A CONTROLLER DESIGN GUARANTEEING PRECISE TRAJECTORY FOLLOWING FOR A ROBOTIC MANIPULATOR

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ABSTRACT

A controller synthesis procedure for precise tracking of reference inputs in the sense of spheres is applied to a 3 d.o.f. robotic manipulator. This methodology applies to a class of nonlinear systems with input uncertainty and parameter uncertainty. The 3 d.o.f. manipulator to be controlled is subjected to varying payloads and is required to follow specified joint trajectories to within prespecified tolerances. The design procedure above lends itself naturally to this type of control problem. The appeal of such a design procedure lies on a special decomposition which exploits linear control theory on the one hand and facilitates a separate treatment of the effects of nonlinearities and the uncertainties on the other.

1. INTRODUCTION

Various approaches have been pursued to develop controller criteria based on state feedback concept that forces the closed-loop poles to be at suitable locations in the complex left-half plane, depending on the design specifications. Some of these approaches are based on Lyapunov stability theory [1], adaptive control [6], classical control concept [4], geometric notions [9], servomechanism theory [2], and functional analysis [8], [11], [12].

However, many of these deal with qualitative aspects of system behavior, not quantitative ones. Here, on the other hand, the primary research objective is to employ a controller synthesis procedure that facilitates the treatment of uncertain dynamical systems in a quantitative framework. These controller design schemes were introduced by Horowitz [4] essentially in the frequency domain. These controller criteria are developed for assuring system performance specified by acceptable range of rise time, overshoot, and settling time in the presence of parameter variations and disturbance inputs.

Recently quantitative controller criteria for precision tracking in nonlinear uncertain systems were developed by Barnard and Jayasuriya [5]. Their approach to the problem is one of nonlinear functional analysis based on Banach space concepts together with equation comparison and fixed point techniques. Information available about the uncertainties of the model is assumed to belong to certain prescribed compact sets.

In this paper the control design procedure is motivated by the requirement that the manipulator joint variables must follow given trajectories within prespecified tolerances. The problem is formulated by augmenting a Luenberger type nonlinear observer [7] to the system state equations to generate a state feedback controller enabling the construction of the joint input torques and forces. Expressing the augmented equations in an operator setting and comparing it with a nominal system (i.e., without uncertainties) form the nucleus of the latter formulation. Utilization of the local form of the Banach contraction mapping fixed point theorem and the above comparison lead to a result which primarily is a separation of the linear design from the effect of nonlinearities and uncertainties in the form of computable operator norms (specifically norms in $L_\infty[0, \infty)$), together with a crucial inequality. This inequality basically states that an operator norm must be forced to satisfy an upper bound by proper tuning of controller and observer gain matrices. The satisfaction of the latter inequality guarantees "precise" trajectory following with prespecified tolerances β_{o1}, β_{o2}, and β_{o3}. The tracking tolerances are expressed in terms of the $L^n_\infty[0, \infty)$ norm, i.e., if $y(t) \in L^n_\infty[0, \infty)$ and $y_o(t) \in L^n_\infty[0, \infty)$ are the actual and nominal output respectively, then the error $\|y - y_o\|$ is given by

$$\|y - y_o\|_{L^n_\infty} = \max_i \, [\text{ess.sup} \, |y_i(t) - y_{oi}(t)|], \quad i = 1, \ldots, n.$$

II. DESIGN CRITERIA

Design criteria for precise tracking requirement apply to systems modeled by state and output equations of the form

$$\dot{x}(t) = Ax(t) + Bu(t) + Dw(t) + f(x(t),\gamma,t)$$
$$y(t) = Cx(t) \tag{1}$$

where $x(t) \in R^n$, $u(t) \in R^m$, $\gamma \in \Gamma \subset R^a$, $t \in T = [0, \infty)$, $w(t) \in R^d$, $f : R^n \times R^a \times T \rightarrow R^n$, $y(t) \in R^b$ and A, B, C and D are constant matrices of appropriate dimensions. A Luenberger type nonlinear state observer of the form

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + g(\hat{x}(t),t) + GC(\hat{x}(t)-x(t)) + V_1v(t)$$
$$u(t) = V_2v(t) + H\hat{x}(t) \tag{2}$$

where $\hat{x}(t) \in R^n$, $u(t) \in R^m$, $t \in T = [0, \infty)$, $v(t) \in R^m$, $g : R^n \times T \rightarrow R^n$ and G, H, V_1 and V_2 are constant matrices of appropriate dimensions, is employed to implement nonlinear state feedback control.

