Translational Control of a One Link Flexible Arm

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Abstract

This is a study of the translational end-point control of a single link flexible arm with a tip mass. The beam is mounted on a translational mechanism driven by the ball screw, whose rotation is controlled by DC servomotor. The problem of shifting the end-point from its initial position to the commanded position is studied analytically both for the open-loop control subjected to some path funtions and for the closed-loop control using the feedback of the tip information.

1. INTRODUCTION

The requirements for more precise positional accuracy, higher speed and productivity in recent years have accelerated the research activities on the lightweight robot. Such arms are superior to rigid ones in that the high-speed operation can be realized with less energy consumption. It should be noted however that their operation is interfered with the flexural vibration caused from the back of the arm stiffness. One then has to control the arm so that the vibration is not induced and the high-speed positioning is achieved.

When one moves the endpoint of the arm from its initial position to the desired position, it may be effective to use the information of the tip directly as the feedback signal. Cannon and Schmitz[1] studied the closed-loop endpoint control of a flexible pinned-free beam using the modal expansion method. They showed some experimental results on the controlled arm tip displacement. Tahara and Chonan [2] investigated the closed loop control of a one link flexible arm with a tip mass using the tip displacement and velocity as the feeback signals. They obtained the theoretical results and compared them with the experimental ones. All of these papers are concerned with rotational control of the flexible arm.

In this paper, the translational control of a one link flexible arm with a tip mass is studied. The beam is mounted on a translational mechanism driven by the ball screw, whose rotation is controlled by the DC servomotor. The problem of shifting the end point from its initial position to the commanded position is studied analytically both for the open loop control to have the base subjected to the given path functions and for the closed loop control using the tip information. The governing equation of the system is constructed based on the Bernoulli-Euler theory. The solutions for the governing equations are obtained by applying both the method of the Laplace transform and the method of numerical inversion proposed by Weeks[3]. The step function and some path functions are introduced as the input to the system. The responses of these inputs to the two types of control system are compared. The numerical results are presented here.

2. GOVERNING EQUATIONS

2.1 Closed loop control

Figure 1 shows the one-link flexible arm studied in this analysis. It is composed of a motor, reduction gears, a ball-screw mechanism, an arm of length L and an end point payload. The base of the arm is translated by the ball screw driven by the motor. As shown in the figure, the x-axis is placed along the axis of the arm and the y-axis normal to the x-axis. The z-axis is perpendicular to both the x- and y-axes. M is the bending moment, and Q denotes the shear force. One assumes that the arm deflection is seen only in the xy-plane. In this case, denoting the lateral displacement of the arm by w(x,t) the equation of motion of the arm by Bernoulli-Euler Theorv is written as

$$(1)\quad EI\left(1+C\frac{\partial}{\partial t}\right)\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} = 0$$

where E is the Young's modulus, the mass density, C the internal damping coefficient, A the sectional area, I the moment of inertia, and t time. Assuming that the arm is resting statically at t=0, one has the Laplace transformed equation of (1) as

$$(2)\quad EI(1+Cs)\frac{\partial^4 W(x,s)}{\partial x^4} + \rho A s^2 W(x,s) = 0$$

where W(x,s) is the Laplace transform of w(x,t). By introducing

$$(3)\quad \zeta^4 = -\rho A s^2/EI(1+Cs)$$

equation (2) is simplified as

$$(4)\quad \frac{\partial^4 W(x,s)}{\partial x^4} - \zeta^4 W(x,s) = 0$$

The general solution of equation (4) is given by

$$(5)\quad W(x) = k_1\sin\zeta x + k_2\cos\zeta x + k_3\sinh\zeta x + k_4\cosh\zeta x$$

where k_1, k_2, k_3 and k_4 are unknown to be determined from the boundary conditions.

