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End-Point Control of a Flexible Arm under Base Fluctuation

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A theoretical study is presented for the end-point holding control of a one-link flexible arm, whose base is subjected to a lateral fluctuation. The arm is clamped on a rigid hub mounted directly on the shaft of d.c. servomoter. The tip position is measured by a gap sensor fixed in space isolated from the system vibration. The arm is controlled so as to make the end point stay precisely at its initial position even if the base is fluctuated.

1 INTRODUCTION

With the development of technology, lightweight manipulators are being introduced in various technological fields. In this case however, due to the flexible oscillations appeared, the assumption of rigid manipulator is no longer applicable. To endure good performance of the manipulator, it is necessary to develop a control strategy that takes into account the flexibility of the manipulator, and many papers on it have been published during the past years. Cannon and Schmitz (1984), Skaar and Tucker (1986), Yuh (1987), Yoshida, Shimogo and Inoshe (1988), Tahara and Chonan (1988), Yamaura and Ono (1988) and Yigit, Scott and Ulsoy (1988) studied the open-loop and the closed-loop end-point controls of single-link flexible manipulators. As for the multi-link flexible arm, Book, Maizza Neto, and Whitney (1975), Fukuda and Arakawa (1987), Ower and Van De Vegte (1987), Lee and Wang (1988) and Chonan and Umeno (1989) investigated the in-plane positioning of two-link arms with distributed flexibility. In all those papers attention has been directed to the control of flexible arms working in quiet circumstances. Robots performing tasks in environments hazardous to human workers are sometimes under the influence of some sort of disturbance. The manipulator working in the vicinity of a forging machine is a typical example of this kind, where the manipulator task is disturbed by the floor vibration. It appears, to the authors' knowledge, that no work has been done on the control of a flexible arm under external disturbances. It is in this context that the subject discussed in this paper is that of the end-point position control of a one-link flexible arm, whose base is subjected to a lateral fluctuation. The arm is controlled so as to make the end point stay precisely at its initial position even if the base is fluctuated. The control strategy is tested by means of simulations for the one-link flexible-arm prototype in the authors' Laboratory of Tohoku University. It is shown that PD control using the tip sensing and base torquing is sufficient to make the arm tip stay at its initial position precisely.

2 FORMULATION OF THE PROBLEM

Figure 1 shows a slender flexible arm whose one end is fixed onto the vertical shaft of a d.c. servomotor via a hub of radius r. The other end ($x=L$) is loaded with a mass (payload). M and Q are the bending moment and the shear force acting on the arm cross-section. The problem that follows is the control of the arm so that the tip stays precisely at its initial position even if the base is distributed by the lateral fluctuation $f(t)$. The torque T applied by the motor rotates the arm in the horizontal plane.

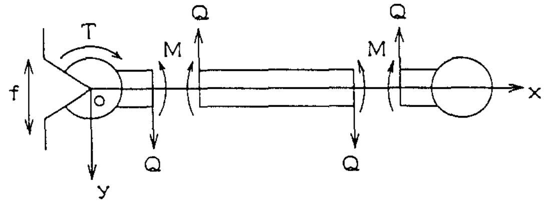

Fig.1 Geometry of problem and co-ordinates.

One assumes that Bernoulli-Euler theory is adequate to describe the flexural motion. In this case, the equation of motion of the arm is

$$EI(1+c\partial/\partial t)(\partial^4/\partial x^4)w(x,t)+\rho A(\partial^2/\partial t^2)w(x,t)=0 \quad (1)$$

where E is the Young's modulus, ρ is the mass density, I is the moment of inertia, and A is the cross-sectional area, c is the internal damping coefficient of the arm, and t is time.