Combining equations (1) and (2), in an operator formulation, yields system equations of the form (see, [5] for details)

$$\dot{z}(t) = LN_\gamma z(t) + LB_o v(t) + LB_1 w(t) + q(t)$$
$$y(t) = C_o z(t) \tag{3}$$

where $z(t) = \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix} \in R^{2n}$, $B_o = \begin{bmatrix} BV_2 \\ BV_2+V_1 \end{bmatrix}$,

$B_1 = \begin{bmatrix} D \\ 0 \end{bmatrix}$,

$$(Lz)(t) = \int_0^t e^{R(t-\tau)} z(\tau)\, d\tau,$$

$C_o = [\ C \quad 0\ ]$,

$(N_\gamma z)(t) = m(z(t),\gamma,t)$, $\quad q(t) = e^{Rt}z(0)$,

and $R = \begin{bmatrix} A & BH \\ -GC & A+BH+GC \end{bmatrix}$.

Next, by considering a corresponding dynamical system which is completely known (i.e. free of uncertain elements), the nominal operator equations can be written as

$$\dot{z}_o = LN_o z_o(t) + LB_o v_o(t) + q(t)$$
$$y_o = C_o z_o \tag{4}$$

where (v_o, y_o, z_o) is a known combination of a specified nominal output y_o, and corresponding solutions z_o, v_o, with v_o serving as a nominal command input relative to y_o, and N_o is the nonlinear map given by

$$(N_o z)(t) = \begin{bmatrix} g(x(t),t) \\ g(\hat{x}(t),t) \end{bmatrix}$$

To compare the actual and nominal systems for any uncertain combination (w, v, z, y) satisfying equation (3) and a known combination (v_o, y_o, z_o) satisfying equations (4), the differences

$$z - z_o = L(N_\gamma z - N_o z_o) + LB_1 w$$
$$y - y_o = C_o(z - z_o)$$

are transformed into the fixed point formulation

$$\tilde{z} = T\tilde{z} = P(I-LK)^{-1}LQ^{-1}Q(N_\gamma P^{-1}\tilde{z} - N_o P^{-1}\tilde{z}_o - KP^{-1}(\tilde{z} - \tilde{z}_o)) + P(I-LK)^{-1}LQ^{-1}QB_1W_i^{-1}\tilde{w} + \tilde{z}_o$$
$$\tilde{y} - \tilde{y}_o = EC_oP^{-1}(\tilde{z} - \tilde{z}_o) \tag{5}$$

where $T : \tilde{L}_\infty^{2n} \rightarrow \tilde{L}_\infty^{2n}$, P, E and Q are arbitrary nonsingular weighting matrices, and $\tilde{z} = Pz$, $\tilde{z}_o = Pz_o$, $\tilde{w} = W_i w$, $\tilde{y} = Ey$, $\tilde{y}_o = Ey_o$. Here the nominal input v_o and the actual input v are assumed to be exactly the same.

Then, based on the fixed-point formulation utilizing the local form of the Banach fixed-point theorem, the following result gives sufficient conditions on command input v_o and design elements G, H, V_1, V_2 and g that insure servotracking in the sense of input-output spheres. The norm considered throughout the paper is the L_∞-norm and is denoted by $\| \cdot \|$.

THEOREM : Let f and g be continuous, and let H, K and G be assigned so that the eigenvalues of the matrices R and R+K are in the left-hand complex plane. Let (v_o, z_o, y_o) be a combination satisfying the equations (4), and let (w, v, z, y) be any combination satisfying equations (3). Then for any disturbances w in the specified sphere

$$S_{\beta_i} = \{ w \in L_\infty^{2d} : \| w \| \leq \beta_i \}$$

there exists a unique response z in the specified β_o neighbourhood

$$S_{\beta_o} = \{ z \in L_\infty^{2n} : \| P(z-z_o) \| \leq \beta_o \}$$

provided
$$\varsigma \leq \frac{\beta_o}{\rho_1\beta_o + \rho_o\beta_i + \rho_2} \tag{6}$$

where $\varsigma = \| P(I-LK)^{-1}LQ^{-1} \|$

$$\rho_o = \| QB_1 \|$$

$$\rho_1 = \sup_{\substack{z,z' \in S_o \\ z \neq z' \\ \gamma \in \Gamma}} \frac{\| Q[N_\gamma z - N_\gamma z' - K(z - z')] \|}{\| P(z - z') \|}$$

$$\rho_2 = \sup_{\gamma \in \Gamma} \| Q(N_\gamma z_o - N_o z_o) \|$$

The inequality (6) will be referred to as the primary design criterion for precision tracking. Some important design features of this criterion are