Next, the boundary conditions of the arm displacement and force equilibrium at x=0 give

$$(6)\quad \frac{\partial w(0,t)}{\partial x} = 0$$

$$(7)\quad M_0\frac{\partial^2 w(0,t)}{\partial t^2} + C_s\frac{\partial w(0,t)}{\partial t} = R + Q$$

where M_0 is the mass of the pedestal, C_s the damping coefficient between the base and the ball screw, R the reaction force between the pedestal and the screw, and Q the shear force from the arm. The moment equilibriums around the motor shaft and the reduction gears shown in figure are obtained as follow

$$(8)\quad J_1\frac{\partial^2\theta_1}{\partial t^2} + \epsilon_1\frac{\partial\theta_1}{\partial t} = T - Fr_1$$

$$(9)\quad (J_2 + J_3)\frac{\partial^2\theta_2}{\partial t^2} + \epsilon_2\frac{\partial\theta_2}{\partial t} = Fr_2 - P_t r_3$$

where T is the motor torque applied to the gear, F the transfer force at the gear, P_t the radial force acting on the ball screw from the pedestal, then θ_1 and θ_2 are the rotation angles of the motor shaft and the ball screw, J_1, J_2 and J_3 the polar moment of inertia of the motor shaft, the gear and the ball screw, ϵ_1 and ϵ_2 are the damping coefficient between the pinion and motor, the gear and ball screw, r_1, r_2, and r_3 the radius of the pinion, gear and ball screw respectively.

The force equilibrium for the ball screw is, from figure 1,

$$(10)\quad P_t\cos\phi = R\sin\phi + (P_t\sin\phi + R\cos\phi)\mu$$

where μ is the friction coefficient.

The displacement relation between the pedestal and the rotation of the screw, and the geometrical relation between the motor and the reduction gear give

$$(11)\quad \theta_2 = w(0,t)/(r_3\tan\phi)$$

$$(12)\quad \theta_1 = (r_2/r_1)\theta_2$$

Next, the equilibrium conditions of the moment and the lateral forces at the endpoint of the arm, x=L, lead to

$$(13)\quad J_p\frac{\partial^2}{\partial t^2}\{\frac{\partial w(L,t)}{\partial x}\} = M = -EI(1+C\frac{\partial}{\partial t})\frac{\partial^2 w(L,t)}{\partial x^2}$$

$$(14)\quad M_p\frac{\partial^2 w(L,t)}{\partial t^2} = -Q = EI(1+C\frac{\partial}{\partial t})\frac{\partial^3 w(L,t)}{\partial x^3}$$

where J_p is the polar moment of inertia of the payload, M_p the mass of the payload. Equations (6)-(14) constitute the boundary conditions for the problem under consideration. One first transforms equations (6), (13), and (14) with respect to time t and substitutes equation(5), which gives

$$(15)\quad k_1 + k_3 = 0$$

$$(16)\quad J_p s^2\zeta(k_1\cos\zeta L - k_2\sin\zeta L + k_3\cosh\zeta L + k_4\sinh\zeta L) + EI(1 + Cs)\zeta^2(-k_1\sin\zeta L - k_2\cos\zeta L + k_3\sinh\zeta L + k_4\cosh\zeta L) = 0$$

$$(17)\quad M_p s^2(k_1\sin\zeta L + k_2\cos\zeta L + k_3\sinh\zeta L + k_4\cosh\zeta L) - EI\zeta^3(1 + Cs)(-k_1\cos\zeta L + k_2\sin\zeta L + k_3\cosh\zeta L + k_4\sinh\zeta L) = 0$$

Next, one arranges the equations (8)-(12). Eliminating the parameter F from equation (8), and (9) give

$$(18)\quad P_t = \frac{r_2}{r_1 r_3}\{T - (J_1\frac{\partial^2\theta_1}{\partial t^2} + \epsilon_1\frac{\partial\theta_1}{\partial t})\} - \frac{1}{r_3}(J_2 + J_3)\frac{\partial^2\theta_2}{\partial t^2} + \epsilon_2\frac{\partial\theta_2}{\partial t}$$

Substituting equations (18) into equation (10), together with the use of equations (11) and (12), one has

$$(19)\quad R = P_t/\tan(\phi + \Phi) = \frac{r_2 T}{r_1 r_3\tan(\phi+\Phi)} - \frac{(r_2/r_1 r_3)^2}{\tan\phi\tan(\phi+\Phi)}\{J_1\frac{\partial^2 w(0,t)}{\partial t^2} + \epsilon_1\frac{w(0,t)}{t}\} - \frac{1}{r_3^2\tan\phi\tan(\phi+\Phi)}\{(J_2+J_3)\frac{\partial^2 w(0,t)}{\partial t^2} + \epsilon_2\frac{\partial w(0,t)}{\partial t}\}$$