One next considers the boundary conditions of the arm. One end of the arm is clamped on a rigid hub of radius r which is mounted on the vertical shaft of the motor. The geometrical consideration brings in

$$w(r,t)=r(\partial/\partial x)w(r,t)+f(t) \quad (2)$$

Here, $f(t)$ is the fluctuation in the displacement at the base of the arm. The equilibrium of moment around the motor shaft is

$$\begin{aligned}J_m(\partial^3/\partial t^2\partial x)w(r,t)=\;&EI(1+c\partial/\partial t)(\partial^2/\partial x^2)w(r,t)\\&-\varepsilon(\partial^2/\partial t\partial x)w(r,t)\\&-rEI(1+c\partial/\partial t)(\partial^3/\partial x^3)w(r,t)\\&+T\end{aligned} \quad (3)$$

Here, J_m is the polar moment of inertia of the motor shaft, and ε is the armature damping coefficient. The torque T applied by the motor is given by

$$T=K_t i_a \quad (4)$$

where i_a is the armature current and K_t the torque constant of the motor.

At the other end ($x=L$), the arm is fitted with a payload of mass M_p and moment of inertia J_p. The equilibria of moment and force in this case are

$$\begin{aligned}J_p(\partial^3/\partial t^2\partial x)w(L,t)=\;&-EI(1+c\partial/\partial t)(\partial^2/\partial x^2)w(L,t)\\&-r_pEI(1+c\partial/\partial t)(\partial^3/\partial x^3)w(L,t)\end{aligned} \quad (5)$$

$$M_p(\partial^2/\partial t^2+r_p\partial^3/\partial t^2\partial x)w(L,t)=EI(1+c\partial/\partial t)(\partial^3/\partial x^3)w(L,t) \quad (6)$$

Here, r_p is the distance between the arm tip and the gravity center of payload.

To put the arm motion under control, one has to prescribe the armature current i_a in the motor circuit. As mentioned about, the motor is driven so as to make the arm tip stay precisely at its initial position even if the base is disturbed by the lateral fluctuation. To this end, one compares the initial tip position $w_d(=0)$ with the actual position which is measured by a sensor fixed in space isolated from the system vibration. The tip position error is used, together with the estimated tip velocity, as the basis for applying torque to the arm base through the motor. The equation of i_a to be controlled is

$$\begin{aligned}(L_a/R_a)(d/dt)i_a+i_a&+(K'/R_a)(\partial^2/\partial t\partial x)w(r,t)\\&=[G_d+G_v(\partial/\partial t)]\\&\times\{w_d-[1+r_p(\partial/\partial x)]w(L,t)\}\end{aligned} \quad (7)$$

Here, L_a is the motor inductance, R_a is the circuit resistance, and K' is the back electromotive force constant, G_d and G_v are the displacement and the velocity feedback gains of the servo loop.

Equations (1)-(7) are the governing equations for the problem under consideration. In the following, one will introduce the method of Laplace transform w.r.t. time to solve the equations. First one transforms equation (1) assuming that the arm is resting statically at $t=0$. Solving the resulted equation, one has the transformed displacement in the form

$$W(x,s)=\alpha\sin\xi x+\beta\cos\xi x+\gamma\sinh\xi x+\delta\cosh\xi x \quad (8)$$

where

$$\xi^4=-\rho As^2/EI(1+cs)$$

Here, s is the Laplace transform parameter, α to δ are unknowns to be determined from the boundary conditions. Substituting equation (8) into the transformed equations of (2)-(6), and further introducing the transformed current I_a obtained from equation (7) into the resulted equations, one has a system of simultaneous algebraic equations of the form

$$[a_{ij}][\alpha,\beta,\gamma,\delta]^T=[F(s),0,0,0]^T \quad (9)$$

where $F(s)$ is the transformed lateral displacement of the arm base, $a_{ij}, i,j=1,\cdots,4$, are given in the Appendix. Substituting α to δ determined from equation (9) into equation (8) results in

$$W(x,s)=(\Delta_\alpha\sin\xi x+\Delta_\beta\cos\xi x+\Delta_\gamma\sinh\xi x+\Delta_\delta\cosh\xi x)(1/\Delta)F(s) \quad (10)$$

Here, $\Delta=\det(a_{ij})$; Δ_α to Δ_δ are the Δ's with the first to fourth columns replaced by $[1,0,0,0]^T$.

