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Iterative Adaptive Control of Partially Known System Under Tight Servo Constraints

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A new sufficient condition for the convergency of an iterative adaptive control algorithm is presented, in which a parameter estimator of the system together with an inverse system model to generate the control signal at each iteration. Also the result is extended to discrete time domain and a similar sufficient condition is derived.

1. Introduction

Adaptive techniques have been used when the plant dynamics are not fully known, but the output trajectory is not specified.

The iterative control method, which was proposed by Uchiyama[1] and later elaborated as a more formal theory by Arimoto et al.[2], also has been receving a lot of attention as a means of controlling uncertain dynamic systems. The proposed simple algorithm which is called 'Betterment' process is of the form

$$u_{k+1}(t) = u_k(t) + [\Gamma(t)\ 0][\dot{e}_k^T(t)\ e_k^T]^T \ , 0 \le t \le T$$

here $u_k(t)$ is the control at k-th iteration and $e_k(t)$ denotes the output error. This control method was reported to be applicable, for example, for controlling of a non-linear robot manipulator with repetitive tasks. In Arimoto et al.[2] the iterative control strategy was proved to be convergent if the controller gain $\Gamma(t)$ is properly chosen so that an inequality relation involving system matrices holds. Thus their approach may be unsuccessful unless certain specific knowledge about the system dynamics is given. We may say that the use of identical gain function $\Gamma(t)$ at each iteration implies that the 'Betterment' process itself is not 'adaptive' and that since any accepted knowledge is contained only in $u_k(t)$, the iterative algorithm can be easily disturbed by external noise. In fact, when we use the iterative algorithm, we must perform different trials for any new trajectory and their algorithm is sensitive to external noise.

Oh et al.[3] proposed a new iterative control method for a class of linear periodic continuous systems, in which the term corresponding to the controller gain $[\Gamma(t)\ 0]$ is determined adaptively in conjunction with the built-in system parameter estimator. Their method, in which the accepted knowledge is distributed both to $u_k(t)$ and to $[\Gamma(t)\ 0]=[\bar{B}_k(t)\ -\bar{B}_k(t)\bar{A}_k(t)]$, is not only efficient in convergence but also robust to disturbances in comparison with existing methods in[2][5]. A drawback of the method in [3] is, however, that the inequality checking condition is expressed in terms of linear operator norm. As a consequence, it is not easy to examine if the system and the estimatated system satisfy the sufficient condition and is not obvious how the method can be applied to a class of discrete-time dynamic systems.

In this paper, a refined form of a sufficient condition for convergency is provided when adaptive iterative control method is adopted as in [3]. The condition is more specific than the result in [3] and expressed in terms of system matrix and estimated system matrix. This result enables us to know if the system under control satisfies the sufficient condition and the assumption is weaker than that in [3]. Also a discrete-time version of the algorithm for digital control of a class of linear periodic discrete system as well.

In the sequel, given a matrix B, B^T denotes the transpose of B and B^+ implies the generalized inverse of B, respectively[7]. For an n-dimensional vector x, $\|x\|$ denotes the Euclidean norm and $\|x\|_\infty$ implies the sup norm,i.e.,

$$\|x\|_\infty = \max_{1\le i\le n} |x^i| \text{ when } x = (x^1,\cdots,x^n)$$

For an $n\times r$ matrix G whose entries are defined as g^{ij},

$$\|G\|_\infty = \max_{1\le i\le n} \{\sum_{j=1}^{r} |g^{ij}|\}$$

Also given a time function $h:[0,T] \to R^n$, let

$$\|h(\cdot)\|_\lambda = \sup_{t\in[0,T]} e^{-\lambda t} \|h(t)\|_\infty$$

2. Iterative Adaptive Control Method for a Class of Linear Periodic Continuous Systems

In this section, a new sufficient condition for convergency is discussed when the algorithm in [3] is adopted. Thus firstly the same problem, parameter estimator and iterative control algorithm are presented as those in [3] and then the convergency is shown.

