DIRECT COMPUTATION OF MARGINAL OPERATING CONDITIONS FOR VOLTAGE COLLAPSE

Kyung Jae Lee, Tay Ho Jung*
Research Center,
Korea Electric Power Cor.

Abstract

Voltage collapse is a serious concern to the electirc utility industry. It is common to associate steady-state stability with the ability of the transmission system to transport real power and to associate voltage collapse with the inability to provide reactive power at the necessary locations within the system. An algorithm to directly calculate the critical point of system voltage collapse was presented by the authors. The method (based on the ordinary power flow equations and explicit requirement of singularity of the Jacobian matrix) is basically one degree of freedom with proper load distribution factors.

This paper suggests a modified algorithm to increase the degree of freedom, introducing the nonlinear programming technique. The objective function is a distance measure between the present operating point and the closest voltage collapse point. Knowledge of the distance and the most vulnarable bus from the voltage collapse point of view may be used as a useful index for the secure system operation.

Introduction

The power system operating environment of today has substantially increased the difficulty of maintaining an acceptable system voltage profile. Problems associated with the steady-state stability and voltage collapse of electric power systems have become increasingly important and have received singnificant attention from many researchers. Low voltages can result in loss of stability and voltage collapse, and ultimately to cascading power outages. Voltage difficulties have been associated recently with major incidents in several countries [1,2]. Several factors have contributed to this situation, including the use of higher voltage transmission lines, the relative insufficiency of reactive power reserves that result from the use of large generating units and from the shift in power flow patterns associated with transmission economy and generator availability. Research efforts have resulted in several different methodologies for the coordination and utilization of the reactive power and voltage control resources of a system.
Venikov et al [3] recognized the singnificance of a degeneracy in the Jacobian matrix with respect to the steady-state stability of a power system. They observed that, under certain conditions, a change in the sign of the determinant of the Jacobian matrix during a continuous variation of parameters coincides with the movement of a real characteristic root of the linearized swing equations across the imaginary axis into the right half of the complex plane.
Tamura et al [4] have confirmed that multiple flow solutions are likely to apper under heavy load conditions. This seems to be related to voltage instability, especially when a pair of solutions are located close each other.
Barbier and Barret [5] suggested an approximate method to calculate critical value of bus voltage as a threshold value. Using a maximum transfer condition and reduced bus admittance matrices, bus voltage stability is checked in a static manner.
Carpentier et al [6] defined a proximity indicator for voltage collapse for a bus, an area or the complete system, as a vector of ratios dQ/dD, where dQ is the incremental generated reactive power at a generator when a given reactive load demand increases by dD. When any element of this vector of ratios becomes infinite, voltage collapse is said to occur. Optimal power flows are proposed to evaluate these indicators.
Kessel and Glavitsch [7] proposed a different type of indicator to express the risk of voltage collapse. The indicator uses information from a normal power flow and it can be obtained with resonable computational effort. They uesd a hybrid model, where partial

inversion of the bus admittance matrix for the load buses necessary to generate a hybrid matrix.
Tranuchit and Thomas [8] proposed the minimum singular value of the Jacobian of the descriptor network equations as a voltage security index. Instead of performing singular value decomposition for every change of system operating conditions, they established incremental linear relationship between parameters (dP's and dQ's) and increments of the minimum singular value.
Jarjis and Galiana [9] suggested a method for the analysis of voltage stability that does not rely upon power flow or optimal power flow simulations. It is based on the concept of the feasibility regions of power flow maps and the feasibility margins. This is an exact method. However, the procedure does not lend itself to an application in larger systems due to the enormous computational requirements.
H .G. Kwatny et al [10] presented a precise definition of static stability and voltage collapse based upon static bifurcation theory. Static bifurcations of the power flow equations were analyzed using the Liapunov-Schimit reduction and Tavlor series expansion of the resulting reduced bifurcation equations. It was shown that static bifurcations of the power flow equations were associated with either divergence-type instability or loss of causality.
Sekine and Yokoyama [11] proposed an eigenvalue method to detect voltage instability. They showed that voltage instability is influenced by multiple solutions and by various dynamic characteristics of loads and control equipment. It is also shown that a static var compensator at the receiving end of long distance transmission line improves the voltage stability.
DeMarco and Bergen [12], recognizing recent developments in the theory of large deviations within nonlinear systems, used a set of stochastic differential equations to represent the power system model, where the load demand is modeled as random white noise with zero mean. He related the voltage cooapse to the phenomenon where the system trajectory of the stochastic dynamics leaves any bouded region in finite time with probability one.
Alvarado and Jung [19] proposed a simple and elegant algorithm to directly calculate the critical point without numerical difficulties. The method used the power flow equations, and a requirement of the singularity of Jacobian matrix. This algorithm is very efficient to find out the voltage collapse conditions, but restricted to one degree of freedom with a proper load distribution factor.