To get the final result, one needs to fix the function $F(s)$. Two cases are considered. The first case is the one in which the base is fluctuated as

$$\begin{aligned}f(t)&=F_0\sin^2\omega t,\quad 0\leq t\leq\pi/2\omega\\&=F_0 \quad\quad ,\ \pi/2\omega<t\end{aligned} \quad (11)$$

The solution to this input is

$$\begin{aligned}W(x,s)/F_0=\;&(\Delta_\alpha\sin\xi x+\Delta_\beta\cos\xi x+\Delta_\gamma\sinh\xi x+\Delta_\delta\cosh\xi x)(1/\Delta)\\&\times\{(2\omega)^2/2s[s^2+(2\omega)^2]\}\\&\times[1+\exp(-\pi s/2\omega)]\end{aligned} \quad (12)$$

The second case is the one in which the fluctuation is given by

$$f(t)=F_0\sin^2\omega t,\quad 0\leq t\leq \pi/\omega$$
$$=0\qquad ,\quad \pi/\omega<t \tag{13}$$

The corresponding solution is

$$W(x,s)/F_0=(\Delta_\alpha \sin\xi x+\Delta_\beta \cos\xi x$$
$$+\Delta_\gamma \sinh\xi x+\Delta_\delta \cosh\xi x)(1/\Delta)$$
$$\times\{(2\omega)^2/2s[s^2+(2\omega)^2]\}$$
$$\times[1-\exp(-\pi s/\omega)] \tag{14}$$

In the numerical examples that follow, equations (12) and (14) are inversed numerically by using the computer, following the method proposed by Weeks (1966).

3 NUMERICAL RESULTS AND DISCUSSIONS

The control strategy described thus far has been examined by means of simulations for the single-link flexible-arm prototype in the Department of Mechanical Engineering of Tohoku University. The arm is an aluminum beam with a rectangular cross-section of thickness B and width H. The end-point load is a disk of radius r_p and mass M_p. In this case, the moment of inertia of payload about the diameter is given by $Jp=M_p r_p{}^2/4$. The physical parameters for the assembled system are as follows.

Flexible arm

$E=6.57\times10^{10}$ Pa

$\rho=2.67\times10^3$ kg/m^3

$c=1.19\times10^{-4}$ s

$B=1.19\times10^{-2}$ m

$H=1.99\times10^{-3}$ m

$L=5.00\times10^{-1}$ m

D.C.servomotor

$J_m=4.90\times10^{-6}$ kgm^2

$\varepsilon=2.60\times10^{-3}$ kgm^2/s

$L_a=1.80\times10^{-3}$ H

$R_a=4.70\Omega$

$K'=1.77\times10^{-5}$ Vs/rad

$K_t=6.77\times10^{-2}$ Nm/A

$r=2.00\times10^{-2}$ m

Tip load

$M_p=4.53\times10^{-2}$ kg

$r_p=2.00\times10^{-2}$ m (15)

Figure 2 shows the base fluctuations given by equations (11) and (13). (a) is an example of the transient lateral shift of the base,and (b) a typical impulsive fluctuation.

Figures 3 through 6 show the response of the arm when the fluctuation is given by Figure 2(a). Figure 3 is the variation of the tip displacement with an increasing displacement feedback gain G_d, with the velocity feedback gain G_v fixed at 0.7 As/m. The position $w=0$ is the commanded, initial tip position. The motor rotates the arm base so that the arm tip remains at its initial position. Figure shows that the recovering of the tip to the commanded position is not improved even if the displacement feedback gain is increased, further an excess value of the gain even worsens the convergence of the tip vibration. Figure 4 shows the variation when the velocity feedback gain is increased. It is evident that the arm tip comes to the initial position more rapidly with an increase of G_v. However, the excess value induces a higher mode vibration on the arm as observed from the bottom figure, which then interrupts on the convergence of tip fluctuation. Hence it is said that there are optimum values for the feedback gains G_d and G_v to achieve rapid shift of the arm tip to its commanded position.

