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An Approach for Remote Measurement of Instantaneous Flowrate by Making Use of Hydraulic Pipeline Dynamics

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ABSTRACT This paper describes *a remote measurement method* for estimating unsteady flowrate through a pipeline. By this method, instantaneous flowrate at the remote location along a pipeline (distance L) from flowmeters is measured by making use of dynamic characteristics between two cross sections of the circular pipeline. Using this method, instantaneous flowrate is accurately measured at a location where it is difficult to setup flowmeters. The estimated flowrate waveforms by the method are compared with directly measured ones by cylindrical choke-type instantaneous flowmeter. The validity of the method is established.

NOMENCLATURE

a = inner radius of pipeline, c = velocity of sound in fluid, d = choke diameter, dp = differential pressure across choke, F^{-1} = inverse Fourier transform, f = frequency, Δf = sampling frequency interval, J_0, J_1 = Bessel functions of the first kind, L = pipe length between upstream and downstream section, l = choke length, p = pressure, q = flowrate, s = Laplace operator, t = time, Δt = sampling time interval, $Z_o(s)$ = characteristic impedance of pipeline, $\lambda(s)$ = propagation constant, ρ = density of fluid, ν = kinematic viscosity of fluid, ω = angular frequency.

subscripts

u = upstream section of pipeline, d = downstream section of pipeline.

INTRODUCTION

It is indispensable to measure unsteady flowrate accurately through an arbitrary cross section of hydraulic pipelines and components in order to clarify and improve the dynamic characteristics in hydraulic systems.

Recently, several methods for measuring unsteady flowrate in real time are proposed and developed. (a) A laser Doppler velocimeter-type flowmeter in which unsteady flowrate is estimated by the measured center-line velocity and dynamic relationship between flowrate and center-line velocity under unsteady laminar flow [1],[2],[3]. (b) A cylindrical choke-type instantaneous flowmeter, which is computed unsteady flowrate in real time by making use of the measured differential pressure across the choke and a mathematical model of dynamic characteristics between flowrate and differential pressure in high frequencies region [4], [5]. (c) A measurement method for unsteady flowrate by making use of an analog electronic circuit whose transfer characteristics is equivalent to the approximate transfer function between pressure drop along a long pipeline and mean velocity [6].

In many practical applications, following problems are yet to be solved in the measurement of unsteady flowrate. It is considered that installing a flowmeter at a location, where we are desirous of measuring instantaneous flowrate, is difficult and sometimes is impossible because of the restriction of installation space, the size of flowmeter under high pressure condition. In addition, flow is disturbed due to the installation of a flowmeter, so that the dynamic behaviors of hydraulic components may be affected. In these cases, it is desirable to accurately estimate instantaneous flowrate at the desired location using measured flowrate at the other remote location (distance L) connected to a pipeline.

In this paper, we propose two approaches for estimating instantaneous flowrate at a remote location from flowmeters in real time through an arbitrary cross section of a pipeline under unsteady laminar flow. The estimated flowrate waveforms are compared with directly measured results for rectangular wave inputs.

PRINCIPLE

With reference to a hydraulic pipeline system as shown in **Fig.1**, we consider axisymmetric, slightly compressible, viscous and unsteady laminar liquid flow (e.g. hydraulic oil flow). The hydraulic pipeline dynamics between flowrate and pressure variables at two cross sections (upstream and downstream section) of a pipeline are known to be well described in the forms of transfer matrix (four-terminal networks representations), which are related to the transfer functions in the frequency domain [7].

