + H_{245}\dot{q}_4\dot{q}_5 + H_{246}\dot{q}_4\dot{q}_6 + H_{256}\dot{q}_5\dot{q}_6 \\ - H_{124}\dot{q}_1\dot{q}_4 - H_{125}\dot{q}_1\dot{q}_5 - H_{126}\dot{q}_1\dot{q}_6 + H_{245}\dot{q}_4\dot{q}_5 + H_{246}\dot{q}_4\dot{q}_6 + H_{256}\dot{q}_5\dot{q}_6 \\ H_{124}\dot{q}_1(\dot{q}_2 + \dot{q}_3) - H_{145}\dot{q}_4\dot{q}_5 - H_{146}\dot{q}_4\dot{q}_6 - H_{245}(\dot{q}_2 + \dot{q}_3)\dot{q}_5 - H_{246}(\dot{q}_2 + \dot{q}_3)\dot{q}_6 - H_{456}\dot{q}_5\dot{q}_6 \\ H_{125}\dot{q}_1(\dot{q}_2 + \dot{q}_3) + H_{145}\dot{q}_4\dot{q}_5 - H_{156}\dot{q}_4\dot{q}_6 + H_{245}(\dot{q}_2 + \dot{q}_3)\dot{q}_4 - H_{256}(\dot{q}_2 + \dot{q}_3)\dot{q}_6 - H_{456}\dot{q}_4\dot{q}_6 \\ H_{126}\dot{q}_1(\dot{q}_2 + \dot{q}_3) + H_{146}\dot{q}_4\dot{q}_6 + H_{156}\dot{q}_5\dot{q}_6 + H_{246}(\dot{q}_2 + \dot{q}_3)\dot{q}_4 + H_{256}(\dot{q}_2 + \dot{q}_3)\dot{q}_5 + H_{456}\dot{q}_5\dot{q}_6 \end{bmatrix} \quad (1)$$

where

$$\begin{aligned} D_{11} &= 0.8521 + 1.914 \cdot C_2^2 - 0.2447 \cdot C_{23}^2 \\ D_{14} &= 0.0023 \cdot C_{23} \\ D_{15} &= 0.0005 \cdot S_4 S_{23} \\ D_{16} &= 0.0001 \cdot (C_5 C_{23} - S_5 C_4 S_{23}) \\ D_{22} &= 2.4511 \\ D_{23} &= 0.3921 \\ D_{25} &= 0.0005 \cdot C_4 \\ D_{26} &= 0.0001 \cdot S_5 S_4 \\ D_{44} &= 0.0023 \\ D_{46} &= 0.0001 \cdot C_5 \\ D_{55} &= 0.0005 \\ D_{66} &= 0.0001 \\ H_{112} &= -3.828 \cdot S_2 C_2 - 0.04 \cdot S_{23} C_{23} \\ H_{113} &= -0.4894 \cdot S_{23} C_{23} \\ H_{124} &= -0.0023 \cdot S_{23} \\ H_{126} &= -0.0001 \cdot (C_5 S_{23} + S_5 C_4 C_{23}) \\ H_{145} &= 0.0005 \cdot C_4 S_{23} \\ H_{146} &= 0.0001 \cdot S_5 S_4 S_{23} \\ H_{156} &= -0.0001 \cdot (S_5 C_{23} + C_5 C_4 S_{23}) \\ H_{245} &= -0.0005 \cdot S_4 \\ H_{246} &= 0.0001 \cdot S_5 C_4 \\ H_{256} &= 0.0001 \cdot C_5 S_4 \\ H_{456} &= -0.0001 \cdot S_5 \end{aligned}$$

In the above  $S_i = \sin(q_i)$ ,  $C_i = \cos(q_i)$ ,  $S_{ij} = \sin(q_i + q_j)$ , and  $C_{ij} = \cos(q_i + q_j)$ . This is a simple but exact dynamic equation of a robot designed according to the ideal robot design guidelines. It is greatly simplified equation in comparison with that of the conventional PUMA robot and thus more suitable for on-line computer control. In addition, it shows some enhanced dynamic characteristics. Examination of the resultant dynamic equations of the ideally designed robot show the following distinct characteristics: (1) Diagonal terms of the inertia matrix become constant except that of the first joint. (2) Centrifugal terms and gravity loading terms are eliminated. (3) Complexity of the off-diagonal terms of the inertia matrix and Coriolis terms becomes to be greatly simplified. Therefore, the ideally designed robot seems to have some desirable dynamic characteristics with respect to the joint input torques and coupling between the joints, which will be examined in the next section.

