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Stability of the Robot Compliant Motion Control, Part I : Theory

Sungkwun Kim

Samsung Semiconductor and
Telecommunications Co., Ltd.

Abstract: This two-part paper presents a control method that allows for stable interaction of a robot manipulator with the environment. In part 1, we focus on the input ouput relationships (unstructured modeling) of the robot and environment dynamics. This analysis leads to a general condition for stability of the robot and environment taken as a whole. This stability condition, for stable maneuver, prescribes a finite sensitivity for robot and environment where sensitivity of the robot(or the environment) is defined as a mapping forces into displacement. According to this stability condition, smaller sensitivity either in robot or in environment leads to narrower stability range. In the limit, when both systems have zero sensitivity, stability cannot be guaranteed. These models do not have any particular structure, yet they can model a wide variety of industrial and research robot manipulators and environment dynamic behavior. Although this approach of modeling may not lead to any design procedure, it will allow us to understand the fundamental issues in stability when a robot interacts with an environment.

1. Introduction

Most assembly operations and manufacturing tasks require mechanical interactions with the environment or with the object being manipulated, along with "fast" motion in free and unconstrained space. In constrained maneuvers, the interaction force must be accommodated rather than resisted. Two methods have been suggested for development of compliant motion. The first approach is aimed at controlling force and position in a nonconflicting way [Mason. 1981; Paul, Shimano, 1976; Raibert, Craig, 1981; Whitney, 1977]. In this method, force is commanded along those directions constrained by the environment, while position is commanded along those directions in which the manipulator is unconstrained and free to move. The second approach is focused on developing a relationship between the interaction forces and the manipulator position [Hogan, 1985; Kazerooni, 1986b; 1986c; Salisbury, 1980]. By controlling the manipulator position and specifying its relationship with the interaction forces, a designer can ensure that the manipulator will be able to maneuver in a constrained space while maintaining appropriate contact forces. This chapter describes an analysis on the stability of the robot and environment taken as a whole when the second method is employed to control the robot compliancy. This analysis leads to a general condition for stability of the robot and environment taken as a whole. This stability condition, for stable maneuver, prescribes a finite sensitivity for robot and environment where sensitivity of the robot(or the environment) is defined as a mapping forces into displacement. Using structured model, the stability condition for the direct drive robots has been achieved in terms of the Jacobian and robot tracking controller. This condition has been verified via simulation and experiment on the Minnesota direct drive robot.

In Section 2, the motivation of compliant motion control is addressed. In Section 3, the unstructured model of the robot and environment is discussed. In section 4, a general criterion has been derived using the unstructured models for the robot and environment to guarantee the stability of robot manipulators in constrained maneuvers. In Section 5, the compensator to be designed is discussed and analyzed.

2. Motivation

The following scenario reveals the crucial need for adaptive compliance control(Impedance Control) [Hogan, 1985; Kazerooni, 1986b,c] in manufacturing. Consider an assembly operation by a human worker. There are some parts to be assembled on a table. Each time the worker decides to pick up a part, she/he can contact the table with non-zero speed, that is, the parts can be swept up by the person. The worker also assembles the parts with a non - zero speed; meaning the parts are assembled while still moving. The ability of the human hand to encounter the unknown and unstructured environment with non-zero speed, allows for a higher speed of operation. This ability in human beings indicates the existence of a compliance control mechanism in biological systems. This mechanism guarantees the "stability" of contact forces in constrained maneuvering, in addtion to high speed maneuvering in unconstrained environment. With the existing state of technology there is no integrated sensory robotic assembly system that can encounter an unstructured environment as a human worker can. No existing robotic assembly system is faster than a human hand. The compliancy in the human hand allows the worker to encounter the environment with non-zero speed. The above example does not imply that we choose to imitate human level physiological/psychological behavior as our model to develop an overall control system for manufacturing tasks such as assembly and finishing processes. We stated this example to show 1) A reliable and optimum solution for simple manufacturing tasks such as assembly does not yet exist; 2) the existence of and efficient, fast compliance control system in human beings that allows for superior and faster performance, We bilieve that compliance control is one of the key issues in development of high speed manufacturing operations.