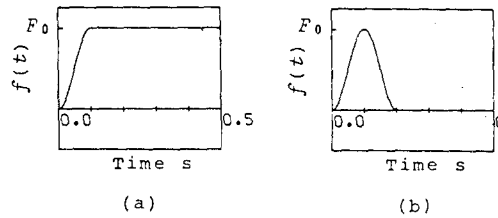

Fig.2 Fluctuation of arm base. $\omega=5\pi$; (a) $f(t)=F_0\sin^2\omega t$ $(0\leq t\leq\pi/2\omega)$, $=F_0$ $(\pi/2\omega<t)$; (b) $f(t)=F_0\sin^2\omega t$ $(0\leq t\leq\pi/\omega)$, $=0$ $(\pi/\omega<t)$.

Figure 5 shows the time variation of tip displacement when the payload mass M_p is increased, where λ is the mass ratio of the payload to the arm given by $\lambda=M_p/\rho A(L-r)$. It is founded that an increased of M_p reduces the amplitude of tip fluctuation but promotes the duration of the vibration.

Figure 6 shows the time response of the arm tip with the rate constant ω taken as a parameter. The speed of shift at the arm base becomes greater with an increase of ω . It is seen that the settling time of arm is not affected by ω . A higher structural mode appears and the tip vibration amplitude becomes greater with an increase of ω .

Figures 7 to 10 show the time history of the tip displacement when the arm base is fluctuated as equation (13). The tip comes to the final position in the similar manner as the one observed in figures 3 to 6 for a variation of the parameter G_d, G_v, λ and ω , which means that the response of the arm tip controlled by the present algorithm is not much affected by the type of disturbance fluctuating the base of the arm.

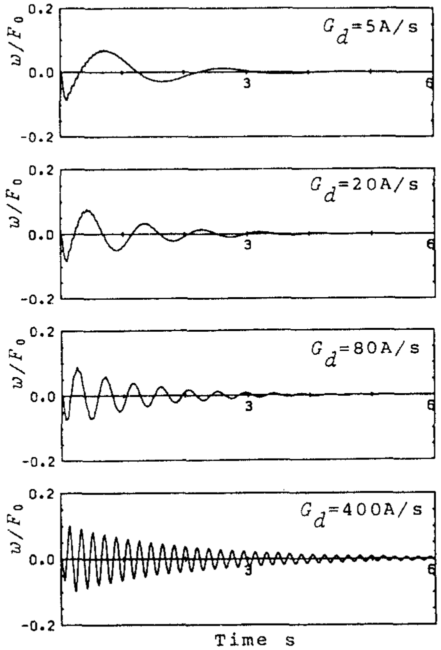

Fig.3 Variation of arm tip displacement with an increase of displacement feedback gain G_d. G_v=0.7As/m; $\omega=5\pi$; $f(t)=F_0\sin^2\omega t$ $(0\leqq t\leqq\pi/2\omega)$, $=F_0$ $(\pi/2\omega<t)$.