### 3. EVALUATION OF THE DYNAMIC CHARACTERISTICS

#### 3.1 The Joint Input Torque Characteristics

To see the joint input torque characteristics, simulation studies are performed for both systems, the conventional PUMA robot and the ideally designed one. As representative trajectories three circular trajectories on the X-Y, Y-Z, and Z-X plane along a globe in the work space are adopted as shown in Fig. 2. The center and diameter of the globe are (500mm, 0, 0) and 500mm, respectively. The orientation of the wrist is aligned with the X-axis direction and the velocity is 0.5 m/sec at the end of the wrist. Fig.3 shows resultant joint torques for the Y-Z plane circular trajectory. The joints 1 and 6 show no distinct differences between the two systems, since the parameters of the link 6 and the inertia terms of the links 2 and 3 are not modified. For the conventional PUMA robot there are gravity loadings at the joints 2, 3, and 5. The gravity loadings are dominant torque terms and dependent on the robot configuration. This feature appears in Fig. 3 where the torques of the joints 2, 3, and 5 of the conventional PUMA robot vary according to the robot configuration. However, the ideally designed PUMA robot has no gravity loadings via the mass balancing conditions. Therefore, the joint input torques are not much dependent on the variance of the robot configuration in comparison with the conventional one. In addition, the magnitude of the joint input torques of the ideally designed robot are less than those of the conventional one. This reveals a possibility of energy saving or increasing of the maximum payload through an ideal robot design.

#### 3.2 The Coupling Evaluation

An articulated type robot has greatly coupled dynamics due to its nature of construction, which deteriorates the robot performance. As have been noticed in the previous section, the ideally designed robot seems to have reduced coupling. To evaluate this, let us define a coupling measure as a summation of peak-to-peak joints torques for a given configuration due to sinusoidal motion of one joint when the other joints have no motion. That is,

$$CM_i = \sum_{j=i} \tau_p(i,j) \quad (2)$$

where  $CM_i$  is a coupling measure of a joint  $i$  and  $\tau_p(i,j)$  is a peak-to-peak torque of a joint  $i$  due to a sinusoidal motion of the joint  $j$ . This measure reflects the

magnitude of the coupling torques of a joint due to motion of the other joints. Fig. 4 shows the resultant coupling measure along the trajectory defined in the previous section. In this simulation  $0.05\sin(2\pi t)$  is used as a sinusoidal reference input. The joints 2 and 3 of the ideally designed PUMA robot show small and configuration invariant coupling torques. This results from that dominant inertia coupling of the joints are constant and centrifugal and gravity torques are eliminated in the ideally designed PUMA robot. For the joint 1, the main coupling torque is due to the couplings between the joints 2 and 3, which can be completely decoupled via the mass balancing condition. As a result, the ideally designed PUMA 560 has negligible coupling for the joint 1. For the joints 4 and 6 of an ideally designed PUMA robot also shows small magnitude and variance of the coupling torques. These features are desirable for the robot control and performance enhancement.

#### 4. CONCLUSIONS

The ideal robot design guidelines have been applied to the PUMA 560 robot. The redesigned robot has greatly simplified dynamics and thus it is more suitable for on-line computer control. Examination of the resultant dynamic equations of the ideally designed robot show the following distinct characteristics: (1) Diagonal terms of the inertia matrix become constant except that of the first joint. (2) Centrifugal terms and gravity loading terms are eliminated. (3) Complexity of the off-diagonal terms of the inertia matrix and Coriolis terms becomes to be greatly simplified. Therefore, the ideally designed robot seems to have some desirable dynamic characteristics with respect to the joint input torques and coupling between the joints. Due to eliminated gravity loadings, the redesigned robot becomes to have small magnitude and variance of the joint input torque. This reveals a possibility of energy saving or increasing of the maximum payloads. The redesigned robot also shows small coupling torques between the joints in comparison with the conventional one. This is desirable for the enhancement of the robot performance, although actual evaluation of the performance enhancement of the redesigned robot will be different according to the robot control algorithm.