(i) A Larger upper bound for linear operator L can be allowed by a proper selection of a nonlinear design function g. The norm values of ρ_1, ρ_2 and ς play a vital role in the design procedure. ρ_1 can be regarded as a measure of the severity of nonlinearity and the uncertainty of the system, ρ_2 as a measure of the maximal difference of operator N_γ and operator N_o at z_o, and ς as a measure of the quantitative trackability for the system. A function g should be assigned properly so that the values of ρ_1 and ρ_2 will allow a larger upper bound for the linear operator norm $\| P(I-LK)^{-1}LQ^{-1} \|$.

(ii) A Quantitative pole-placement is defined by (6). That is, a proper selection of eigenvalues of the matrix R will enable one to satisfy the operator norm condition. To achieve an optimum design, the eigenvalues must be placed such that the operator norm is as close as possible to the threshold value, $\beta_o/(\rho_1\beta_o + \rho_o\beta_1 + \rho_2)$.

III. 3 D.O.F. MANIPULATOR

We consider the three degrees-of-freedom manipulator shown in Fig.1. This manipulator has a rotational joint and a translational joint in the (x,y) plane. Moreover the arm can be lifted along the vertical z-axis thus defining the third degree of freedom. The kinetic equations for this robot configuration follow directly from an application of Lagrange's equations and take the following form [3].

$$M(\varphi(t),\gamma)\ddot{\varphi}(t) = -f(\varphi(t), \dot{\varphi}(t), \gamma) + u(t) \qquad (7)$$

where $\varphi(t)=[r(t), \theta(t), z(t)]^T$ specifies the configuration at time t in a cylindrical frame of reference, γ is the payload uncertainty and the dot denotes time derivatives. u(t) represents the generalized forces and is given by

$$u(t) = [F_r, T_\theta, F_z]^T$$

where F_r is the radial force, T_θ is the torque and F_z is the vertical force associated with the coordinates r, θ, and z respectively. $M(\varphi(t),\gamma)$ and $f(\varphi(t),\dot{\varphi}(t),\gamma)$ are given belows.

$$M(\varphi(t),\gamma) = \begin{bmatrix} M_{12} & 0 & 0 \\ 0 & M_{12}r^2(t) - M_1\ell r(t)+k & 0 \\ 0 & 0 & M_{12} \end{bmatrix}$$

$$f(\varphi(t),\dot{\varphi}(t),\gamma) = \begin{bmatrix} -M_{12}r(t)\dot{\theta}^2(t)+\frac{1}{2}M_1\ell\dot{\theta}^2(t) \\ \{2M_{12}r(t)\dot{r}(t)-M_1\ell\dot{r}(t)\}\dot{\theta}(t) \\ 0 \end{bmatrix} \qquad (8)$$

where $M_{12} = M_1 + M_2$, M_1 and M_2 are the arm mass and the payload mass respectively, ℓ is the length of the arm AB. J the net moment of inertia of the arm and the swivel joint is given by

$$J = J_{M_1} + J_{M_3} = \frac{1}{2} M_3 r_z^2 + \frac{1}{3} M_1 \ell^2$$

where, J_{M_1} and J_{M_3} are the moments of inertia of the swivel and the arm respectively, about the z-axis. M_3 and r_z are the mass and the radius of the swivel.

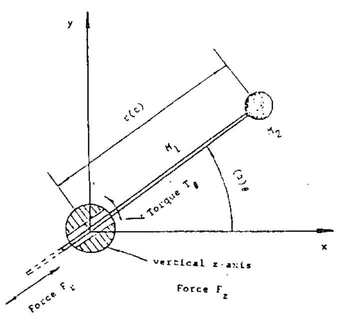

Fig. 1. 3 d.o.f. of robot manipulator

Equations (8) depict a highly coupled nonlinear set of equations. By employing the state dependent transformation

$$u(t) = M(\varphi(t),\gamma)v(t) \qquad (9)$$

on the input u(t), the equations of motion (7) are transformed into

$$\ddot{\varphi}(t) = -M(\varphi(t),\gamma)f(\varphi(t),\dot{\varphi}(t);\gamma)+v(t) \qquad (10)$$

The inertia matrix $M(\varphi(t),\gamma)$ is clearly invertible for all $t \in [0, \infty)$ which follows from the positive definitness of the mass matrix of a manipulator.