Consider the linear continuous periodic system described by

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) \tag{1}$$

$$x(0) = \xi^0 \tag{2}$$

where x and u are an nx1 state vector and mx1 control vector, respectively. The nxn matrix function $A(t)$ and the nxm matrix function $B(t)$ are assumed to contain unknown parameters but are known to be continuous and periodic with period T such that

$$A(t+T) = A(t)$$
$$B(t+T) = B(t)$$

Now consider the following problem(P1)

Problem(P1) :
Let $x_d(t)$, $0 \le t \le T$, denote the given desired state trajectory. Let $\epsilon^* > 0$ be a given tolerance bound. Find a control function $u(t)$, $0 \le t \le T$, such that the corresponding state trajectory $x(t)$ of the linear system in (1) with initial condition (2) satisfies

$$p(x(t)) = \|x(t) - x_d(t)\|_\infty \le \epsilon^* \ , 0 \le t \le T \qquad (3)$$

In order to solve the problem(P1) stated above we adopt parameter estimator and the iterative control algorithm proposed by Oh et al.[3] which are given below.

Parameter Estimator :
Let the matrices $A(t)$ and $B(t)$ in (1) be written as

$$A(t) = \begin{bmatrix} a^1(t) \\ . \\ . \\ a^n(t) \end{bmatrix} \text{ and } B(t) = \begin{bmatrix} b^1(t) \\ . \\ . \\ b^n(t) \end{bmatrix}$$

Define 1x$(n+m)$ vectors $\theta^i(t)$ and $\psi(t)$ be as

$$\theta^{i^T}(t) = [a^i(t) \ b^i(t)]$$
$$\psi(t) = [x^T(t) \ u^T(t)]^T \qquad (4)$$

For each fixed time $\hat{t} \in [0,T]$, let

$$x^T(\hat{t}) = [x^1(\hat{t}), x^2(\hat{t}), \cdots, x^n(\hat{t})]$$
$$y(\hat{t}) = \dot{x}(\hat{t})$$

such that the ith component of $y(\hat{t})$ is given by

$$y^i(\hat{t}) = \dot{x}(\hat{t}), \text{ for } i = 1,2,\cdots,n$$

Then the system in (1) can be described by

$$y^i(\hat{t}) = \theta^{i^T}(\hat{t})\psi(\hat{t}) \quad , \text{for } i = 1,2,3,\cdots,n$$

Let the estimated parameter vector $\bar{\theta}_k^i(t)$ in the kth operation be given by

$$\bar{\theta}_k^i(\hat{t}) = \bar{\theta}_{k-1}^i(\hat{t}) + F_k^i(\hat{t})[y_k^i(\hat{t}) - \theta_{k-1}^{i^T}(\hat{t})\psi_k(\hat{t})] \qquad (5)$$

where

$$F_k^i(\hat{t}) = \frac{S_{k-1}^i(\hat{t})\psi_k(\hat{t})}{\dfrac{1}{\alpha_k^i(\hat{t})} + \psi_k^T(\hat{t})S_{k-1}^i(\hat{t})\psi_k(\hat{t})} \qquad (6)$$

and

$$S_k^i(\hat{t}) = S_{k-1}^i(\hat{t}) - \frac{S_{k-1}^i(\hat{t})\psi_k(\hat{t})\psi_k^T(\hat{t})S_{k-1}^i(\hat{t})}{\dfrac{1}{\alpha_k^i(\hat{t})} + \psi_k^T(\hat{t})S_{k-1}^i(\hat{t})\psi_k(\hat{t})} \qquad (7)$$

Algorithm :
Let the initial condition $u_0(t)$, $0 \le t \le T$, be given as an m-vector continuous function. Also let the initial modeled system matrices $A_0(t)$ and $B_0(t)$, $0 \le t \le T$, be given as continuous matrices on $[0,T]$. Let

$$e_k(t) = x_d(t) - x_k(t), \ 0 \le t \le T \qquad (8)$$
$$\bar{u}_k(t) = \bar{B}_k^+(t)[\dot{e}_k(t) - \bar{A}_k(t)e_k(t)] \qquad (9)$$
$$u_{k+1}(t) = u_k(t) + \bar{u}_k(t) \qquad (10)$$

The structure of the above algorithm is schematically shown in Fig.1 and is shown to be convergent in the following statement.