This paper extends the former algorithm developed in [19] introducing nonlinear optimization techniques to increase the degree of freedom. Monitoring the distance from the present operating point to the most vulnarable bus or buses (from the voltage collapse point of view) will be an essential role for the secure operation of powewr systems.

System Representation and Problem Formulation

This section presents system models and mathematical formulations. Voltage collapse will be described for a two terminal system then extended into a general multi-machine and multi-load systems.

Two Bus System with a Constant Voltage Source

A very simple power system for understanding voltage collapse phenomena is shown in Figure 1. In this system a constant voltage generator supplies power to a load via a single lossless transmission line with its series admittance $-jB$ and shunt admittance jB_{sh}. Assume that the source voltage V_1 is $1.0\angle 0$, and the load bus voltage is $V_2\angle\alpha$. The load has real and reactive powers P and Q respectively. The system and line condition may be the Thevenin equivalent of a system as seen by a load.
From the ordinary power flow equations, we obtain the following system equations for P and Q,

$$P = -B\ V_2\ \mathrm{SIN}(-\alpha) \quad (1)$$

$$Q = B\ V_2\ \mathrm{COS}(\alpha) - (B-B_{sh})\ V_2^2 \quad (2a)$$

$$Q = P(\sqrt{1-pf^2})/pf \quad (2b)$$

Eliminating α and Q from equations (1) and (2), we obtain:

$$P^2 - 2pf\sqrt{1-pf^2}(B-B_{sh})\ V_2^2P + pf^2((B-B_{sh})^2\ V_2^4 - B^2V_2^2) = 0 \quad (3)$$

Equation (3) represents the relationship between load bus voltage and active power to be supplied to the load under steady-state conditions. Figure 2 shows the loci of P vs V_2 parameterizing the power factor under the assumption of $B = 5.0$ and $B_{sh} = 0.1$ pu.
The following are observations from Figure 2:

For a given load power factor, there exists a critical point beyond which the power cannot be transmitted. We call the power corresponding to this critical point a steady-state stability limit and the bus voltage to this limit value a critical voltage.

For each active power level P, except at the critical point, there are two operating points for each load bus voltage V_2. One is a high voltage operating point (we call this point the stable operating point), and the other one is lower than the critical voltage (we call it a unstable operating point) [12]. At the critical point dP/dV_2 becomes zero. If the load attempts to increase beyond the critical point, then the system will not have a feasible operating point and system collapse may result.

The critical values vary according to system conditions, such as rescheduling of generation power, generation bus voltage, and power factor of the load buses.

If instead of controlling the power factor we assume that the reactive power of the load bus is controlled, the reactive power Q is represented by:

$$Q - (B-B_{sh})\,V_2^2 + \sqrt{B^2 V_2^2 - P^2} = 0 \qquad (4)$$

where Q is the required amount of the reactive compensation. Figure 3 depicts the loci of Q vs for a given P under the same assumptions.
The results are similar to those in Figure 2. For a given fixed value of P, no feasible solution is expected if the reactive power compensation is not sufficient. If for any reson the system is opreated at the lower voltage operating point, then the bus voltage magnitude will decrease gradually along the contour of constant P, in spite of increasing reactive power compensation. If the load is supplied by transformers with on load tap changers, they will try to raise the voltage at the terminals of the load, which has the effect of reducing it's apparent impedance as seen from the system, and therefore of lowering voltage still further until the on-load tap changers reach their limit. This is the phenomenon of voltage collapse discussed in [5].