Further, substituting equations (14) and (19) into equation(7) and transforming the resulted equation with respect to time, give

$$(20)\quad M_0 s^2 W(o,s) + C_s s W(0,s) = \frac{r_2 T(s)}{r_1 r_3\tan(\phi+\Phi)} - \frac{2\pi(r_2/r_1)^2}{r_3 p\tan(\phi+\Phi)}\{J_1 s^2+\epsilon_1 s)W(0,s) - \frac{2\pi}{r_3 p\tan(\phi+\Phi)}\{(J_2 + J_3)s^2 + \epsilon_2 s\}W(0,s) - EI(1 + Cs)\frac{\partial^3}{\partial x^3}W(0,s)$$

where $p(=2\pi r_3\tan\phi)$ is the lead, ϕ the lead angle, and Φ the friction angle of the ball screw, respectively.

In the following, to shift the endpoint of the arm from its initial position to the commanded position by the amount w_d, w_d is compared with the actual tip position $w(L,t)$, which is measured by a gap sensor fixed in space. The position error (w_d-w) is then amplified and put into the motor in order to control the arm. In this case, the equation governing the motor current i_a given by

$$(21)\quad \frac{L_a}{R_a}\frac{di_a}{dt} + i_a + \frac{K_b}{R_a}\frac{r_2}{r_1 r_3\tan\phi}\frac{\partial w(0,t)}{\partial t} = G_d\{ w_d(L,t) - w(L,t)\} - G_v\frac{\partial w(L,t)}{\partial t}$$

where G_d is the displacement feedback gain, G_v the velocity feedback gain, K_b the back electromotive force constant, L_a the motor inductance, and R_a the resistance of the circuit. The torque applied to the gear from the motor is given by

(22) $T = K_t i_a$

where K_t is the torque constant of the motor. Transforming equation (21) and (22) with respect to t and combining give the transformed torque T(s) written in term of the transformed commanded position W_d. Substituting T(s) into equation(20) and arranging, one has

$$
\begin{aligned}
(23)\quad & [\{M_o s^2 + C_s s + \frac{2\pi(r_2/r_1)^2}{r_3 p\tan(\phi+\Phi)}(J_1 s^2 + \epsilon_1 s) \\
& + \frac{2\pi}{r_3 p\tan(\phi+\Phi)}\{(J_2 + J_3)s^2 + \epsilon_1 s)\}(\frac{L_a}{R_a}s + 1) \\
& + \frac{2\pi K_t K_b (r_2/r_1)^2 s}{R_a r_3 \tan(\phi+\Phi)}](k_2 + k_4) + \frac{r_2 K_t (G_d + G_v s)}{r_1 r_3 \tan(\phi+\Phi)} \\
& *(k_1 \sin\zeta L + k_2 \cos\zeta L + k_3 \sinh\zeta L + k_4 \cosh\zeta L) \\
& + EI(1+Cs)\zeta^3(\frac{L_a}{R_a}s + 1)(-k_1 + k_3) \\
& = \frac{r_2}{r_1 r_3}\frac{K_t G_d W_d(L,s)}{\tan(\phi+\Phi)}
\end{aligned}
$$

Eqs.(15)-(17) and (23) constitute a simultaneous algebraic equation of the unknown k_1, k_2, k_3 and k_4. Using eq.(15) gives further simplified form.

(24)

$$
\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}
\begin{bmatrix} k_1 \\ k_2 \\ k_4 \end{bmatrix}
=
\begin{bmatrix} G_d W_d(L,s)\{K_t r_2/r_1 r_3 \tan(\phi+\Phi)\} \\ 0 \\ 0 \end{bmatrix}
$$

where the a_{ij} are follow

$$
\begin{aligned}
a_{11} &= \frac{r_2 K_t (G_d + G_v s)}{r_1 r_3 \tan(\phi+\Phi)}(\sin\zeta L - \sinh\zeta L) \\
&\qquad - 2EI(1 + Cs)\zeta^3 (L_a s/R_a + 1)
\end{aligned}
$$

