GLR APPROACH TO FAILURE DIAGNOSIS IN A LINEAR SYSTEM WITH DECENTRALIZED ESTIMATORS

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ABSTRACT : A systematic way of failure diagnosis in a linear system with decentralized estimators is developed. The generalized likelihood ratio (G.L.R) approach to failure detection and identification is used for designing a diagnosis system in each subsystem based on the innovation analysis. For the simplicity of the theoretical formulation, a design scheme of failure diagnosis is developed for the system decomposed into two subsystems. To demonstrate the effectiveness of our approach, several simulation studies have been carried out on a third-order linear system which is constructed of a second-order damped oscillator and a first-order lag.

1. INTRODUCTION

Recently, it has received growing attention to use a decentralized optimization of large-scale or compound systems. Therefore it is important subject in practice of the system maintenance to establish a technique of failure diagnosis for such systems. For the linear systems with decentralized estimators, following failure modes can be considered :

(1) failure in over all system
(2) dynamics failures in subsystems
(3) sensor failures in subsystems
(4) transmission errors in information exchange between the subsystems,

and etc.

It is known that the diagnosis problems for all of these failure modes are not possible to solve. Such solvability depends on structures of the decentralized systems and on statistical properties of information obtained from each subsystem.([1])

In this paper, we intend to develop a systematic way of failure diagnosis in a linear system with decentralized estimators. The state estimation for each subsystem is performed by using standard Kalman filter, in which the estimation results of subsystems are exchanged each other. Suppose that a dynamics failure which is characterized by an abrupt change in the system equation has occurred in a subsystem. Then the failure effect will be reflected in the filtering results of all subsystems due to the interaction between them. For the simplicity of the theoretical formulation, we will consider as the object to be diagnosed a linear system which is decomposed into two subsystems. A GLR approach ([2]) to failure detection and identification (FDI) is used for the design of diagnosis system based on the innovation analysis. The resulting diagnosis system leads to a decentralized decision system to check the operating mode in each subsystem and to recognize a failed subsystem by using the GLR test. The effectiveness of our approach has been confirmed through simulation studies on a third-order system.

2. SYSTEM DESCRIPTION AND FAILURE MODELLING

As the object to be diagnosed, consider a linear system which is decomposed into two subsystems described by the following state space models:

Subsystem 1 :

$$\begin{cases} X_1(k+1) = \Phi_1(k)X_1(k) + \Phi_{12}(k)X_2(k) + w_1(k) \\ Y_1(k) = H_1(k)X_1(k) + v_1(k), \qquad k = 1, 2, \cdots \end{cases} \tag{1}$$

$$w_1(k) \sim \mathcal{N}(0, Q_1(k)), \quad v_1(k) \sim \mathcal{N}(0, R_1(k)) \tag{2}$$

Subsystem 2 :

$$\begin{cases} X_2(k+1) = \Phi_2(k)X_2(k) + \Phi_{21}(k)X_1(k) + w_2(k) \\ Y_2(k) = H_2(k)X_2(k) + v_2(k), \qquad k = 1, 2, \cdots \end{cases} \tag{3}$$

$$w_2(k) \sim \mathcal{N}(0, Q_2(k)), \quad v_2(k) \sim \mathcal{N}(0, R_2(k)) \tag{4}$$

where X(k) is the state vector and Y(k) is the observation vector with appropriate dimension respectively. w(k) and v(k) are the system and the observation noise vectors which are assumed to be independent white gaussian distributed variables. The transition matrices $\Phi_{12}(k)$ and $\Phi_{21}(k)$ denote the interaction between the subsystem 1 and 2. The failure mode is assumed to be modelled by an abrupt additive change in the state of a subsystem at an instant of θ

$$X_i(k) \Rightarrow X_i(k) + \delta_{k,\theta}\ , \quad i \in [1, 2] \tag{5}$$

where $\delta_{k,\theta}$ denotes the delta function, and θ stands for a failure on set time. For the simplicity of the analysis, we will further assume that the simultaneous failure in subsystems never occur.