$$\begin{pmatrix} P_u(s) \\ Q_u(s) \end{pmatrix} = \begin{bmatrix} \cosh\{\lambda(s)\cdot L\} & Z_o(s)\sinh\{\lambda(s)\cdot L\} \\ \sinh\{\lambda(s)\cdot L\}/Z_o(s) & \cosh\{\lambda(s)\cdot L\} \end{bmatrix} \cdot \begin{pmatrix} P_d(s) \\ Q_d(s) \end{pmatrix} \quad (1)$$

$$\lambda(s) = \frac{s}{c}\left\{1 - \frac{2J_1(ja\sqrt{s/\nu})}{ja\sqrt{s/\nu}J_0(ja\sqrt{s/\nu})}\right\}^{-1/2} \quad (2)$$

$$Z_o(s) = \frac{\rho c}{\pi a^2}\left\{1 - \frac{2J_1(ja\sqrt{s/\nu})}{ja\sqrt{s/\nu}J_0(ja\sqrt{s/\nu})}\right\}^{-1/2} \quad (3)$$

where $\lambda(s)$ is referred to propagation constant and $Z_o(s)$ is the characteristic impedance of the pipeline, for a dissipative pipeline model which is included viscous effect dependent on frequencies [8]. $P(s)$ and $Q(s)$ denote Laplace transforms of the pressure and flowrate derivations at the upstream and downstream section of the pipeline,respectively.

By utilizing the dynamic relationships of the pipeline, it is possible that one unknown variable among $P_u(s)$, $Q_u(s)$, $P_d(s)$, and $Q_d(s)$ in Fig.1 is estimated by measuring and using other two variables (pressure or flowrate at a section of the pipeline.

Based on the above mentioned principle, two approaches are provided to estimate instantaneous flowrate without installing a flowmeter at a desired location to measure instantaneous flowrate. Hence, we propose following two approaches (*i.e. remote instantaneous flowrate measurement method and quasi-remote instantaneous flowrate measurement method*) for estimating instantaneous flowrate at the remote location from flowmeters in real time under unsteady laminar flow.

1) *Remote instantaneous flowrate measurement method*: instantaneous flowrate at one section of a pipeline (for instance $q_u(t)$) can be estimated by utilizing the hydraulic pipeline dynamics and the informations of pressure and flowrate ($p_d(t)$ and $q_d(t)$) at the other remote end (distance L) of the pipeline.

2) *Quasi-remote instantaneous flowrate measurement method*: instantaneous flowrate (for instance $q_u(t)$) is estimated in real time using the measured values of pressure at a desired location to get flowrate information ($p_u(t)$) and flowrate at the other remote location ($q_d(t)$). In this case, it is necessary to measure the pressure at a flowrate estimating point.

IMPLEMENTATION OF REMOTE INSTANTANEOUS FLOWRATE MEASUREMENT

In this section, we mainly investigated remote measurement for instantaneous flowrate. As stated before, it is possible that upstream flowrate $q_u(t)$ is estimated from measured downstream pressure $p_d(t)$ and flowrate $q_d(t)$ and using dynamic relationship between upstream and downstream section of the pipeline, or downstream flowrate $q_d(t)$ using informations of upstream pressure $p_u(t)$ and flowrate $q_u(t)$ in a similar manner, shown in Fig.1. We, here, establish the former method in detail. From equation (1), the estimating upstream flowrate $Q_u(s)$ is given

$$Q_u(s) = G_a(s)\cdot P_d(s) + G_b(s)\cdot Q_d(s) \quad (4)$$

$$G_a(s) = \sinh\{\lambda(s)\cdot L\}/Z_o(s) \quad (5)$$

$$G_b(s) = \cosh\{\lambda(s)\cdot L\} \quad (6)$$

Upstream flowrate $q_u(t)$ in the time domain can be given theoretically by transforming equation (4) into the time domain. By taking inverse Laplace transform of equation (4), upstream flowrate $q_u(t)$ is given by the convolution integral with the weighting functions and the measured downstream pressure $p_d(t)$ and flowrate $q_d(t)$, respectively. It is, however, impossible to obtain easily the weighting functions (equivalent to response to the unit impulse input) given by the inverse Laplace transform of transfer functions in equations (5),(6).