$$
\begin{aligned}
a_{12},\ a_{13} &= [M_o s^2 + C_s s + \frac{2\pi(r_2/r_1)^2}{r_3 p\tan(\phi+\Phi)}(J_1 s^2 + \epsilon_1 s) \\
&\quad + \frac{2\pi\{(J_2+J_3)s^2+\epsilon_2 s\}}{r_3 p\tan(\phi+\Phi)}](\frac{L_a}{R_a}s+1) + \frac{2\pi K_t K_b s(r_2/r_1)^2}{R_a r_3 p\tan(\phi+\Phi)} \\
&\quad + \frac{r_2 K_t (G_d + G_v s)}{r_1 r_3 \tan(\phi+\Phi)}[\ \cos\zeta L\ ,\ \cosh\zeta L\ ]
\end{aligned}
$$

$$
\begin{aligned}
a_{21} &= J_p s^2 \zeta(\cos\zeta L - \cosh\zeta L) - EI(1 + Cs)\zeta^2 \\
&\qquad (\sin\zeta L + \sinh\zeta L)
\end{aligned}
$$

$$a_{22} = -J_p s^2 \zeta \sin\zeta L - EI(1+Cs)\zeta^2 \cos\zeta L$$

$$a_{23} = J_p s^2 \zeta \sinh\zeta L + EI(1+Cs)\zeta^2 \cosh\zeta L$$

$$
\begin{aligned}
a_{31} &= M_p s^2 (\sin\zeta L - \sinh\zeta L) \\
&\qquad + EI(1 + Cs)\zeta^3(\cos\zeta L + \cosh\zeta L)
\end{aligned}
$$

$$a_{32} = M_p s^2 \cos\zeta L - EI(1 + Cs)\zeta^3 \sin\zeta L$$

$$a_{33} = M_p s^2 \cosh\zeta L - EI(1+Cs)\zeta^3 \sinh\zeta L$$

After having k_1, k_2 and k_4 from equations (24) one has the transformed displacement represented by W_d as

$$
\begin{aligned}
(25)\quad \frac{W(x,s)}{W_d(L,s)} &= \frac{1}{D_t}\{d_1(\sin\zeta x - \sinh\zeta x) + d_2 \cos\zeta x \\
&\qquad + d_3 \cosh\zeta x\}\frac{r_2 K_t G_d}{r_1 r_3 \tan(\phi+\Phi)}
\end{aligned}
$$

where

$d_1 = a_{22}a_{33} - a_{23}a_{32}$
$d_2 = a_{23}a_{31} - a_{21}a_{33}$
$d_3 = a_{21}a_{32} - a_{22}a_{31}$
$D_t = \det(a_{ij})$, (i,j = 1, 2, 3)

W_d is stored in the computer memory through the keyboard. In this case, the input is represented as

(26) $w_d = w_d^* H(t)$

where w_d^* is the commanded position of the arm tip, and H(t) the Heaviside step function. Subtituting equation (26) into equation (25) one finally has

$$
\begin{aligned}
(27)\quad \frac{W(x,s)}{W_d^*} &= \frac{1}{D_t}\{\ d_1(\sin\zeta x - \sinh\zeta x) \\
&\qquad + d_2 \cos\zeta x + d_3 \cosh\zeta x\}\frac{r_2}{r_1 r_3 \tan(\phi+\Phi)}\frac{K_t G_d}{s}
\end{aligned}
$$

2.2 Open loop control

Next, one obtain the solution for the arm driven by the open-loop base translation. The boundary conditions in this case are given by

(28) $\frac{\partial w(0,t)}{\partial x} = 0$

(29) $w(0,t) = w_d(t) = W_d^* P(t)$

where P(t) is the path function applied to the arm base. Proceeding in the same way as in the case of the closed-loop control, one has

$$
(30)\quad
\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}
\begin{bmatrix} k_1 \\ k_2 \\ k_4 \end{bmatrix}
=
\begin{bmatrix} W_d(s) \\ 0 \\ 0 \end{bmatrix}
$$

where b_{ij} are

$b_{11} = 0$
$b_{12} = b_{13} = 1$
$b_{ij} = a_{ij}$, $i = 2, 3$ and $j = 1,2,3$.

The input-output relation in this case is

$$
\begin{aligned}
(31)\quad \frac{W(x,s)}{W_d^*} &= \frac{1}{D_t}\ \{\ d_1(\sin\zeta x - \sinh\zeta x) \\
&\qquad + d_2 \cos\zeta x + d_3 \cosh\zeta x\ \}P(s)
\end{aligned}
$$

In the numerical examples, one calculates the inverse integral of equations (27) and (31) by the numerical method proposed by Weeks.