Our purpose is to construct a failure diagnosis system to check the operating mode in each subsystem and to recognize a failed subsystem as fast as possible when a failure occurred. Such a FDI system leads to a decentralized decision system. We thus consider to apply the GLR test to perform the FDI as an on-line procedure.

3. DECENTRALIZED STATE ESTIMATIONS

Let us introduce following three hypotheses according to the system operating modes:

$$\begin{cases} H^0\ ;\ \textit{Normal situation} & (k \le \theta) \\ H^1(\theta)\ ;\ \textit{Failure occurrence in subsystem 1} & \\ \qquad \textit{at an instant of } \theta & (k - M \le \theta \le k) \\ H^2(\theta)\ ;\ \textit{Failure occurrence in subsystem 2} & \\ \qquad \textit{at an instant of } \theta & (k - M \le \theta \le k) \end{cases}$$

where k denotes the current stage and M is a data window which is properly selected by considering the trade-off between the allowable computational labour and the required decision accuracy.

The state estimation is executed in each subsystem under the hypothesis H^0 by using the standard Kalman filter as the decentralized state estimator. The results in subsystem 1 are given as follows :

3.1 State Estimations in the subsystem 1 :

$$\bar{X}_1(k+1) = \Phi_1(k)\,\hat{X}_1(k) + \Phi_{12}(k)\,\hat{X}_2(k) \tag{6}$$

$$\hat{X}_1(k+1) = \bar{X}_1(k+1) + K_1(k+1)\,\gamma_1(k+1) \tag{7}$$

$$\gamma_1(k+1) = Y_1(k+1) - H_1(k+1)\,\bar{X}_1(k+1) \tag{8}$$

$$K_1(k+1) = \bar{S}_1(k+1)\,H_1(k+1)^T \{H_1(k+1)\,\bar{S}_1(k+1)\,H_1(k+1)^T + R_1(k+1)\}^{-1} \tag{9}$$

$$\bar{S}_1(k+1) = \Phi_1(k)\,\hat{S}_1(k)\,\Phi_1(k)^T + \Phi_{12}(k)\,\hat{S}_2(k)\,\Phi_{12}(k)^T + Q_1(k) \tag{10}$$

$$\hat{S}_1(k+1) = \bar{S}_1(k+1) - K_1(k+1)\,H_1(k+1)\,\bar{S}_1(k+1) \tag{11}$$

$$\gamma_1(k+1) \sim \mathcal{N}(0, H_1(k+1)\,\bar{S}_1(k+1)\,H_1(k+1)^T + R_1(k+1)) \tag{12}$$

where $\hat{X}_1(k)$ and $\bar{X}_1(k+1)$ are the estimate of $X_1(k)$ and the prediction of $X_1(k+1)$ using the observation sequence up to k, $\{ Y_1(1), \cdots, Y_1(k) \}$, respectively and $\hat{S}_1(k)$, $\bar{S}_1(k+1)$ are the corresponding error covariance matrices. In this decentralized estimator, we assume that the estimation results $\hat{X}_2(k)$ and $\hat{S}_2(k)$ are given as the transmitted information from the estimator in subsystem 2.

When the hypothesis $H^0(\theta)$ is true, let us denote all of variables in the decentralized estimator by subscript ^{0}, like $\hat{X}_1(k)^0$, $\bar{X}_1(k+1)^0$, etc. We then have following expressions from eqs(1)-(5) when the hypothesis $H^1(\theta)$ is true :

$$\begin{aligned} X_1(k) &= X_1(k)^0 + A_1(k:\theta)\,\nu_1 \\ Y_1(k) &= Y_1(k)^0 + H_1(k)\,A_1(k:\theta)\,\nu_1 \\ \bar{X}_1(k) &= \bar{X}_1(k)^0 + P_1(k:\theta)\,\nu_1 \\ \hat{X}_1(k) &= \hat{X}_1(k)^0 + F_1(k:\theta)\,\nu_1 \\ \gamma_1(k) &= \gamma_1(k)^0 + G_1(k:\theta)\,\nu_1\,. \end{aligned} \tag{13}$$