Fig.2,3 show frequency characteristics of transfer functions calculated from equation (5),(6). Because frequency characteristics of transfer functions are already known, alternative procedure to estimate unsteady flowrate using the application of fast Fourier transform (FFT) is offered.

Fig.4 shows a calculation procedure for estimating upstream flowrate $\hat{q}_u(t)$ in the time domain. For directly measured downstream pressure $p_d(t)$ and flowrate $q_d(t)$, frequency spectra $P_d(jn\omega)$ and $Q_d(jn\omega)$ of the measured data are determined by following fast Fourier transform.

$$P_d(jn\omega) = \Delta t \cdot \sum_{k=0}^{N-1} p_d(k\Delta t)\cdot exp(-2\pi jnk/N) \quad (7)$$

$$Q_d(jn\omega) = \Delta t \cdot \sum_{k=0}^{N-1} q_d(k\Delta t)\cdot exp(-2\pi jnk/N) \quad (8)$$

Since $p_d(k\Delta t)$ and $q_d(k\Delta t)$ is a real functions with reference to time, frequency spectra $P_d(jn\omega)$, $Q_d(jn\omega)$ are complex functions in which the real part is even and imaginary part is odd with respect to ω. The angular frequencies are determined from $\omega = 2\pi f$, where $f = n\cdot\Delta f$, $n = 0, 1, 2, \ldots, n-1$, and Δf denotes the sampling frequency interval.

Also, convolution integral in the time domain is equivalent to multiplication in the frequency domain, therefore convolution integral can be written as the product of $G_a(jn\omega)$ and $P_d(jn\omega)$, as the product of $G_b(jn\omega)$ and $Q_d(jn\omega)$, respectively.

As shown in **Fig.4**, the estimated upstream flowrate $\hat{q}_u(t)$ in the time domain can be obtained by the summation of the values, which is given by the following inverse Fourier transform of the $Q_p^*(jn\omega)$ and $Q_q^*(jn\omega)$.

$$\hat{q}_u(k\Delta t) = F^{-1}\{Q_p^*(jn\omega)\} + F^{-1}\{Q_q^*(jn\omega)\} \quad (9)$$

$$\begin{aligned} q_p(k\Delta t) &= F^{-1}\{Q_p^*(jn\omega)\} \\ &= \Delta f\cdot\sum_{n=0}^{N-1} Q_p^*(2\pi\Delta f)\cdot exp(-2\pi jnk/N) \quad (10) \end{aligned}$$

$$\begin{aligned} q_q(k\Delta t) &= F^{-1}\{Q_q^*(jn\omega)\} \\ &= \Delta f\cdot\sum_{n=0}^{N-1} Q_q^*(2\pi\Delta f)\cdot exp(-2\pi jnk/N) \quad (11) \end{aligned}$$

$$(k = 0, 1, 2, 3, \ldots, N-1)$$

where F^{-1} represents inverse Fourier transform, $Q_p^*(jn\omega)$ and $Q_q^*(jn\omega)$ are complex conjugates of $Q_p(jn\omega)$ and $Q_q(jn\omega)$, respectively. In practical computation, the fast Fourier transform(FFT) procedure based on Sande-Tukey algorithm is employed [9].

EXPERIMENT

Experimental setup

Fig.5 shows a schematic diagram of experimental setup. Overall experimental setup consists of hydraulic pipeline system, which is included straight long copper pipeline with circular cross section (inner radius a=0.7 [cm], pipe length L=58 [cm], 170 [cm]) installed cylindrical choke-type instantaneous flowmeters (abbreviated as CCFM) at the both ends of the pipeline, and numerical operating system. The CCFM used here has been confirmed to respond to the change of flowrate in high frequencies above 200 [Hz] in previous author's research works [4],[5].