3. Numerical Results and Discussion

Figure 2 shows the block diagram for the closed loop control system. One first inputs the program into the computer so as to have the path function, the signal of which is changed into the analogous voltage by the D/A converter, and then the adjusted signal is amplified and converted to a current signal through the power amplifier. Since the current signal is fed back to the motor armature, the base of beam is moved throuh the ball screw mechanism translationally. The physical parameters of the system to be considered in the numerical example are as follows.

E	= 6.517	x 1E+10	(Pa)
ρ	= 2.6476	x 1E-03	(kg/m^3)
C	= 1.19	x 1E-04	(s)
B	= 1.2	x 1E-02	(m)
H	= 2.0	x 1E-03	(m)
J_m	= 4.9	x 1E-06	($kg\text{-}m^2$)
ϵ	= 2.604	x 1E-03	(kgm^2/s)
K_t	= 6.7647	x 1E-02	(N-m/A)
K_b	= 1.772	x 1E-05	(V/rad)
L_a	= 1.8	x 1E-03	(H)
R_a	= 4.7		(ohm)

The system has both the displacement and the velocity feedback loops as shown in the figure2. The total displacement feedback gain G_d and the velocity feedback gain G_v in the system are given by

$$G_d = K_g * AD * f_d * Da * K_p$$
$$G_v = K_g * AD * f_v * DA * K_p$$

where
- K_g = gap sensor gain
- AD = A/D converter gain
- f_d = displacement feedback gain in computer
- f_v = velocity feedback gain in computer
- DA = D/A converter gain
- K_p = Power amplifier gain

Figure 3(a) shows the tip displacement in the case of the open loop system when the arm base is translated in the form of $W_d H(t)$ by using these physical parameters. Where H(t) is the Heaviside Unit step function. As shown in figure, the tip displacement first bigins to move with large overshoot. After that, this is decreased like the shape of the exponential decrement function. In a way, figure 3(b) shows the tip displacement of the closed loop system with the same physical parameters and step input function.

Figure 4 shows the displacement of the arm tip and that of the base when the motor cuttent is controlled by the feedback signal given in equation (21). As shown in the figure, the arm base first begins to move and then the tip follows the base deflection. The base translates in the same direction until the tip reaches the reference position. After that the tip has an overshoot due to the inertial force, and the base starts to move reversely in the opposite direction in order to reduce the overshoot of the tip. In this way, the base moves like a shuttle so that the vibration of the tip may be reduced quickly.

Next, one introduces two types of path functions which can give the smooth movement of tip from the initial position to the disired position without the feedback within the some limited time. The path functions used in this paper are shown in Table 1.

Table 1. Path functions used in this paper

(a) $P(t) = \sin^2 wt$		$0 < t \le \pi/2w$
$= 1$		$t > \pi/2w$
(b) $F(t) = 3(t/T_f)^3 - 2(t/T_f)^2$		$0 < t \le T_f$
$= 1$		$t > T_f$

Figure 5 shows the displacement, velocity and acceleration properties for the path functions (a) and (b). Those functions have smooth variations in the displacement. It was found that the endpoint follows the translational movement of base smoothly when one drives the arm base using those path functions.

Figure 6 shows the variations of the tip displacement depanding on the values of angular velocity of the open loop system with path function(a). When one gives the smooth path for the arm base to be taken about 2 sec from the staring position to the desired one, the movement of the tip coincides with that of the base very well. This means that we can use the open loop system instead of the closed loop system when it is not desired for the tip to have to move to the desired position within short time. The getting faster the arm base reaches to the desired position, the tip has bigger overshoot of tip displacement. At last, when the time of base to be reached to the desired positon is shorter, that may get the responses like the step input.

Figure 7 shows the variations of the tip displacement with path function(b). This also assumes same aspects of the path function(a). But, the time to be reached to the desired position smoothly is a little bit longer than that of the path function(a). That is, the path function $P(t) = \sin^2 wt$ brings a short settling time on the arm tip.