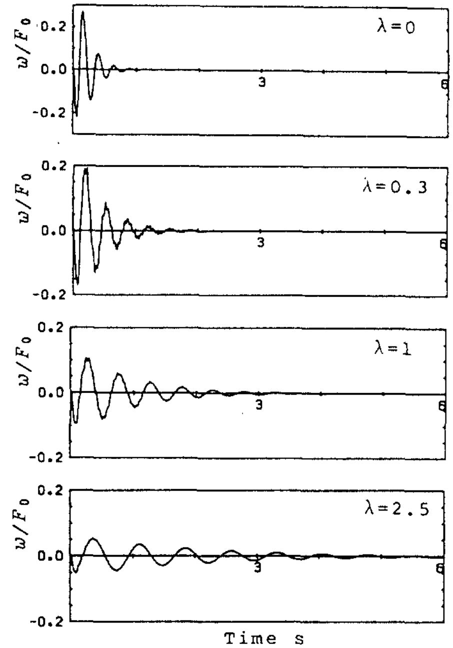

Fig.5 Variation of arm tip displacement with an increase of payload mass M_p. Physical parameters (except M_p) are as shown in equation (15). $\lambda=M_p/\rho A(L-r)$; G_d=50A/m; G_v=0.7As/m; $\omega=5\pi$; $f(t)=F_0\sin^2\omega t$ $(0\leqq t\leqq\pi/2\omega)$, $=F_0$ $(\pi/2\omega<t)$.

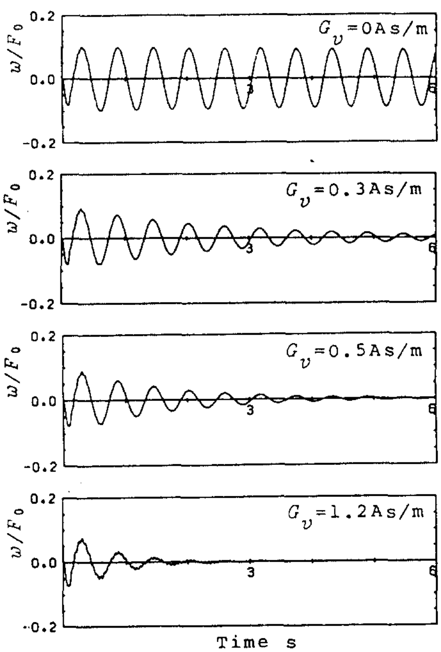

Fig.4 Variation of arm tip displacement with an increase of velocity feedback gain G_v. G_d=50A/m; $\omega=5\pi$; $f(t)=F_0\sin^2\omega t$ $(0\leqq t\leqq\pi/2\omega)$, $=F_0$ $(\pi/2\omega<t)$.

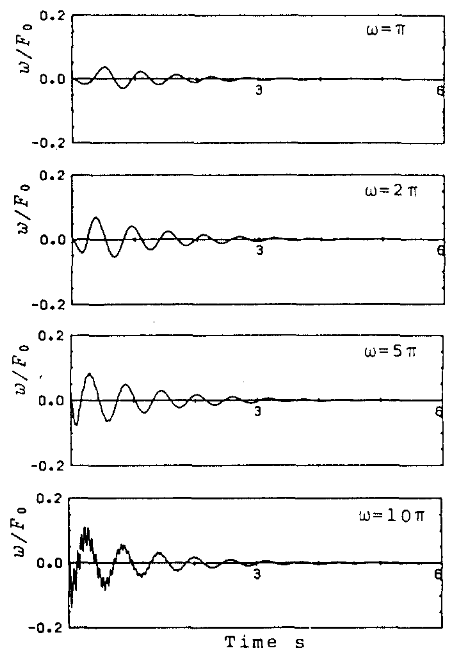

Fig.6 Variation of arm tip displacement with an increase of rate constant ω. G_d=50A/m; G_v=0.7As/m; $f(t)=F_0\sin^2\omega t$ $(0\leqq t\leqq\pi/2\omega)$, $=F_0$ $(\pi/2\omega<t)$.