Multi-Machine and Multi-Load Power Systems

The relationship between load bus voltage and active power for a very simple two node system is derived in the previous section. Now we want to generalize this concept to an n+1 bus system. Let's suppose that the system has m generation buses one of which is swing bus and n-m+1 load buses. Buses with both a load and a generator or both a load and reactive sources connected will be considered as generating buses. Furthermore, we assume that buses with generators or reactive sources are ordered first and with the swing bus being the first bus. Analogous to the voltage/power relation in (4) are the following power flow equations [13],

$$f_i = P_i - \sum_{k=1}^{n} V_i V_k (g_{ik}\ \mathrm{COS}(\alpha_i-\alpha_k) + b_{ik}\ \mathrm{SIN}(\alpha_i-\alpha_k)) = 0 \qquad (5)$$

$$g_i = Q_i - \sum_{k=1}^{n} V_i V_k (g_{ik}\ \mathrm{SIN}(\alpha_i-\alpha_k) - b_{ik}\ \mathrm{SIN}(\alpha_i-\alpha_k)) = 0 \qquad (6)$$

for i = 2,..., n+1. Where (P_i,Q_i) is the net complex power entering into bus i, V_i is the voltage magnitude of bus i, α_i is the voltage angle difference of bus i from a slack bus, and $g_{ik}+jb_{ik}$ is the (i,j) element of bus admittance matrix. These equation can be written in compact form as,

$$f(x,p) = 0 \qquad (7)$$

where,

$$f = [f_2,\ldots,f_{n+1},g_2,\ldots,g_{n+1}]^T : R^{4n} \rightarrow R^{2n}$$

$$x = [\,'_2,\ldots,'_{n+1},V_2,\ldots,V_{n+1}]^T \in R^{2n}$$

$$p = [P_2,\ldots,P_{n+1},Q_2,\ldots,Q_{n+1}]^T \in R^{2n}$$

The superscript T denotes transposition. A number of ways have been developed to find a solution of equation (7) for any specified parameter vector p [13]. Our goal here is not to discuss solution methods, but to investigate the phemonena at the critical conditions. We amy use the chain rule of differentiation and the continuation method [9]. Assume x and p are parameterized by an arbitrary scalar t, and the locus of (7) is defined as the set of points (x,p) such that f(x,p) = 0. Then,

$$df(x,p)/dt = f_x(dx/dt) + f_p(dp/dt) = 0 \qquad (8)$$

where, $f_x = \partial f/\partial x$ (2nx2n Jacobian matrix), dx/dt (2nx1 vector), $f_p = \partial f/\partial p$ (2nx2n identity matrix), and dp/dt is a 2n x 1 vector respectively. Conditions in equation (8) must be satfsfied for all (x,p) on the trajectory of feasible solutions. We assume implicitly the continuity and differentiability of all functions involved. Solutions x(t) to equation (7) are obtained by solving the following differ rential equation set,

$$f_x(dx/dt) = -(dp/dt) \qquad (9)$$

We now investigate changes in equilibrium for variations in the parameters P. Suppose only one load (say load i) is changing. At the critical point, dP_i/dV_i becomes zero because the active power at bus i can be increased further and so do the remaining elements of dP/dV_i. Under these circumstances, we may write equation (9) as:

$$f_x(dx/dV_i) = -dp/dV_i = 0. \qquad (10)$$

Since dx/dV_i is nonzero, then it must be the case that the Jacobian matrix is singular. This represents the extension from the voltage/power sensitivity problem in the two bus system to the general n+1 bus system. We propose an efficient direct method for finding the power flow solutions with a requrement of singularity of the Jacobian matrix. Let:

$$f_x y = 0 \qquad (11)$$

$$f(x,p) = 0 \qquad (12)$$

$$y_1 = 1.0 \qquad (13)$$

where y is a 2nx1 transform vector. Equation (13) insures that the vector y is nontrivial. Equation (11) together with equation (13) establishes the singularity of the Jacobian. Equation (12) requires that they satisfy the power flow equations.

Solving the above equations simultaneously for both x and y for the specified p, these solutions lead to one of the critical points of the system.In equatons (11), (12) and (13), there are 4n+1 equations with 4n variables and 2n parameters. Therefore, only 2n-1 (instead of 2n) parameters can be specified independently and one parameter is dependent on the others (one degree of freedom). The above discussions are related how to find a critical condition using the singularity of the Jacobian with one degree of freedom. However, it is frequently desirable to calculate the closest critical condition from the present operating point with more degrees of freedom. This problem can be resolved with the idea of minimizing a distance using the following nonlinear optimization technique:

$$\text{Minimize:}d = \sum_{i=1}^{k} (p_i - p_i^*)^2 \qquad (14)$$

Subject to:

$$f_x y = 0$$
$$f(x,p) = 0$$
$$y_1 = 1.0$$

where d is the square of distance between the present state and the critical condition, p_i^* is the present value of i^{th} parameter, and k is the number of parameters to be changed.