In general, manipulation consists of two categories. In the first category, the manipulator end-point is free to move in all directions. In the second, the manipulatpr end-point interacts mechanically with the environment. Most assembly operations and manufacturing tasks require mechanical interactions with the environment or with the object being manipulated, along with "fast" motion in free and unconstrained space. Therefore the object of the control task on this robot is to develop a control system such that the robot will be capable of "handling" both types of maneuvers without any hardware and software switches.

The design objective is to provide a stabilizing dynamic compensator for the robot manipulator such that the following design specifications are satisfied.

(1) The robot end-point follows an input-command vector, r, when the robot manipulator is free to move.

(2) The contact force, f, is a function of the input command vector, r,when the robot is in contact with the environment.

The first design specification allows for free manipulation when the robot is not constrained. If the robot encounters the environment, then according to the second design specification, the contact force will be a function of the input command vector. Thus, the system will not have a large and uncontrollable contact force. Note that r is an input command vector that is used for both unconstrained and constrained maneuverings. The end-point of the robot will follow r when the robot is unconstrained, while the contact force will be a function of r(preferably a linear function for some bounded frequency range of r) when the robot is constrained.

Note that the above notation does not imply a force control technique [Whitney, 1977]. We are looking for a controller that guarantees the tracking of the input-command vector when the robot is not constrained, as well as the relation of the contact-force vector with the same input-command vector when the robot encounters an unknown environment.

3. Unstructured Model of the Robot and Environment

The general from of the non-linear dynamic equations of a robot manipulator with positioning controller is given by two non-linear vector functions G and S in equation (1).

$$\mathbf{y} = \mathbf{G}(\mathbf{e}) + \mathbf{S}(\mathbf{d}) \qquad (1)$$

where:

d = n x 1 vector of the external force on the robot end-point

e = n x 1 input trajectory vector

G = robot dynamics with positioning controller

S = robot manipulator sensitivity

y = n x 1 vector of the end-point position

e is the n x 1 input trajectory vector that the robot manipulator accepts via its position controller. The fast that most manipulators have some kind of positioning controller is the motivation behind our approach. Also a number of methodologies exist for the development of the robust positioning controllers for direct and non-direct drive robot manipulators. Using any controller design method [Vidyasagar and Spong, 1985], one can always arrive at operator G such that it maps the input command vector, e, to the robot end-point position, y. The motion of the robot in response to imposed forces on the end-point is caused by either structural compliance in the robot or the positioning controller compliance. S represents this compliancy. Note that robot manipulators with positioning controllers are not infinitely stiff in response to external forces (also called disturbances). Even though the positioning controllers of robots are usually designed to follow the trajectory commands and reject the disturbances, the robot end-point moves somewhat in response to imposed forces on the robot end-points. Although d and e affect the robot in a nonlinear fashion, equation (1) assumes that the motion of the robot end-point is a linear addition of both effects. No assumptions on the internal structure of G and S are made.

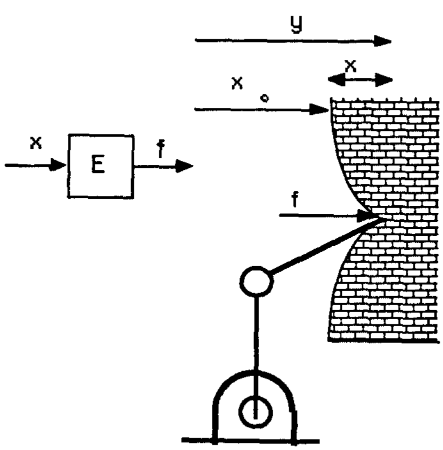

Figure 1 Environment and its dynamics

The environment can be considered from the viewpoint of an unstructured model as shown Figure 1. If one point on the environment is displaced as vector of x, with force vector, f, then the dynamic behavior of the environment is given by equation(2).

$$\mathbf{f} = \mathbf{E(x)} \qquad (2)$$

This equation represents a general mapping from x to f. If one point of the environment is displaced as vector of x, then f is the required force to do such a task. E, represents the environment dynamics, while f and x are n x 1 vector of the contact force and the environment deflection respectively. x_o is the initial location of the point of contact before deformation occurs and y is the robot end point position($x=y-x_o$).