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Table 1. Link parameters of the conventional PUMA 560 robot.

link	m	r <sub>x</sub>	r <sub>y</sub>	r <sub>z</sub>	I <sub>xx</sub>	I <sub>yy</sub>	I <sub>zz</sub>
link 2	17.4	-.363	.006	-.016	.13	.524	.539
link 3	4.8	.0	.014	.07	.066	.086	.0125
link 4	.82	.0	.019	.0	.0013	.0018	.0018
link 5	.34	.0	.0	.0	.0003	.0004	.0003
link 6	.09	.0	.0	.032	.00015	.00015	.00004

Table 2. Link parameters of the ideally designed PUMA 560 robot.

link	m	r <sub>x</sub>	r <sub>y</sub>	r <sub>z</sub>	I <sub>xx</sub>	I <sub>yy</sub>	I <sub>zz</sub>
link 2	17.4	-.5819	.0	.0	.13	.524	.539
link 3	4.8	.0	.0	-.1095	.066	.086	.0125
link 4	.82	.0	.019	.0	.0018	.0018	.0018
link 5	.34	.0	.0	-.0085	.0003	.0003	.0003
link 6	.09	.0	.0	.032	.0001	.0001	.0001

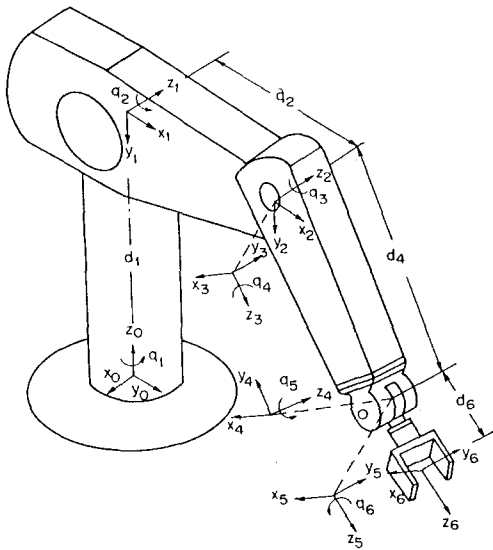


Fig.1 Coordinates system of the PUMA 560 robot.

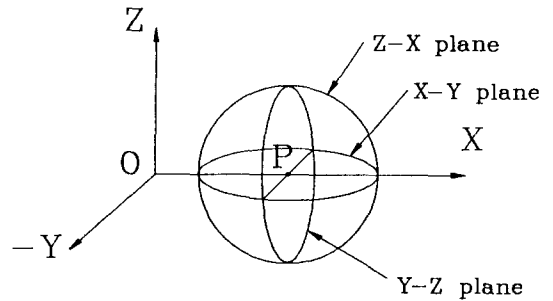


Fig.2 Trajectories used in the simulation.

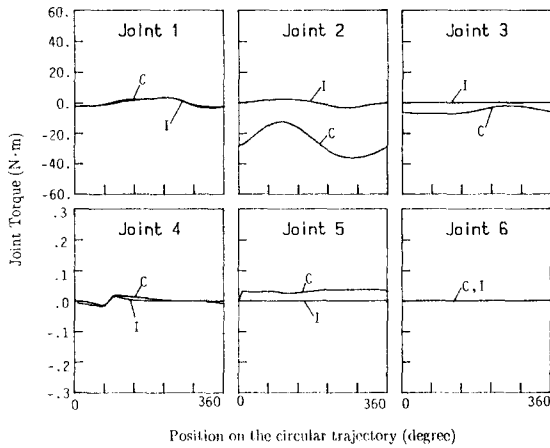


Fig.3 Joint input torques for the circular trajectory on the Y-Z plane.  
 C : Conventional PUMA 560  
 I : Ideally designed PUMA 560

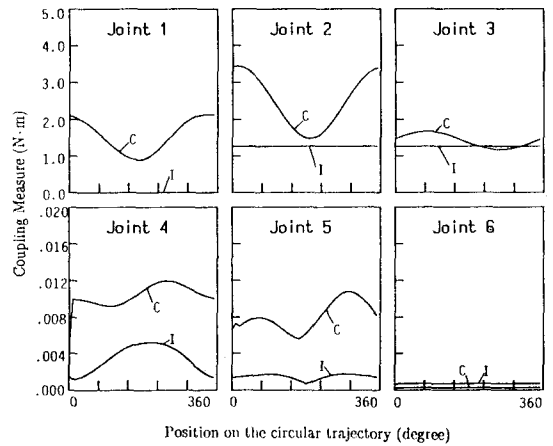


Fig.4 Coupling Measure for the sinusoidal motion of the joints.  
 C : Conventional PUMA 560  
 I : Ideally designed PUMA 560