Now equations (10) can be rewritten in the usual state space form yielding.

$$\dot{x}(t) = \begin{bmatrix} 0 & I_3 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ I_3 \end{bmatrix} v(t) + \begin{bmatrix} 0 \\ f_N(x(t),\gamma) \end{bmatrix}$$

$$y(t) = [\ I_3 \quad 0\ ]\ x(t) \qquad (11)$$

where, I_3 and 0 are 3×3 identity and null matrices respectively,

$x(t) = \begin{bmatrix} \varphi(t) \\ \dot{\varphi}(t) \end{bmatrix} \in R^6$, and $v(t) \in R^3$, and nonlinear term

$$f_N = -M^{-1}(\varphi(t),\gamma)f(\varphi(t),\dot{\varphi}(t),\gamma).$$

Now equations (11) are decoupled with respect to the linear parts and is made use of in executing the design procedure obtained previously in section II. This form clearly allows the arbitrary placement of eigenvalues of each decoupled subsystem.

Design Objective : Our basic design objective is to synthesize a control u(t) in order to achieve the tracking performance specified by the output constraints

$$\| y_i - y_{oi} \| \le \beta_{oi}, \qquad \beta_{oi} > 0, \quad i = 1,2,3$$

despite the payload uncertainty. y_{oi}, i = 1,2,3 are the 3 nominal outputs to be tracked and y_i are the three actual outputs.

Nominal Output : The nominal outputs to be tracked are

$y_{o1} = 0.8 - 0.8\, e^{-3t}(\cos(t) + 3\sin(t))$ for the radial displacement of the arm,

$y_{o2} = t^2 e^{-t}$ for the angular rotation of the arm,

$y_{o3} = 0.5 - 0.5\cos(t)$ for the vertical motion of the arm.

Output Spheres : Output sphere specifications are

$$\beta_{o1} = \beta_{o2} = \beta_{o3} = 0.1$$

Thus the tracking specifications call for precise tracking of the nominal outputs given above upto an accuracy of 0.1 m in y_1, 0.1 rad in y_2 and 0.1 m in y_3.

Bounded Uncertainty : We consider the payload M_2 to be the primary uncertainty and assume that

$$M_2 \in \Gamma = [0, 20] \text{ kg}.$$

In order to design a controller as outlined in the previous section (see, Fig.2), a threshold value as specified

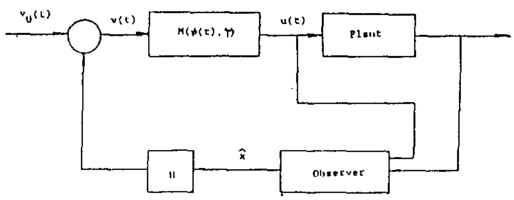

Fig. 2 Controller structure with an observer

in equation (6) needs to be computed first. This requires the computation of several norm quantities as described in the theorem. We use the following data for all computations. M_1 = 40 kg, M_2 = [0, 20] kg, M_3 = 100 kg, ℓ = 1 m, r(t) = [0.0, 1.0] m, z(t) = [0.0, 1.0] m, θ(t) = [0, π] rad, and r_z = 0.1 m.

The weighting matrices P, Q, and K chosen primarily to yield favorable norm values are

$$K = \begin{bmatrix} 0_6 & 0_6 \\ 0_6 & 0_6 \end{bmatrix}, \quad P = Q = \begin{bmatrix} \Sigma & 0_6 \\ 0_6 & \Sigma \end{bmatrix}.$$

where $\Sigma = \begin{bmatrix} I_3 & 0_3 \\ 0_3 & \frac{1}{|\lambda_i|_{max}} I_3 \end{bmatrix}$. and $|\lambda_i|_{max}$ is the maximum absolute value of the eigenvalues of matrix R. I_3 is the 3x3 identity matrix, 0_3 and 0_6 respectively are 3x3 and 6x6 null matrices.

Then, ρ_o = ‖ QB_i ‖ = 0, since B_1 = 0 due to no external disturbance.