The general block diagram of the robot and environment can be considered as in Figure 2. For the purposes of our analysis, it is convenient to consider E to be an odd function; however, in physical application, f may be zero if x is negative. For example, in the grinding of a surface, therobot can only push the surface. If one considers positive f for "pushing" and negative f for "pulling", in this class of manipulation, then robot manipulator and the environment are in contact with each other only along those directions. In some applications such as turning a bolt,the interaction force can be either positive or negative, meaning that the interaction torque can be clockwise or counter-clockwise. The nonlinear discriminator block diagram in Figure2 is drawn with a dashed-line to represent the above concept - the bolck is present when the interaction forces can only be compressive.

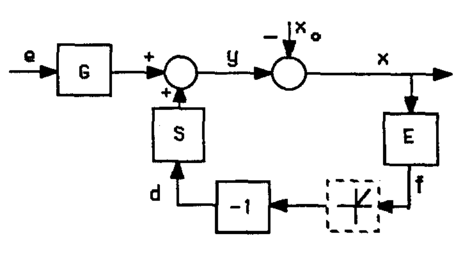

Figure2 The block diagram of the robot and environment

Figure 3 shows the system when compliance compensator, H, is incorporated in the control structure. The input to the compensator, H is the contact force. The output of the compensator is subtracted from the input command vector, r. Note that when the robot is in interaction with the environment, f = -d and $x = y - x_o$. There are two feedback loops in the system. For brevity, we refer to the SE loop as the natural loop because this represent the internal manipulator and environment dynamics and to the HE loop as the compliance loop because it contains the compliance compensator.

If the robot and the environment are not in contact, then the dynamic behavior of the system reduces to y = G(r). When the robot and the environment are in contact, then the value of the contact force and the end-point position of robot are given by f and y where the following equations are true:

$$y = G(e) - S(f) \quad (3)$$

$$f = E(x) \quad (4)$$

$$e = r - H(f) \quad (5)$$

If all the operators of Figure3 are linear transfer function matrices,

$$f = [E^{-1} + S + GH]^{-1} G\ r \quad (6)$$

$$y = [I_n + SE + GHE]^{-1} G\ r \quad (7)$$

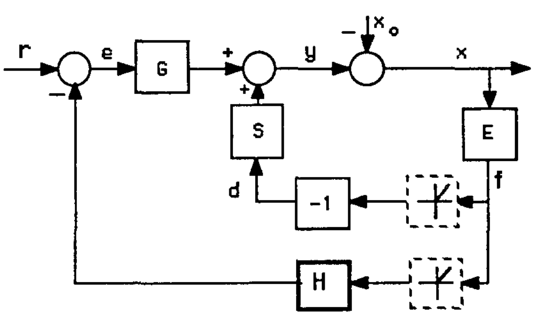

Figure3 Addition of a compliance compensator to the robot and environment dynamics

4. Stability Condition

The objective of this section is to arrive at a sufficient condition for stability of the system in Figure3. This sufficient condition leads to the introduction of a class of compensators, H, that can be used to develop compliancy in the system of Figure3. In the approach that follows, we first simplify the control structure in Figure 3 by substituting a function, V, which is the mapping from the command position, e, to the contact force, f. Next, we use the small Gain Theorem to derive the general stability condition. Then we show the stability condition when H is chosen as a linear operator (transfer function matrix) while V is a nonlinear operator.

The V operator is defined as a mapping from e to f and is a general relationship that encompasses the G,S, and E operators within Figure 3. If G,S, and E represent linear, multivariable transfer functions, and H = 0, the transfer function, V, from equation(6) is given by

$$V = E[I_n + SE]^{-1} G \quad (8)$$

Moreover, the V operator can be obtained from simulation or experiment. The following proposition (using the Small Gain Theorem references [Vidyasagar, 1978; Vidyasagar, Spong, 1985]) states the stability condition of the closed-loop system shown in Figure4.