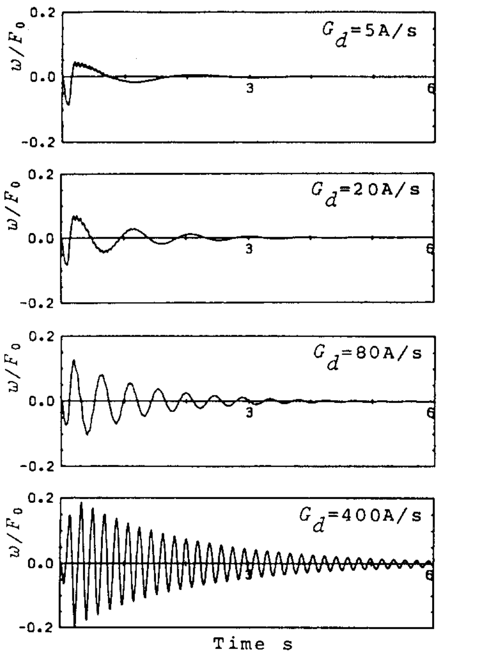

Fig.7 Variation of arm tip displacement with an increase of displacement feedback gain G_d. G_v=0.7As/m; $\omega=5\pi$; $f(t)=F_0\sin^2\omega t$ ($0\leq t\leq\pi/\omega$), $=0$ ($\pi/\omega<t$).

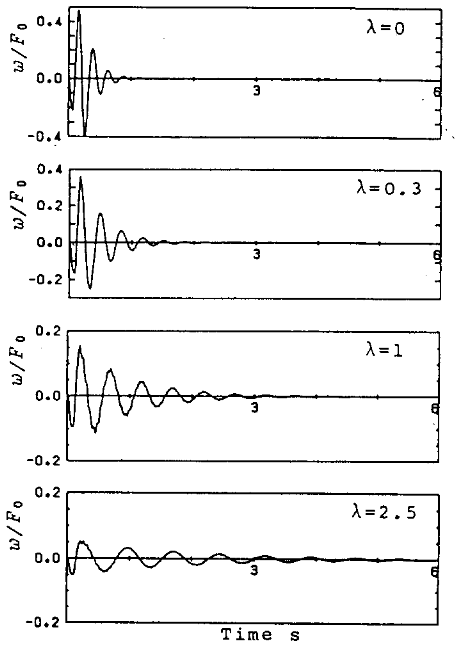

Fig.9 Variation of arm tip displacement with an increase of payload mass M_p. Physical parameters (except M_p) are as shown in equation (15). $\lambda=M_p/\rho A(L-r)$; G_d=50A/m; G_v=0.7As/m; $\omega=5\pi$; $f(t)=F_0\sin^2\omega t$ ($0\leq t\leq\pi/\omega$), $=0$ ($\pi/\omega<t$).

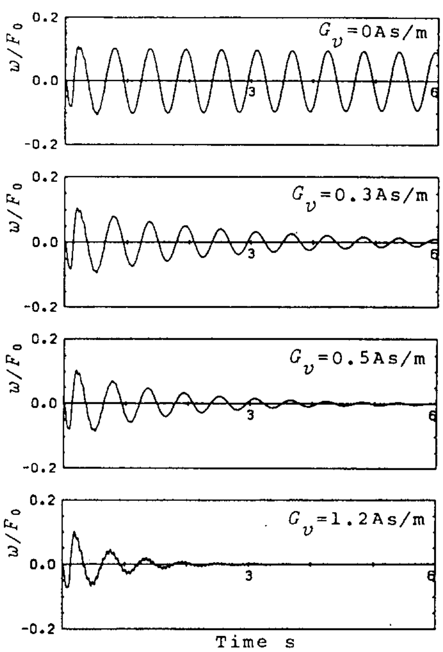

Fig.8 Variation of arm tip displacement with an increase of velocity feedback gain G_v. G_d=50A/m; $\omega=5\pi$; $f(t)=F_0\sin^2\omega t$ ($0\leq t\leq\pi/\omega$), $=0$ ($\pi/\omega<t$).