Computation of ρ_1 involves the calculation of gradients of the nonlinearity with respect to the state vector x, and is given by

$$\rho_1 = \max \{ \max \| \nabla_x f_{N1}^T \|, \max \| \nabla_x f_{N2}^T \| \}$$
$$= 21.0$$

To calculate $\rho_2 = \sup_{\gamma \in \Gamma} \| Q_o(N_\gamma z_o - N_o z_o) \|$, we need to select a nominal nonlinear function g(x) to cancel as much as possible the uncertain effects of f_N. We choose g(x) to be of the same form as $f_N(x,\gamma)$ with γ replaced by γ_o, where γ_o are the arithmetic means of the uncertain parameters. Thus,

$$\rho_2 = \sup \| Q_o(N_\gamma z_o - N_o z_o) \|$$
$$= \sup \left\| \begin{bmatrix} I_3 & 0_3 \\ 0_3 & \frac{1}{|\lambda_i|_{max}} I_3 \end{bmatrix} \begin{bmatrix} 0_3 \\ f_N - f_o(x) \end{bmatrix} \right\|$$

On substitution of numerical values, it follows that

$$\rho_2 = \frac{6}{|\lambda_i|_{max}}$$

Now assembling all of the above computations we compute threshold given in equation (6)

$$\frac{\rho_o}{\rho_o\beta_o + \rho_o\beta_i + \rho_2}$$
$$= \frac{0.1}{6.0/|\lambda_i|_{max} + (21.0)(0.1)} \qquad (12)$$

Now it only remains to fine a set of eigenvalues for the system matrix

$$R = \begin{bmatrix} A & BH \\ -GC & A+BH+GC \end{bmatrix}$$

so that the norm ‖ $P(I - LK)^{-1}LQ^{-1}$ ‖ is less than the upper bound (12). Based on the numerical scheme previously outlined, we obtain the spectra

$$\sigma(A+BH) = \{ -47.0 \pm j\,49.0, -50.0 \pm j\,53.0, -53.0 \pm j\,51.0 \}$$

$$\sigma(A+GC) = \{ -110.0 \pm j\,111.0, -113.0 \pm j\,114.0, -115.0 \pm j\,113.0 \}$$

yielding

$$H = \begin{bmatrix} -4160 & 0 & 0 & -94 & 0 & 0 \\ 0 & -5309 & 0 & 0 & -100 & 0 \\ 0 & 0 & -5410 & 0 & 0 & -106 \end{bmatrix}$$

and

$$G = \begin{bmatrix} -220 & 0 & 0 & -24421 & 0 & 0 \\ 0 & -226 & 0 & 0 & -25765 & 0 \\ 0 & 0 & -230 & 0 & 0 & -25994 \end{bmatrix}^T$$

In our experience placement of the eigenvalues as shown in Fig. 3 on ± 45° lines yield the smallest value of the operator norm for a given magnitude of eigenvalue.

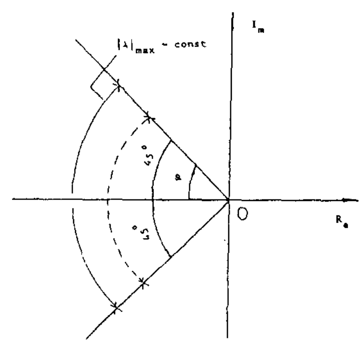

Fig. 3 Placement of eigenvalues

With the above spectra, we obtain the upper bound

$$\frac{\rho_o}{\rho_o\beta_o + \rho_o\beta_i + \rho_2} = 0.045$$

and the critical norm of the operator

$$\| P(I - LK)^{-1}LQ^{-1} \| = 0.041$$

which clearly satisfies inequality (6).

With H known we can now compute the nominal command input functions $r_{oi}(t)$, i = 1, 2, 3. Thus it follows that the design specified by matrices G, H, the nonlinear function g(x) and the nominal inputs r_{oi}, i = 1, 2, 3, guarantee the required tracking performance according to the theorem. The validity of the theorem is also confirmed by simulation results.

Figure (4 - 6) show simulations for M_2 = 20kg. Figure 4 shows the nominal output y_{o1} and the actual output y_1. There is hardly any difference in the two graphs. This clearly demonstrates the tracking accuracy. Fig. 5 shows the errors $e_i = y_i - y_{oi}$, i = 1, 2, 3. These errors are of the order of 10^{-3} which is quite conservative in comparison with the imposed output sphere ρ_o = 0.1. This conservativeness is not surprising due to the generality of the inputs and the nonlinearity admissible in $L_\infty[0, \infty)$. The required control inputs are shown in Fig. 6. Fig. (7) - (9) show simulations for a sinusoidally varying uncertainty $M_2 = 10 + 10 \sin(10t)$. Figure 7 shows the nominal output y_{o2} and the actual output y_2. Figure 8 shows the error $y_2 - y_{o2}$ and the control input u_2 is shown in Figure 9. The latter uncertainty is considered just for the sake of demonstrating that the methodology is valid for any uncertainty in a given band.