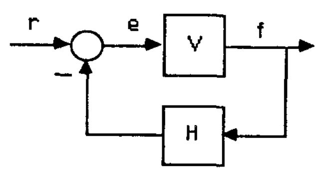

Figure4 Manipulator and the environment with force feedback compensator, H

If conditions I, II and III hold:

I. V is a L_2 - stable operator, that is

a) $V: L^n_2 \rightarrow L^n_2$ (9)

b) $\| V(e) \|_2 \leq \alpha_4 \| e \|_2 + \beta_4$ (10)

II. H is chosen such that mapping H is L_2-stable, that is

a) $H: L^n_2 \rightarrow L^n_2$ (11)

b) $\| H(f) \|_2 \leq \alpha_5 \| f \|_2 + \beta_5$ (12)

III. and $\alpha_4 \alpha_5 < 1$ (13)

then the closed-loop system(Figure 4) is L_2 -stable. The proof is given in reference [Kazerooni, Tsay; 1988].

where $\|\cdot\|$ represents the P-norm of a function(see Appendix A). Substituting for $\|f\|_2$ from inequality (10) into inequality (12) results in inequality(14) (Note that f = V(e))

$$\| H(V(e)) \|_2 \leq \alpha_4\alpha_5 \| e \|_2 + \alpha_5\beta_4 + \beta_5 \quad (14)$$

$\alpha_4\alpha_5$ in inequality (14) represents the gain of the loop mapping, H(V(e)). The third stability condition requires that H be chosen such that the loop mapping, H(V(e)), is linearly bounded with less than a unity slope.

If H is chosen as a linear operator (the impulse response) while the other operator is nonlinear, then:

$$\| HV(e) \|_2 \leq \gamma \| V(e) \|_2 \quad (15)$$

where: $\gamma = \sigma_{max}(N)$ (16)

σ_{max} indicates the maximum singular value[1], and N is a matrix whose ij-th entry is $\sup_\omega(Hij)$. In other words, each member of N is maximum value of (Hij) over all $\omega \in [0,\infty)$. Considering inequality (15), inequality (14) can be rewritten as:

$$\| HV(e) \|_2 \leq \gamma \| V(e) \|_2 \leq \gamma\, \alpha_4 \| e \|_2 + \gamma\beta_4 \quad (17)$$

Comparing inequality (17) with inequality (14), to guarantee the closed loop stability, $\gamma\, \alpha_4$ must be smaller than unity, or, equivalently:

$$\gamma < \frac{1}{\alpha_4} \quad (18)$$

To guarantee the stability of the closed loop system, H in Figure 4 must be chosen such that its "size", as indicated by γ, is smaller than the reciprocal of the "gain" of the forward loop mapping, as indicated by α_4.

[1] The maximum singular value of a matrix A, σ_{max} (A) is defined as:

$$\sigma_{max}(A) = \max \frac{[AZ]}{[Z]}$$

where Z is a non-zero vector and $[\cdot]$ denotes Euclidean norm.

When all the operators of Figure 4 are linear transfer function matrices, inequality (18) can be used as a sufficient condition for stability.

$$\sigma_{max}[H] < \frac{1}{\sigma_{max}[\, E(SE + I_n)^{-1}G\,]} \text{ for all } \omega \in [0,\infty) \quad (19)$$

Inequality(19) can also be derived using the multivariable Nyquist criterion [Kazerooni, 1986a]. Similar to the nonlinear case, H must be chosen such that $\sigma_{max}[H]$ is smaller than the reciprocal of the maximum singular value of the forward loop mapping in Figure 5 to guarantee the stability of the closed loop system.

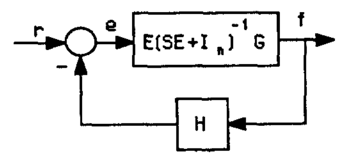

Figure 5 The Simplification of Figure 3 when all the operators are linear transfer function matrices.

5. Compensator Design

The object of this section is to determine the form of the compensating controller, H, so that the deflection of the end effector due to interaction forces between the robot end effector and the environment can be compensated. The stability criteria for chosing the compensator was expressed in equations(18) and (19). For better understanding the nature of the stability of a system, linear case is considered in this section. Since $G \approx I_n$ for all $\omega \in [0, \omega_o]$,(the end point position is approximately equal to the input trajectory vector e), the compensator, H, can be designed by

$$\sigma_{max}[H] < \frac{1}{\sigma_{max}[\, E(SE + I_n)^{-1}]} \text{ for all } \omega \in [0,\omega_o] \quad (20)$$

$$\sigma_{max}[H] < \sigma_{min}[\, E^{-1}(SE + I_n)] \text{ for all } \omega \in [0,\omega_o] \quad (21)$$

In equation(21), an ideal compensator control law can be derived as the inverse of the transfer function ($V = E(SE + I_n)^{-1}$) for the natural feedback loop in Figure 3.