Simulation and Results

A five bus system as shown in Figure 4 is considered. This system has two generators at buses 1 and 3 and three load buses, 2, 4, and 5. The line parameters, bus data, and the base case power flow results for this system are given in Appendices A, B, and C repectively. The equations for this system are symbolically generated from equations (11)-(13). These equations are then simply solved using modern software capable of solving arbitrary sets of nonlinear equations by exact Newton's method with symbolically computed Jacobians, full use of sparse matrix techniques, and pivoting for numerical error control [14-18].

As discussed in the previous section, finding the critical operating condition is equivalent to finding singularity condition for the Jacobian matrix of the power flow equations. Under the base case condition in appendix C, assume that bus 5 gradually increases its real power consumption. Voltage in buses 1 and 3 is assumed constant at 1.04 and 1.02 pu respectively. There are seven system variables; V_2, a_2, V_3, V_4, a_4, V_5 and a_5 in equation (13). There are seven independent active and reactive power flow equations.

To find out the critical voltage and power for bus 5 by the direct method sugested in this paper, fifteen equations are necessary. The eight additional equations come from equations (12) and (14). This yields a complete set of equation to find out a critical operating point with respect to changing P_5. Table 1 shows the results for the critical situation calculated by the new algorithm for the case that all but remain the same as base case condition.

Ordinary power flow calculations also have been performed for different levels of P_5. Two solutions are obtained for each level of P5 by specifying different initial conditions for the bus voltage magnitudes. Figure 5 shows the trajectories of V_4, V_5, a_4, a_5 for the different values of P_5.

The critical real power consumption at bus 5 is about 4.6963 pu. The same value is obtained either by directly solving equations (11)-(13) with the general purpose symbolic equation solving software, or by repetitive solutions of the ordinary power flow equations. Notice that the critical point found by the repetitive power flow calcuations is just an interpolation point because the numerical ill-conditioning in the Newton-Raphson power flow routine does not allow an exact calculation at the bifurcation point due to the singularity of the Jacobian matrix at that point [15]. The new formulation of problem determines this singularity point exactly. The matrices associated with the new formulation do not become singular at the critical point. Furthermore, convergence gets worse for ordinary power flow method as the system condition approaches to the critical point but convergence is excellent for the new method. Bus angles also approach to critical conditions as the voltage magnitudes approach to the critical points. The associated bus voltage angles become infinitely sensitive to small changes in the parameters. This property is generally associatied with loss of steady-state stability. The bifurcation is associated with infinite sensitivity of the associated bus voltage magnitudes with respect to parameter perturbations. This property is considered as the essential feature of voltage collapse [10]. Figure 5 shows that bus angles at buses 4 and 5 increase gradually as the load in the bus increases and they become more than 45 degrees at the critical condition.

To make the problem more general, suppose P_4+P_5 is 5.3024 pu. In this case all other parameters except P_4 and P_5 remain constant as the base case and only P_4 and P_5 are undefined. As shown in Table 2 and Figure 6, there is an agreement between the results calculated directly by the new algorithm and the values calculated as the limit point by the ordinary power flow equations. The maximum discrepancy between two results is than 0.5%. Table 3 shows the closest critical condition to the base case shown in Appendix B and C. This results are calculated using the nonliner optimization technique described in equation (14). In this case, reactive loads at buses 4 and 5 are assumed an half of the real load respectively. The above simulation results show the applicability of the new algorithm. For a given system operating condition, we can check the distance from the critical condition which may cause voltage collapse for the most vulnerable bus or buses in a certain area using this new algorithm. If the situation on the most vulnerable bus or buses are close to the critical condition, then system operators can take proper measures to prevent system instability.

Table 1 Calculation results for P_5 at critical condition.(P_5 = 4.6963 pu)

V_1 =1.04pu	a_1 = 0.0deg
V_2 =0.9161	a_2 =-23.0845
V_3 =1.0200	a_3 =-32.5211
V_4 =0.6841	a_4 =-48.8962
V_5 =0.6322	a_5 =-47.4810

Table 2 Comparison of calculation results at critical condition with direct method and ordinary power flow method.(P_4+P_5 = 5.3024 pu)

	Direct method	Power flow method
P_4+P_5	5.3024 pu.	5.3024 pu.
V_2	0.9231	0.9228
V_4	0.7203	0.7188
V_5	0.6388	0.6366
a_2	-21.497 deg.	-21.566 deg.
a_3	-29.8626	-29.9772
a_4	-40.9550	-41.1040
a_5	-46.4841	-46.6731

Table 3 The closest critical condition from the base case in Appendix B and C.