Figure 8 shows the tip displacement and the base movement subjected to the path function (b) as an input in the closed loop system. As shown in figure, the tip first begins to follow the base movement coincidentlly and reaches the desired position a little later than the base. Of course, the settling time is much shorter than that of the open loop system with same input path function.

4. CONCLUSION

A theoretical study has been presented for the translational path motion control of a one link flexible arm with a tip mass. In this paper the governing equation is available to derived by using the Laplace transform method without the proof the orthogonality. The result obtained for the case of open loop control is that the path function $F(t) = \sin^2 wt$ (for $0 < wt \le \pi/2$, and $=1$ for $wt > \pi/2$) brings a short settling time on the arm tip. And the settling time of the closed loop system is about two times shorter than that of the open loop system when the same path function(b) as an input of arm base is used.

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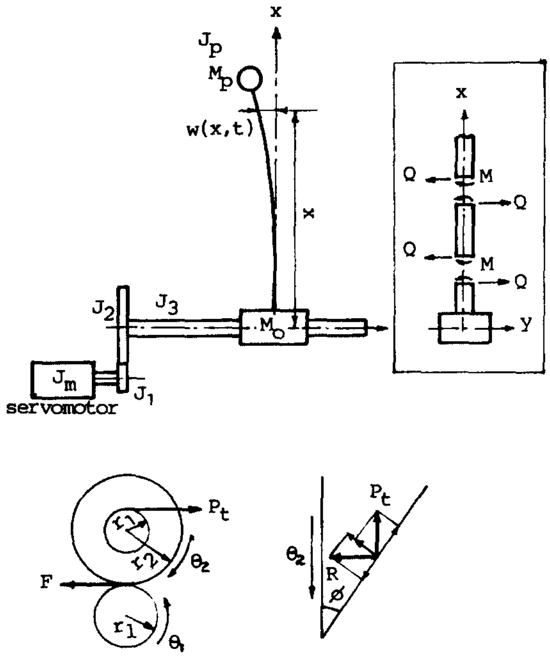

Fig.1 One link flexible arm subjected to the translational motion.

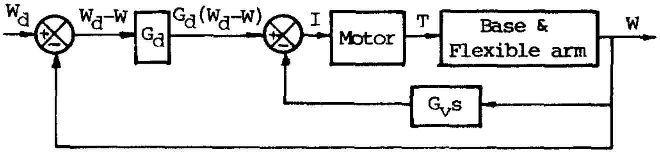

Fig.2 Block diagram of the closed loop control system.

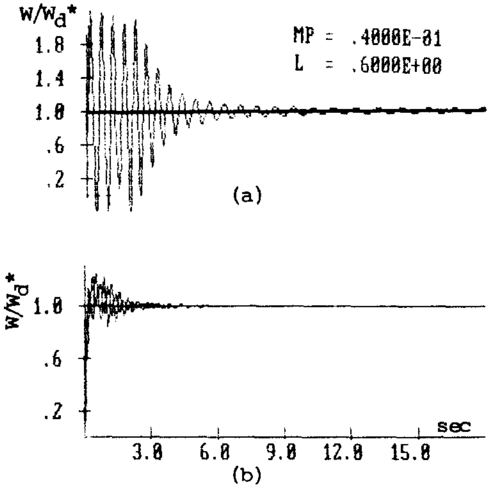

Fig.3 Step responses of the tip for the open loop(a) and the closed loop control(b)

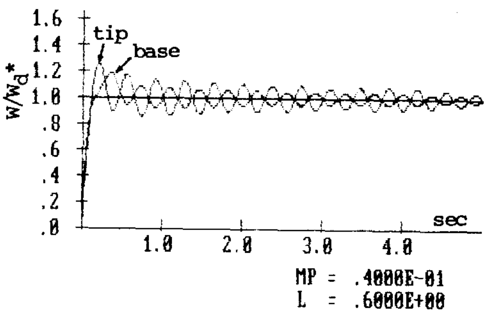

Fig.4 Displacements of the tip and the base for the closed loop control subjected to the step input

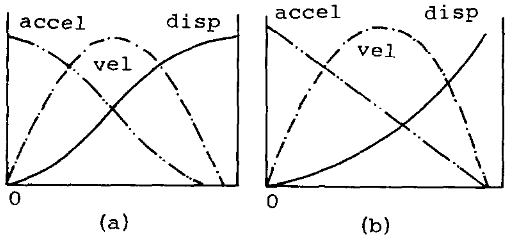

Fig.5 The displacement, velocity, and acceleration properties for the path function (a) and (b).