For more understanding about the compensator to be designed, one degree of freedom system(n=1) is considered. Since in many cases $G \approx 1$ for all $\omega \in [0, \omega_o]$, the stability criterion when n=1 is given by inequality(22).

$$|H| < |(S+1/E)| \quad \text{for all } \omega \in [0, \omega_o] \tag{22}$$

where$[\cdot]$ denotes the magnitude of a transfer function. If the environment is very r id in comparison to the stiffness of the robot structure($1/S <<$ $<< E$), the compensator, H, is given by

$$|H| < |S| \quad \text{for all } \omega \in [0, \omega_o] \tag{23}$$

On the other hand, if the environment is very soft in comparison to the stiffness of the robot($1/S$ E , the compensator, H, can be designed by

$$|H| < |1/E| \quad \text{for all } \omega \in [0, \omega_o] \tag{24}$$

In most manufacturing tasks, the end point of the robot manipulator is in contact with a very stiff environment. Robotic deburring and grinding are examples of practical tasks in which the robot is in contact with hard environment [Kazerooni, 1986d]. There is some compliancy in a robot because of the compliancy of its mechanical structural and the tracking controller. Therefore, the compensator, H, can be designed by equation(23).

As an exaple, if the sensitivity, S, is modeled as $[k_s + d_s s]^{-1}$, where the parameters k_s and d_s are the stiffness and damping of the system, the compensator, H, becomes

$$|H| < \left| \frac{\frac{1}{k_s}}{\frac{d_s}{k_s} s + 1} \right| \quad \omega \in [0, \omega_o] \tag{25}$$

* At a low frequency, $H < \frac{1}{k_s}$

which can also be represented as

$$H = \frac{H_o}{\tau_h s + 1} \tag{26}$$

where H_o is compensator gain, and τ_h is time constant. The stiffness and damping of a robot are nonlinear and time variant, but proper values for H_o and τ_h can be chosen by knowing S. The value of H within $\omega \in [0, \omega_o]$ is the designer's choice and, depending on the task, it can have various values in different directions [Kazerooni, 1986a; 1986b]. For a computer controlled robot the Z transform of the compensator, H, for equation (26) is given Appendix B.

It is clear to show that when the environment is very stiff, the compensator, H, should be less than S. Then one must choose a very small H to satisfy the stability of the system when S is "small". (A good positioning system has "small" S). If S approaches to zero, then no H can be obtained to stabilize the system. To stabilize the system of the very rigid environment and the robot, there must be a minimum compliancy in the robot. Direct drive manipulators, because of the elimination of the transmission system, often have large S. This allows for a wider stability range in constrained manipulation.

We conclude that for stability of the environment and the robot taken as a whole, there must be some compliancy either in the robot or in the environment. The initial compliancy in the robot can be obtained by a non-zero sensitivity function or a passive compliant element such as an RCC. Practitioners always observed that the system of a robot and a stiff environment can always be stabilized when a compliant element (e.g. piece of rubber or an RCC) is installed between the robot and environment. One can also stabilize the system of robot and environment by reducing the robot stiffness. In many commmercial manipulators the stiffness of the robot manipulators can be reduced by decreasing the gain of each actuator positioning loop. This also results in a narrower bandwidth (slow response in the unconstrained maneuvering) for the robot positioning system. The stability criterion alsi shows that no compensator can be found to stabilize the interaction of the ideal positioning system (very rigid tracking robot) with an infinitely rigid environment.

6. Summary

A general stability condition was derived using unstructured models for dynamic behavios of robot manipulators and environment. This unified approach of modeling robot and environment dynamics is expressed in terms of sensitivity functions. The control approach allows not only for tracking the input -command vector, but also for compliancy in the constrained maneuverings. A bound for the global stability of the manipulator and environment has been derived. For stability of the environment and the robot taken as a whole, there must be some initial compliancy either in the robot or in the environment. The compensator to be designed was discussed, an ideal compensator control law can be derived as the inverse of the stiffness of the system of a robot and its environment.