V_1 =1.04 pu.	a_1 =0.0 deg.
V_2 =0.9536	a_2 =-11.4203
V_3 =1.0200	a_3 =-12.6303
V_4 =0.5542	a_4 =-34.9802
V_5 =0.8550	a_5 =-12.2554

Conclusions

A new formulation can be used to find the critical operating point for voltage collapse. This method used both a singularity of the Jacobian matrix criterion and nonlinear optimization techniques. The resulting method shows a numerically stable behavior. Futhermore the method is able to detect voltage collapse on any bus or buses under specified system states. This method may enable system operators to more rapidly and accurately monitor the proximity of the present operating point to a critical point and to take proper measures to prevent system instablity well before it occures without using any indices for voltage collapse.

REFERENCES

[1] W. R. Lachs, "Voltage Collapse in EHV Power Systems," IEEE. PES Winter Meeting, New York, NY, Feb. 1978.
[2] H. D. Chang, F. F. Wu, "On Voltage Stability," IEEE ISCAS, Vol. 3, May 1986.
[3] V. A. Venikov, V. A. Stroev, V. I. Idelchick, V.I.Tarasov, "Estimation of Electrical Power System Steady-State Stability in Load Flow Calculations," IEEE Transactions on Power Apparatus and Systems, Vol. PAS-94, May/June 1975.
[4] Y. Tamura, H. Mory, S.Iwamoto, "Relationship Between Voltage Instability and Multiple Load Flow Solutions in Electric Power Systems," IEEE PES Summer Meeting, San Fransisco, California, July 1982.
[5] C. Barbier, J. P. Barret, "An Analysis of Phenomena of Voltage Collapse on a Transmission System," Reveu General d'Electricite, Tome 89, No.10, Oct. 1980.
[6] J. Carpentier, R. Girard, E. Scano, "Voltage Collapse Proximity Indicators computed from an Optimal Power Flow," Proceedings of the 8 th Power Systems Computation Conference, Helsinki, Finland, Aug. 1984.
[7] P. Kessel, H. Glavitsch, "Estimating the Voltage Stability fo a Power System," IEEE Transactions on Power Delivery Vol. PWRD-1, No. 3,July 1986.
[8] A Tranuchit, R. J. Thomas, "Var Support and Voltage Instability in Electric Power Networks," 1986 North American Power Symposium, Cornell University, Ithaca, N. Y., Oct. 1986.
[9] J. Jarjis, F.D. Galiana, "Quantitative Analysis of Steady-State Stability in Power Networks," IEEE Transactions on Power Apparatus and Systems, Vol. PAS-100, Jan. 1981.
[10] H. G. Kwanty, A. K. Pasrija, L. Y. Bahaar, "Static Bifurcations in Electric Power Network; Loss of Steady-State Stability and Voltage Collapse," IEEE Transactions on Circuits and Systems Vol. CAS-33, No.10, Oct. 1986

[11] Y. Sekine, A. Yokoyama, "Dynamic Stability Analysis Taking into Account Load Flow Multisolutions and Load Characterristics," Proceeding of the 8th Power Systems Computation Conference, Helsinki, Finland, Aug. 1984.

[12] C. L. DeMarco and A. R. Bergen, "A Security Measure for Random Load Disturbances in Nonlinear Power Systems, Vol. CAS-34, No. 12, Dec. 1987.

[13] A. R. Bergen, "Power System Analysis," Prentice Hall Inc., 1986.

[14] F. L. Alvarado, "SOLVER-Q Instruction Manual," Version 0.98, Feb. 1987.

[15] F. L. Alvarado and Y. Liu, "A Direct Modeling Method for General Purpose Simulation," IASTED 1987, Santa Barbara, May, 1987.

[16] W. F. Tinney and J. W. Walker, "Direct Solution of Sparse Network Equations by Optimally Ordered Triangular Factorization," Proceedings, IEEE Vol, 55, Nov. 1967.

[17] F. L. Alvarado, "Computational Complexity in Power Systems," IEEE Trans. on Power Apparatus and Systems, Vol. PAS-95, No. 4, Jul/Aug., 1976.

[18] O. Alsac, B. Stott and W. F. Tinney, "Sparsity Oriented Compensation Methods for Modified Network Solutions," IEEE Transactions on Power Apparatus and Systems, Vol. PAS-102, No. 5, May, 1983.

[19] F. L. Alvarado, T. H. Jung, "Direct Detection of Voltage Collapse Conditions," International Conference on Bulk Power System Voltage Phenomena: Voltage Stability and Security, Trout Lodge, Potoshi, Missoury, Sep. 1988.