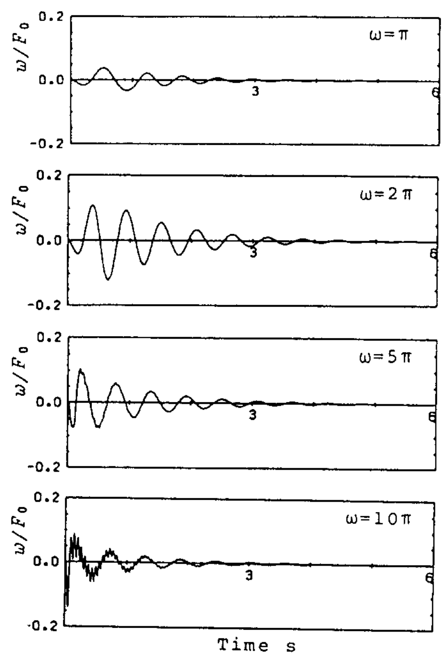

Fig.10 Variation of arm tip displacement with an increase of rate constant ω. G_d=50A/m; G_v=0.7As/m; $f(t)=F_0\sin^2\omega t$ ($0\leq t\leq\pi/\omega$), $=0$ ($\pi/\omega<t$).

4 CONCLUSIONS

A control strategy has been tested for the end point holding of a one-link flexible arm whose base is distributed by an unsteady lateral fluctuation. The flexible-arm prototype at the Department of Mechanical Engineering of Tohoku University has been chosen for developing a simulation study. Results obtained can be summarized as follows.

(1) PD control using the endpoint position sensing and base torquing is sufficient to make the arm tip stay at its initial position even if the base is disturbed by the lateral fluctuation.

(2) For the arm controlled by the present strategy, the time response of the arm tip is not much affected by the type of fluctuation disturbing the base of the arm.

APPENDIX

The a_{ij}'s in equation (9) are given as

$$a_{11}=\sin\xi r-\xi r\cos\xi r$$

$$a_{12}=\cos\xi r+\xi r\sin\xi r$$

$$a_{13}=\sinh\xi r-\xi r\cosh\xi r$$

$$a_{14}=\cosh\xi r-\xi r\sinh\xi r$$

$$\begin{aligned}a_{21}=\{&[(L_a/R_a)s+1](J_m s+\varepsilon)\\&+K_tK'/R_a\}s\xi\cos\xi r\\&+EI[(L_a/R_a)s+1](1+cs)\xi^2\\&\times(\sin\xi r-\xi r\cos\xi r)\\&+K_t(G_d+G_v s)\\&\times(\sin\xi L+\xi r_p\cos\xi L)\end{aligned}$$

$$\begin{aligned}a_{22}=-\{&[(L_a/R_a)s+1](J_m s+\varepsilon)\\&+K_tK'/R_a\}s\xi\sin\xi r\\&+EI[(L_a/R_a)s+1](1+cs)\xi^2\\&\times(\cos\xi r+\xi r\sin\xi r)\\&+K_t(G_d+G_v s)\\&\times(\cos\xi L-\xi r_p\sin\xi L)\end{aligned}$$

$$\begin{aligned}a_{23}=\{&[(L_a/R_a)s+1](J_m s+\varepsilon)\\&+K_tK'/R_a\}s\xi\cosh\xi r\\&-EI[(L_a/R_a)s+1](1+cs)\xi^2\\&\times(\sinh\xi r-\xi r\cosh\xi r)\\&+K_t(G_d+G_v s)\\&\times(\sinh\xi L+\xi r_p\cosh\xi L)\end{aligned}$$

$$\begin{aligned}a_{24}=\{&[(L_a/R_a)s+1](J_m s+\varepsilon)\\&+K_tK'/R_a\}s\xi\sinh\xi r\\&-EI[(L_a/R_a)s+1](1+cs)\xi^2\\&\times(\cosh\xi r-\xi r\sinh\xi r)\\&+K_t(G_d+G_v s)\\&\times(\cosh\xi L+\xi r_p\sinh\xi L)\end{aligned}$$

$$\vdots$$

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