The coefficient matrices A, P, F, G are calculated by the recursive way :

$$\begin{aligned} A_1(\ell+1:\theta) &= \Phi_1(\ell)\,A_1(\ell:\theta) + \Phi_{12}(\ell)\,A_{21}(\ell:\theta) \\ A_{21}(\ell+1:\theta) &= \Phi_2(\ell)\,A_{21}(\ell:\theta) + \Phi_{12}(\ell)\,A_{21}(\ell:\theta) \\ P_1(\ell+1:\theta) &= \Phi_1(\ell)\,F_1(\ell:\theta) + \Phi_{12}(\ell)\,F_{21}(\ell:\theta) \\ P_{21}(\ell+1:\theta) &= \Phi_2(\ell)\,F_{21}(\ell:\theta) + \Phi_{21}(\ell)\,F_1(\ell:\theta) \\ F_1(\ell+1:\theta) &= P_1(\ell+1:\theta) + K_1(\ell+1)\,G_1(\ell+1:\theta) \\ F_{21}(\ell+1:\theta) &= P_{21}(\ell+1:\theta) + K_2(\ell+1)\,G_{21}(\ell+1:\theta) \\ G_1(\ell+1:\theta) &= H_1(\ell+1)\,\{A_1(\ell+1:\theta) - P_1(\ell+1:\theta)\} \\ G_{21}(\ell+1:\theta) &= H_2(\ell+1)\,\{A_{21}(\ell+1:\theta) - P_{21}(\ell+1:\theta)\} \end{aligned} \tag{14}$$

with initial values

$$\begin{aligned} &A_1(\theta:\theta) = I,\quad A_{21}(\theta:\theta) = 0 \\ &P_1(\theta:\theta) = 0,\quad P_{21}(\theta:\theta) = 0 \\ &F_1(\theta:\theta) = K_1(\theta)\,H_1(\theta),\quad F_{21}(\theta:\theta) = 0. \end{aligned} \tag{15}$$

When the hypothesis $H^2(\theta)$ is true, we have

$$\begin{aligned} X_1(k) &= X_1(k)^0 + A_{12}(k:\theta)\,\nu_2 \\ Y_1(k) &= Y_1(k)^0 + H_1(k)\,A_{12}(k:\theta)\,\nu_2 \\ \bar{X}_1(k) &= \bar{X}_1(k)^0 + P_{12}(k:\theta)\,\nu_2 \\ \hat{X}_1(k) &= \hat{X}_1(k)^0 + F_{12}(k:\theta)\,\nu_2 \\ \gamma_1(k) &= \gamma_1(k)^0 + G_{12}(k:\theta)\,\nu_2\,. \end{aligned} \tag{16}$$

Then the coefficient matrices A, P, F, G are calculated by the recursive way :

$$A_2(\ell+1:\theta) = \Phi_2(\ell)\,A_2(\ell:\theta) + \Phi_{21}(\ell)\,A_{12}(\ell:\theta)$$
$$A_{12}(\ell+1:\theta) = \Phi_1(\ell)\,A_{12}(\ell:\theta) + \Phi_{12}(\ell)\,A_2(\ell:\theta)$$
$$P_2(\ell+1:\theta) = \Phi_2(\ell)\,F_2(\ell:\theta) + \Phi_{21}(\ell)\,F_{12}(\ell:\theta)$$
$$P_{12}(\ell+1:\theta) = \Phi_1(\ell)\,F_{12}(\ell:\theta) + \Phi_{12}(\ell)\,F_2(\ell:\theta)$$
$$F_2(\ell+1:\theta) = P_2(\ell+1:\theta) + K_2(\ell+1)\,G_2(\ell+1:\theta)$$
$$F_{12}(\ell+1:\theta) = P_{12}(\ell+1:\theta) + K_1(\ell+1)\,G_{12}(\ell+1:\theta)$$
$$G_2(\ell+1:\theta) = H_2(\ell+1)\,\{A_2(\ell+1:\theta) - P_2(\ell+1:\theta)\}$$
$$G_{12}(\ell+1:\theta) = H_1(\ell+1)\,\{A_{12}(\ell+1:\theta) - P_{12}(\ell+1:\theta)\} \quad (17)$$