Appendix A: Line parameters for five bus test system

Line (bus	to bus)	G(pu)	B(pu)
1	2	1.40056	-5.60224
1	5	1.84118	-7.48352
2	3	1.84118	-7.48352
3	4	0.70028	-2.80112
3	5	1.12985	-4.47675
4	5	0.93372	-3.73483

Appendix B: Bus data for 5 bus test system

Bus NO.	P(pu)	Q(pu)	Charging
1	1.54058	0.72499	0.03600
2	-1.15000	-0.60000	0.01800
3	1.10000	0.78317	0.08200
4	-0.70000	-0.30000	0.03575
5	-0.70000	-0.40000	0.03575

Appendix C: Base case power flow results for the test system

V_1 = 1.04(pu)	a_1 = 0.00(deg)
V_2 = 0.9603	a_2 = -5.974
V_3 = 1.0200	a_3 = -3.22
V_4 = 0.9151	a_4 = -10.078
V_5 = 0.9681	a_5 = -5.248

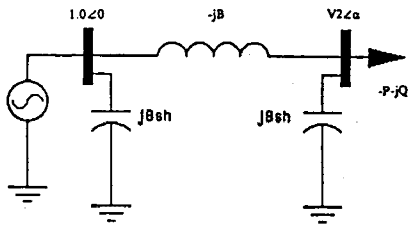

Figure 1. One line diagram of a simple two terminal test system.

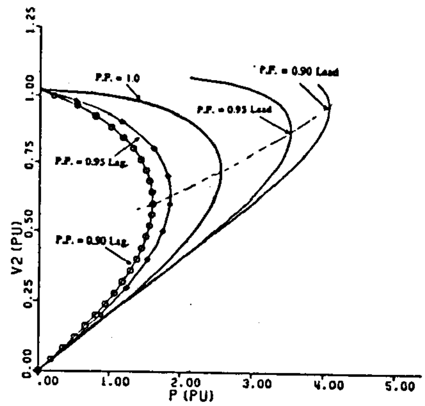

Figure 2. Power transfer capability curves as a function of load bus voltage.

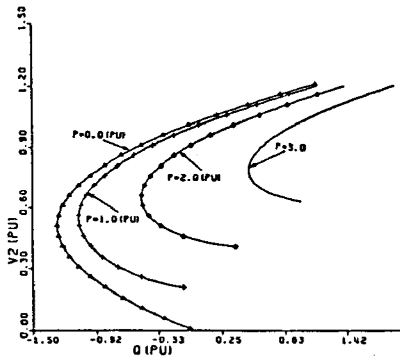

Figure 3. Reactive power requirements to maintain the bus voltage for different levels of P. B=5.0pu, B_{sh}=0.1pu.

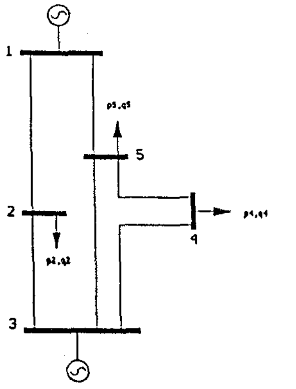

Figure 4. One line diagram of 5 bus test system with 6 branches.

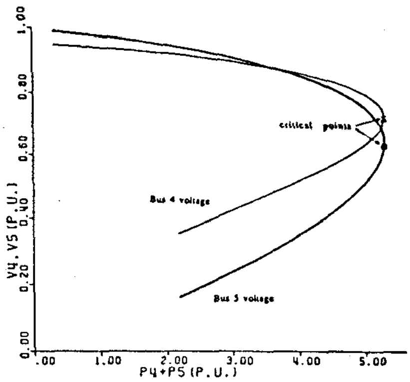

Figure 6. Loci of bus 4 and 5 voltage magnitudes as a function of P_4+P_5.

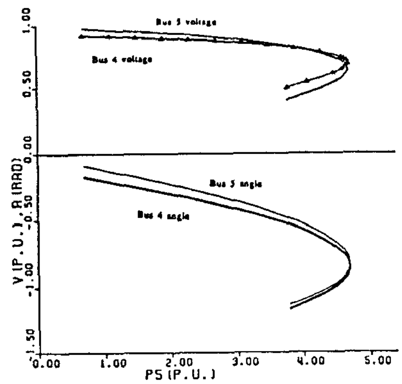

Figure 5. Loci of bus 4 and 5 voltage magnitudes and angles as a function of P_5.