Appendix A : Function Norms

A norm is a function whose range is $\mathbb{R}^{+}$ (positive real), which, roughly speaking, indicates the size of some quantity. In the same way in which a norm can be used to measure the "size" of a vector or a matrix, the norm of a function can be defined. Norms are can play an important role in stability theory by providings measure of the size of various quantities. Here some useful definitions are given in this appendix. For more details see Vidyasagar [1978].

Definition 1: For all $p \in [1,\infty)$, we label as L^n_p the set consistinf of all functions $f=(f_1,f_2,\ldots,f_n)^T$: $[0,\infty) \mapsto \mathbb{R}^n$ such that:

$$\int_0^\infty |f_i|_p \, dt < \infty \qquad \text{for } i = 1,2,\ldots,n$$

Definition 2: For all $T \in [0, \infty)$, the function f^T defined by:

$$f^T = \begin{cases} f & 0 \leqslant t \leqslant T \\ 0 & T < t \end{cases}$$

is called the truncation of f to the interval [0,T]

Definition 3: The set of all functions $f=(f_1,f_2,\ldots,f_n)^T$: $[0,\infty) \mapsto \mathbb{R}^n$ such that $f^T \in L^n_p$ for all finite T is denoted by L^n_{pe} by itself may or may not belong to L^n_p.

Definition 4: The norm on L^n_p is defined by:

$$\|f\|_p = \left\{ \sum_{i=1}^{n} \| f_i\|_p{}^2 \right\}^{1/2}$$

where $\|f_i\|_p$ is defined as:

$$\| f_i\|_p = \left\{ \int_0^\infty w_i | f_i |_p \, dt \right\}^{1/p}$$

where w_i is the weighting factor. w_i is particularly useful for scaling forces and torques of different units.

Definition 5: Let $V_2(\cdot): L^n_{pe} \mapsto L^n_{pe}$. We say that operator $V_2(\cdot)$ is L_p- table, if:

a) $V_2(\cdot): L^n_p \mapsto L^n_p$

b) there exist finite real constants α_2 and β_2 such that:

$$\|V_2(e_2)\|_p \leqslant \alpha_2 \|e_2\|_p + \beta_2 \quad \forall\, e_2 \in L^n_p$$

According to this definition we first assume that the operator maps L^n_{pe} to L^n_{pe}. It is clear that if one does not show that $V_2(\cdot): L^n_{pe} \mapsto L^n_{pe}$, the satisfaction of condition (a) is impossible since L^n_{pe} contains L^n_p. Once the mapping of $V_2(\cdot)$ from L^n_{pe} to L^n_{pe} is established, then we say that the operator $V_2(\cdot)$ is L_p -stable if whenver the input belongs to L^n_p the resulting output belongs to L^n_p. Moreover, the norm of the output is not larger than α_2 times the norm of the input plus the offset constant

Definition 6: The smallest α_2 such that there exists a β_2 so that inequality b of Definition 5 is satisfied is called the gain of the operator $V_2(\cdot)$.

Definition 7: Let $V_2(\cdot): L^n_{pe} \mapsto L^n_{pe}$. The operator $V_2(\cdot)$ is said to be causal if:

$$V_2(e_2)_T = V_2(e_{2T}) \quad \forall\, T < \infty \text{ and } \forall\, e_2 \in L^n_{pe}$$

Appendix B

For a computer controlled robot, compensator, H, is designed in the frequency domain, then transferred to the discrete time (z) domain to be realized digitally. Since the computer has a zero order hold output, the Z transform of the compensator, H, for equation (26) is given by

$$H(z) = \frac{H_0 \, z^{-1}(1 - e^{-T/\tau_h})}{1 - z^{-1}e^{-T/\tau_h}}$$

The difference equation of the compensator can be written by

$$\Delta x_n = e^{-T/\tau_h} \, \Delta x_{n-1} + H_0 (1 - e^{-T/\tau_h}) f_{n-1}$$

where n is positive integer, f is the input to the compensator, and Δx is the output of the compensator. T is the sampling time of the controller.

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