Theorem 1

Consider the linear periodically time-varying continuous system in eqn.(1). If the system in eqn.(1) is totally stable and if the estimated system given in eqn.(5)~(7) satisfies

$$\|I - \bar{B}_k^+(t)B(t)\|_\infty < 1 \qquad (11)$$

then the adaptive iterative controller in eqn.(8)~(10) with $x_k(0) = x_d(0) = \xi^0$ for $k=0,1,\cdots$, yields

$$\lim_{k \to \infty} \|e_k(t)\|_\infty = 0$$

Proof of Theorem 1
Let $u_d(t)$ be a control input that yields the desired output state trajectory $x_d(t)$. It follows from eqns. (1),(8),(9),(10)

$$\begin{aligned} u_d(t) - u_{k+1}(t) &= u_d(t) - u_k(t) - \bar{B}_k^+(t)[\dot{e}_k(t) - \bar{A}_k(t)e_k(t)] \\ &= \{I - \bar{B}_k^+(t)B(t)\}\{u_d(t) - u_k(t)\} - \\ &\quad \bar{B}_k^+(t)\{A(t) - \bar{A}_k(t)\}\{x_d(t) - x_k(t)\} \end{aligned} \qquad (12)$$

Computing the norms of both sides, we obtain,

$$\begin{aligned} \|u_d(t) - u_{k+1}(t)\|_\infty &\le \|I - \bar{B}_k^+(t)B(t)\|_\infty \|u_d(t) - u_k(t)\|_\infty \\ &+ \|\bar{B}_k^+(t)\{A(t) - \bar{A}_k(t)\}\|_\infty \|x_d(t) - x_k(t)\|_\infty \end{aligned} \qquad (13)$$

for all $t \in [0,T]$

Now, because $x_k(0) = x_d(0)$ for all k, we have, for $t \in [0,T]$,

$$\begin{aligned} &\|x_d(t) - x_k(t)\|_\infty \\ &\le \int_0^t \{\|A(\tau)\|_\infty \|x_d(\tau) - x_k(\tau)\|_\infty \\ &+ \|B(\tau)\|_\infty \|u_d(\tau) - u_k(\tau)\|_\infty\} d\tau \end{aligned} \qquad (14)$$

Applying the Bellman-Gronwall lemma[4], we obtain

$$\|x_d(t) - x_k(t)\|_\infty \le \int_0^t \|B(\tau)\|_\infty \|u_d(\tau) - u_k(\tau)\|_\infty e^{a(t-\tau)} d\tau \qquad (15)$$

for all $t \in [0,T]$, where $a = \|A(t)\|_\infty$, for all $t \in [0,T]$

Therefore, combining eqn.(13) and eqn.(15), we see that, with $\delta u_k(t) = u_d(t) - u_k(t)$

$$\begin{aligned} \|\delta u_{k+1}(t)\|_\infty &\le \|I - \bar{B}_k^+(t)B(t)\|_\infty \|\delta u_k(t)\|_\infty \\ &+ \|\bar{B}_k^+(t)\{A(t) - \bar{A}_k(t)\}\|_\infty \\ &\int_0^t \|B(\tau)\|_\infty \|\delta u_k(\tau)\|_\infty e^{a(t-\tau)} d\tau \end{aligned} \qquad (16)$$

Multiplying eqn.(16) by the positive function $e^{-\lambda t}$, we have

$$\begin{aligned} e^{-\lambda t}\|\delta u_{k+1}(t)\|_\infty &\le \|I - \bar{B}_k^+(t)B(t)\|_\infty e^{-\lambda t}\|\delta u_k(t)\|_\infty \\ &+ \|\bar{B}_k^+(t)\{A(t) - \bar{A}_k(t)\}\|_\infty e^{-\lambda t} \int_0^t \|B(\tau)\|_\infty \|\delta u_k(\tau)\|_\infty e^{a(t-\tau)} d\tau \end{aligned}$$

for all $t \in [0,T]$

$$\begin{aligned} &\le \|I - \bar{B}_k^+(t)B(t)\|_\infty e^{-\lambda t}\|\delta u_k(t)\|_\infty \\ &+ \|\bar{B}_k^+(t)\{A(t) - \bar{A}_k(t)\}\|_\infty \\ &b\int_0^t e^{-\lambda\tau}\|\delta u_k(\tau)\|_\infty e^{(a-\lambda)(t-\tau)} d\tau \end{aligned} \qquad (17)$$

for all $t\in[0,T]$, where $b=\|B(t)\|_\infty$, for all $t\in[0,T]$

Therefore

$$\|\delta u_{k+1}(\cdot)\|_\lambda \le \|I-\bar{B}_k^+(t)B(t)\|_\infty \|\delta u_k(\cdot)\|_\lambda + b\,\|\bar{B}_k^+(t)\{A(t) -\bar{A}_k(t)\}\|_\infty [\frac{1-e^{(a-\lambda)T}}{\lambda-a}]\|\delta u_k(\cdot)\|_\lambda \text{ for } \lambda\ne a \tag{18}$$