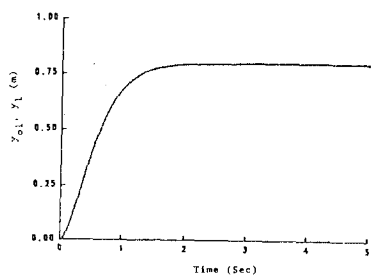

Fig. 4. Nominal output y_{o1} and actual output y_1

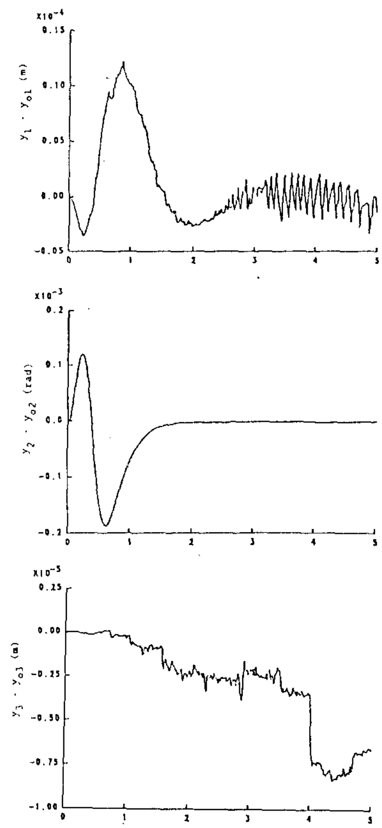

Fig. 5 Error $y_i - y_{oi}$

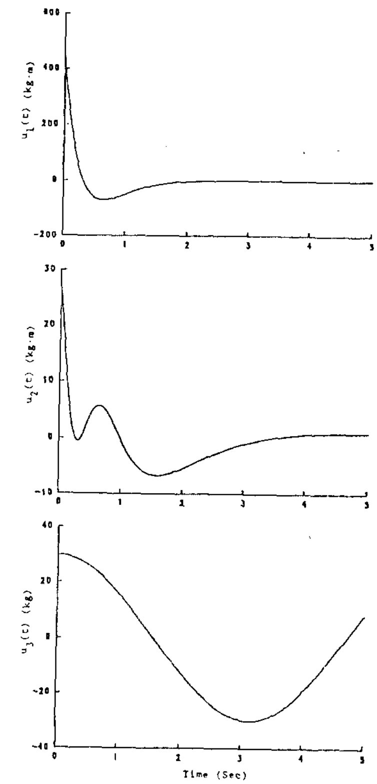

Fig. 6 Control input $u_i(t)$

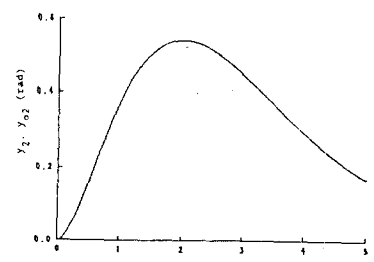

Fig. 7. Nominal output y_{o2} and actual output y_2

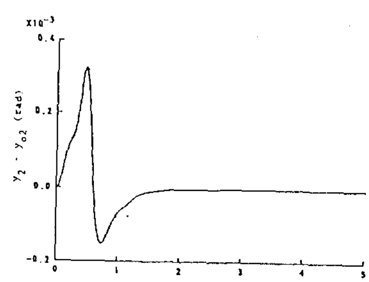

Fig. 8. Error $y_2 - y_{o2}$

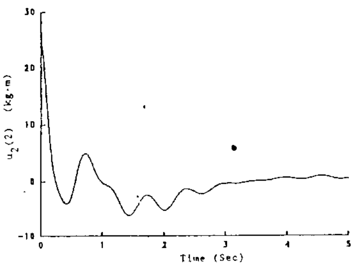

Fig. 9. Control input $u_2(t)$

IV. CONCLUSION

In this paper we considered the design of tracking controller for a 3 d.o.f. manipulator. The full nonlinear dynamic model including uncertainty was treated. The approach to the design was that outlined in Jayasuriya, et. al. [5]. The simulation results clearly demonstrate that the required tracking accuracy is met quite adequately. In fact the design is somewhat conservative. This however is to be expected since any uncertainty whatsoever within the specified bounds is admissible. In particular any L_∞-function with the appropriate sup and inf is admissible. The design procedure outlined is a direct one in the sense that the design specifications are directly satisfied.

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