with initial values

$$A_2(\theta:\theta) = I,\; A_{12}(\theta:\theta) = 0$$
$$P_2(\theta:\theta) = 0,\; P_{12}(\theta:\theta) = 0 \quad (18)$$
$$F_2(\theta:\theta) = K_2(\theta)\,H_2(\theta),\; F_{12}(\theta:\theta) = 0.$$

Applying least squares method to eqs (13) , (16) , we have the estimates of jump quantities ν_1 and ν_2 as follows :

$$\hat{\nu}_1(\theta) = [\sum_{j=\theta}^{k} G_1(j;\theta)^T V_1(j)^{-1} G_1(j;\theta)]^{-1} \times [\sum_{j=\theta}^{k} G_1(j;\theta)^T V_1(j)^{-1} \gamma_1(j;\theta)] \quad (19)$$

$$\hat{\nu}_2(\theta) = [\sum_{j=\theta}^{k} G_{12}(j;\theta)^T V_1(j)^{-1} G_{12}(j;\theta)]^{-1} \times [\sum_{j=\theta}^{k} G_{12}(j;\theta)^T V_1(j)^{-1} \gamma_1(j;\theta)] \quad (20)$$

where

$$V_1(k) = H_1(k)\,\bar{S}_1(k)\,H_1(k)^T + R_1(k).$$

3.2 State Estimations in the subsystem 2 :

The state estimation in the subsystem 2 can be performed in a similar way of the section 3.1.

4. DESIGN OF DECENTRALIZED DIAGNOSIS SYSTEMS WITH GLR TEST

For the design of the FDI system in subsystems, we introduce the generalized likelihood ratio functions defined as follows for the innovation sequence $\{\gamma_1(k)\}$:

$$\ell_1(k:\theta, H^1(\theta)/H^0) \stackrel{\text{def}}{=} 2ln\frac{p(\gamma_1(k-M),\cdots,\gamma_1(k)/H^1(\theta))}{p(\gamma_1(k-M),\cdots,\gamma_1(k)/H^0)} \quad (21)$$

$$\ell_1(k:\theta, H^2(\theta)/H^0) \stackrel{\text{def}}{=} 2ln\frac{p(\gamma_1(k-M),\cdots,\gamma_1(k)/H^2(\theta))}{p(\gamma_1(k-M),\cdots,\gamma_1(k)/H^0)} \quad (22)$$

$$\ell_1(k:\theta, H^1(\theta)/H^2(\theta)) \stackrel{\text{def}}{=} 2ln\frac{p(\gamma_1(k-M),\cdots,\gamma_1(k)/H^1(\theta))}{p(\gamma_1(k-M),\cdots,\gamma_1(k)/H^2(\theta))} \quad (23)$$

They can be calculated from the results of the decentralized estimator in subsystem 1, by using Bayesian analysis of the likelihood function $p(\cdot/H^0)$, $p(\cdot/H^1)$, and $p(\cdot/H^2)$.

Using the results of eqs (19) , (20) , determine $\hat{\theta}$ that maximizes each of likelihood ratios for $k - M \le \theta \le k$. Then the temporary failure diagnosis in subsystem 1 is performed by the following thresholding approach with proper values of η_1 , η_2 and η_3 .

$$\ell_1(k:\hat{\theta}, H^1(\theta)/H^0) \gtrless \eta_1 \rightarrow \begin{cases} H^1(\theta) \\ H^0 \end{cases}$$
$$\ell_1(k:\hat{\theta}, H^2(\theta)/H^0) \gtrless \eta_2 \rightarrow \begin{cases} H^2(\theta) \\ H^0 \end{cases}$$
$$\ell_1(k:\hat{\theta}, H^1(\theta)/H^2(\theta)) \gtrless \eta_3 \rightarrow \begin{cases} H^1(\theta) \\ H^2(\theta) \end{cases}$$