From the assumption $\|I-\bar{B}_k^+(t)B(t)\|_\infty<1$, we can choose $\lambda>0$ large enough so that

$$\|I-\bar{B}_k^+(t)B(t)\|_\infty + b\,\|\bar{B}_k^+(t)\{A(t) -\bar{A}_k(t)\}\|_\infty[\frac{1-e^{(a-\lambda)T}}{\lambda-a}]<1 \tag{19}$$

Thus, $\|\delta u_k(\cdot)\|_\lambda \to 0$ as $k\to\infty$.
By the definition of $\|\cdot\|_\lambda$, we have that

$$\sup_{t\in[0,T]} \|\delta u_k(t)\|_\infty \le e^{\lambda T}\|\delta u_k(\cdot)\|_\lambda$$

Therefore,

$$\sup_{t\in[0,T]} \|\delta u_k(t)\|_\infty \to 0 \text{ as } k\to\infty$$

which means that

$$u_k(t)\to u_d(t) \text{ as } k\to\infty \text{ on } t\in[0,T]$$

Furthermore eqn.(10) implies

$$x_k(t)\to x_d(t) \text{ as } k\to\infty \text{ on } t\in[0,T]$$

This completes the proof

Remark 1
Comparing the above Theorem1 with Theorem1 in Oh et al.[3], it is observed that the inequality relation eqn.(11) is weaker and easier to check as in [2] and is related only to input matrix. In [3], it is assumed that parameter estimation scheme is convergent and as a result of the assumption, it is argued the relation of the operator norms holds the inequality. But from the eqn.(11) and in the process of the proof of the above Theorem1, the fact can be known that even though the parameter estimation scheme doesn't converge to real parameter, the iterative control algorithm converges. For example, $\bar{A}_k(t)$ converges to arbitrary bounded value and $\bar{B}_k(t)$ converges any value which holds eqn.(11).

Remark 2
If we set the output vector $y(t)=I\cdot x(t)$, then the order difference between the input vector and the output vector is 1 and $\dot{y}(t)=\dot{x}(t)=A(t)x(t)+B(t)u(t)$. Thus $B(t)$ is a direct transmission term from $u(t)$ to $\dot{y}(t)$. This observation is in agreement with the assertion made by Sugie et al.[5], where they reported that direct transmission term plays a crucial role in the error convergency proof.

Remark 3
Note that in compared above algorithm with Arimoto's, we can set

$$\dot{u}_k(t) = [\bar{B}_k^+(t) \quad -\bar{B}_k^+(t)\bar{A}_k(t)][\dot{e}_k^T(t)\ e_k^T(t)]^T$$

in the above algorithm and

$$\dot{u}_k(t) = [\Gamma(t)\ 0][\dot{e}_k^T(t)\ e_k^T(t)]^T$$

in Arimoto's and thus know that $\bar{B}_k^+(t)$ plays the same role as the gain $\Gamma(t)$ and $-\bar{B}_k^+(t)\bar{A}_k(t)$ is zero in Arimoto's. In eqn.(16) if $\bar{A}_k(t)$ equals to $A(t)$, then the second term of right hand side will disappear and particularly in Arimoto's this term appears always. Thus it can be expected that if the parameter estimator is guatanted to be convergent, then the above algorithm will be superior to Arimoto's in convergence speed. This obervation can explain the Oh's computer simulation results roughly.

3. Iterative Adaptive Control Method for a Class of Linear Periodic Discrete Systems

In the previous section iterative control algorithm is presented in continuous time domain as in [2]. Since each iteration's data are not memorized continuously, a practical algorithm is implemented only for a finite number of times. Thus the iterative control algorithm is discussed in discrete time domain.