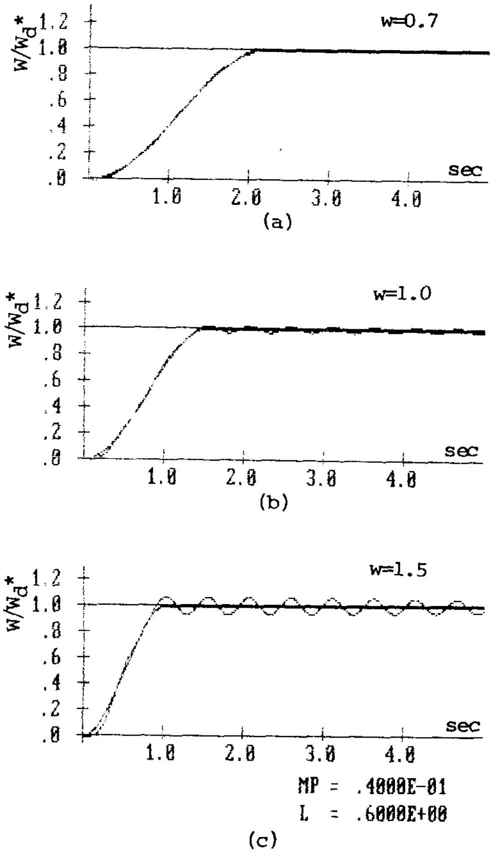

Fig.6 The variations of the tip displacement of the open loop control with a path function (a).

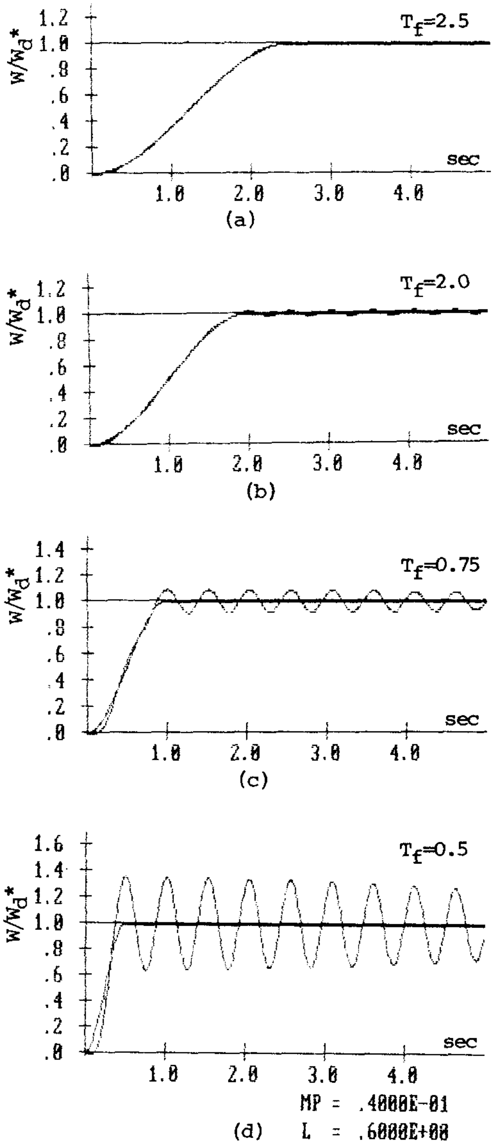

Fig.7 The variations of the tip displacement of the open loop control with a path function (b).

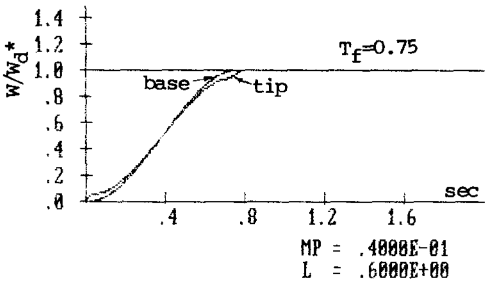

Fig.8 The tip displacement and the base movement subjected to a path function (b) as an input in the closed loop control.