The diagnosis procedure in subsystem 1 (D_1) is shown in Fig. 1 . A similar failure diagnosis system can be designed for the decision system of the subsystem 2 (D_2) by using the innovation sequence $\{\gamma_2(k)\}$. In order to implement the failure diagnosis for the decentralized diagnosis system, the decision results in D_1 and D_2 are coordinated to obtain final decision of the diagnosis. When the decision result in D_1 coincides with D_2, the coodinator indicates the final decision $D = D_1(= D_2)$. But when the results are different, the coordinator accepts the result in which the calculated likelihood function $L(H^i) = -\ln p(\cdot/H^i)$ is smaller than the other. (See Fig. 2)

In this way, the failure diagnosis scheme for the decentralized system seems to be more complicated one compared with that of centralized system. This is because the failure identification becomes essentially difficult due to the decentralized information processings.

5. NUMERICAL EXAMPLE

Simulation studies were carried out on a third-order linear system as shown in Fig. 3 under the following specifications :

$$G_1(s) = \frac{\omega^2}{s^2 + 2\varsigma\omega s + \omega^2} \quad (24)$$

$$G_2(s) = \frac{1}{1 + Ts} \quad (25)$$

where ς is damping factor and ω is natural angular frequency and T is time constant. The values of them are

$$\varsigma = 0.4 \, , \omega = 0.5 \, , T = 1.0 \, .$$

Details of the results will be shown at the presentation of the paper.

REFERENCES

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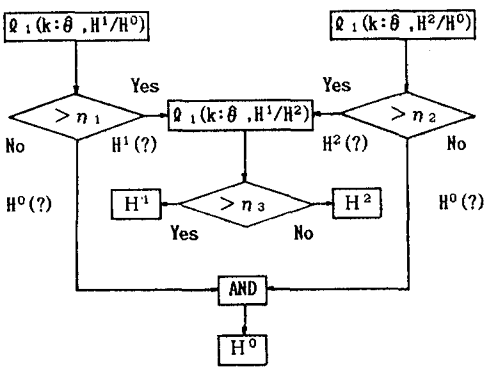

Fig. 1 Diagnosis Procedure in Subsystem 1

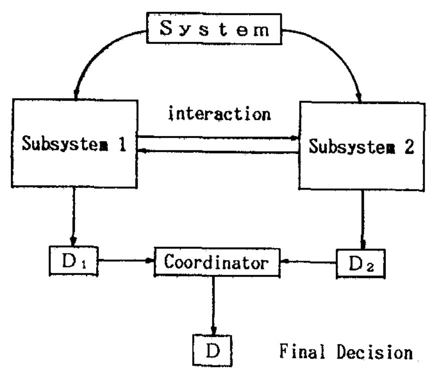

Fig. 2 The Failure Diagnosis for The Decentralized System

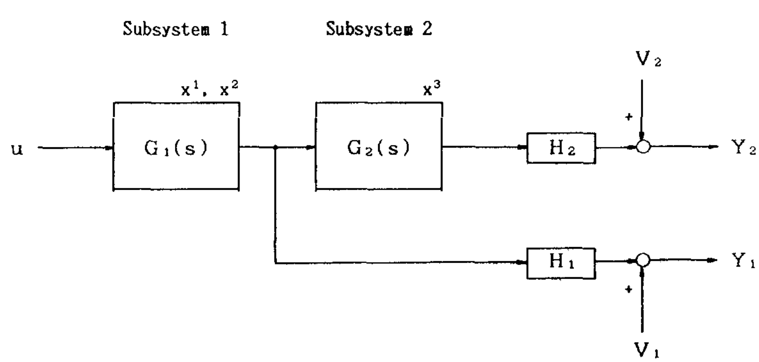

Fig. 3 Block Diagram of A Third-Order System