Consider the linear periodic discrete system described by

$$x(i+1) = A(i)x(i) + B(i)u(i) \tag{20}$$

$$x(0) = \xi^0 \tag{21}$$

where x, u, $A(i)$ and $B(i)$ are assumed to be the same size as in Sec.2. and $A(i)$ and $B(i)$ are assumed to contain uncertain parameters but are to be periodic such that

$$A(i+N) = A(i)$$

$$B(i+N) = B(i)$$

Now consider the problem(P1) in discrete-time domain and in order to solve the problem parameter estimator in [3] is adopted but fixed time $\hat{t}$ is replaced to fixed time $\hat{i}$ and $\theta^{\hat{i}^T}(i)$ and $\psi(i)$ are similarly defined in terms of components of eqn.(20) and the discrete iterative control algorithm is given below.
Let

$$e_k(i) = x_d(i) - x_k(i)\ ,\ i=0,1,\cdots,N \tag{22}$$

$$u_{k+1}(i) = u_k(i) + [\bar{B}_k^+(i) \quad -\bar{B}_k^+(i)\bar{A}_k(i)][e_k^T(i+1)\ e_k^T(i)]^T \tag{23}$$

The above algorithm is shown to be convergent in the following statement.

Theorem 2
Consider the linear periodically time-varying discrete system in eqn.(20). If the system in eqn.(20) is totally stable and if the estimated system satisfies the condition such that eigenvalues of matrix $I-\bar{B}_k^+(i)B(i)$ are all in the unit circle of the complex plane, then the iterative controller in eqns.(22)~(23) with $x_k(0) = x_d(0) = \xi^0$ for $k=0,1,\cdots$, yields

$$\lim_{k\to\infty}\|e_k(i)\|_\infty = 0$$

Proof of Theorem 2
Let $u_d(i)$ be a control input that yields the desired output state trajectory $x_d(i)$. It follows from eqns. (20),(22),(23)

$$\begin{aligned} u_d(i) - u_{k+1}(i) &= u_d(i) - u_k(i) - \bar{B}_k^+(i)\{e_k(i+1)-\bar{A}_k(i)e_k(i)\} \\ &= u_d(i)-u_k(i)-\bar{B}_k^+(i)\{A(i)-\bar{A}_k(i)\}e_k(i) \\ &\quad -\bar{B}_k^+(i)B(i)\{u_d(i)-u_k(i)\} \end{aligned} \tag{24}$$

Let $\delta u_k(i)=u_d(i)-u_k(i)$

$$\begin{aligned} \delta u_{k+1}(i) &= \delta u_k(i)-\bar{B}_k^+(i)\{A(i)-\bar{A}_k(i)\}e_k(i)-\bar{B}_k^+(i)B(i)\delta u_k(i) \\ &= \{I-\bar{B}_k^+(i)B(i)\}\delta u_k(i)-\bar{B}_k^+(i)\{A(i)-\bar{A}_k(i)\}e_k(i) \end{aligned} \tag{25}$$

Consider

$$\begin{aligned} e_i(k) &= x_d(i)-x_k(i) \\ &= A(i-1)\cdots A(1)B(0)\delta u_k(0)+\cdots+B(i-1)\delta u_k(i-1) \end{aligned} \tag{26}$$

Thus

$$\begin{aligned} \delta u_{k+1}(i) = &\{I-\bar{B}_k^+(i)B(i)\}\delta u_k(i) - \bar{B}_k^+(i)\{A(i)-\bar{A}_k(i)\} \\ &\{A(i-1)\cdots A(1)B(0)\delta u_k(0)+\cdots \\ &+B(i-1)\delta u_k(i-1)\} \end{aligned} \tag{27}$$

Expanding eqn.(27) from $i=0$ to $i=N$

$$\delta u_{k+1}(0) = \{I - \bar{B}_k^+(0)B(0)\}\delta u_k(0)$$
$$\delta u_{k+1}(1) = \{I - \bar{B}_k^+(1)B(1)\}\delta u_k(1) - \bar{B}_k^+(1)\{A(1) - \bar{A}_k(1)\}B(0)\delta u_k(0)$$

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$$\delta u_{k+1}(N) = \{I - \bar{B}_k^+(N)B(N)\}\delta u_k(N) - \bar{B}_k^+(N)\{A(N) - \bar{A}_k(N)\}\{A(N-1)\cdots A(i)B(0)\delta u_k(0) + \cdots + B(N-1)\delta u_k(N-1)\}$$

Rewrite above eqns. in matrix eqn. form

$$\begin{bmatrix} \delta u_{k+1}(0) \\ \cdot \\ \cdot \\ \cdot \\ \delta u_{k+1}(N) \end{bmatrix} = \begin{bmatrix} I - \bar{B}_k^+(0)B(0) & 0 & \cdot & 0 \\ * & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 0 \\ * & \cdot & * & I - \bar{B}_k^+(N)B(N) \end{bmatrix} \begin{bmatrix} \delta u_k(0) \\ \cdot \\ \cdot \\ \cdot \\ \delta u_k(N) \end{bmatrix} \tag{28}$$

where $*$ denotes nonzero matrix other than $I - \bar{B}_k^+(i)B(i)$, $i = 0, \cdots, N$

In eqn.(28), if eigenvalues of

$$\begin{bmatrix} I - \bar{B}_k^+(0)B(0) & 0 & \cdot & 0 \\ * & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 0 \\ * & \cdot & * & I - \bar{B}_k^+(N)B(N) \end{bmatrix}$$

are all in the unit circle of the complex plane, then $\delta u_k(i) \to 0$ as $k \to \infty, i = 0, \cdots, N$

Since

$$\begin{bmatrix} I - \bar{B}_k^+(0)B(0) & 0 & \cdot & 0 \\ * & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 0 \\ * & \cdot & * & I - \bar{B}_k^+(N)B(N) \end{bmatrix}$$

is lower triangular block matrix, the above condition is equivalent to the condition such that eigenvalues of $I - \bar{B}_k^+(i)B(i), i = 0, \cdots, N$ are all in the unit circle of the complex plane.

Thus $\delta u_k(i) \to 0$ as $k \to \infty$, $i = 0, \cdots, N$

Therefore from eqn.(26)

$$x_k(i) \to x_d(i) \text{ as } k \to \infty, \; i = 0, \cdots, N$$

This completes the proof.

Remark 4

It can be noted that $B(i)$ is also a direct transmission term from $u(i)$ to $y(i)$ and plays a crucial role in the error convergency proof as in [6].

4. Conclusions

A new sufficient condition for convergency is provided when adaptive iterative control method is adopted as in [3]. Also a discrete-time version of the algorithm for digital control of a class of linear periodic discrete system as well.

It can be observed the sufficient conditions are not related to the matrices $A(i)$ and/or $A(t)$ but to $B(i)$ and/or $B(t)$.

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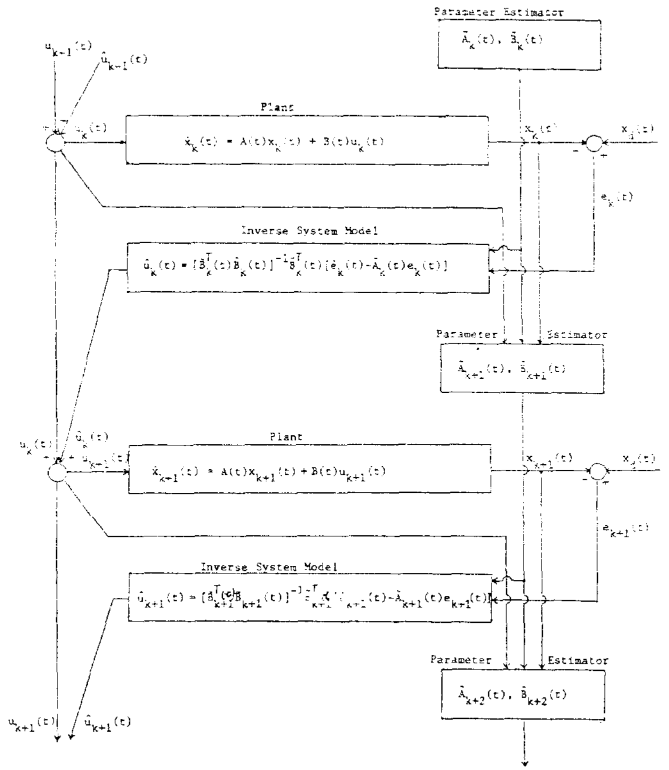

Fig. 1. A schematic diagram of iterative adptive controller