In the CCFM systems, the differential pressure dp due to flow q through chokes are measured by the semiconductor-type pressure transducer (the resonant frequency 100 [Hz]) and fed into a micro-computer system through ultra low drift differential amplifier and A/D converter (12bit, 8 channel). Instantaneous flowrate is estimated in real time, based on a mathematical model described dynamic relationship between flowrate q and differential pressure dp [4],[5]. The sampling frequency of CCFM is set at 5 [kHz] in this experiment.

In this experiment, the CCFM2 located downstream side is installed to get input data in this approach, i.e., downstream pressure $p_d(t)$ and flowrate $q_d(t)$. In order to compare the estimated upstream flowrate $\hat{q}_u(t)$ with directly measured flowrate $q_u(t)$, the CCFM1 is inserted at upstream section of the pipeline. The cylindrical chokes are used same size, choke inner diameter d=2.55 [mm] and choke length l=15 [mm], as shown in **Fig.6**.

The flow rectifier at the front of the CCFM1 is inserted to decrease the influence of flow disturbance in **Fig.5**. To take off the effect of unsteady jet flow caused by an upstream cylindrical choke, a flow rectifier shown in **Fig.6** is inserted at the downstream side of the CCFM1. It was already confirmed experimentally that the flow rectifier is very effective to avoid the flow disturbance caused by unsteady jet flow [10].

The upstream section of pipeline is connected a constant pressure source 3.9 [kPa] and an electro-hydraulic servo valve is set at the downstream manifold to generate a change of flowrate in the hydraulic pipeline system. Oil temperature is measured by thermistor-type thermometer.

In the experiment, oil temperature is maintained 35 ± 1 [°C] during testing and kinematic viscosity of the working fluid is 0.4 [cm^2/s] at 35 [°C].

Experimental results and comparisons

For steady flow, flowrate passing through the cylindrical choke1 and choke 2 is directly measured by volumetric flowmeter, and simultaneously by CCFM1 ($q_u(t)$) and CCFM2($q_d(t)$). The experimental results is shown in **Fig.7**. Flowrate obtained from the measured differential pressure (dp_1 and dp_2) are in good agreement with theoretical values, obtained through a mathematical model of cylindrical choke [4],[5]. It is shown that steady flowrate is measured accurately by CCFM installed at the upstream and downstream section of the pipeline.

Next, stepwise current input is applied to servo valve located at the downstream manifold, to generate unsteady flow in the hydraulic pipeline system.

Fig.8 shows the typical examples of the recorded flowrate and pressure waveforms for rectangular wave input 10 [Hz], in the case of pipe length L=58 [cm]. In **Fig.8**, the top and second waveforms are the pressure $p_d(t)$ and flowrate $q_d(t)$ at the downstream section of the pipeline, which correspond to input data in this approach. The third one is the estimated upstream flowrate waveform $\hat{q}_u(t)$ using this approach, bottom one is directly measured one $q_u(t)$ by CCFM1. The fourth compares the estimated $\hat{q}_u(t)$ with measured upstream flowrate $q_u(t)$ at the same coordinates. In comparison, the estimated upstream flowrate $\hat{q}_u(t)$ is in good agreement with the directly measured one.

Fig.9 compares the estimated result $\hat{q}_u(t)$ with the measured upstream flowrate $q_u(t)$ for rectangular wave input 100 [Hz].

Conversely, it is possible to estimate the downstream flowrate $\hat{q}_d(t)$ by the measured upstream pressure $p_u(t)$ and flowrate $q_u(t)$ in a similar manner. In this case, transfer function between $Q_d(s)$ and $P_u(s)$ is changed $G_a(s)$ into $-G_a(s)$, however, $G_b(s)$ is invariant in equations(5),(6). The estimated and measured waveforms are shown in **Fig.10**.

Fig.11 shows the results for pipe length L=170 [cm], the estimated flowrate waveform is in good agreement with the directly measured one.

CONCLUSION

We proposed two approaches, *remote and quasi-remote instantaneous flowrate measurement method* , using the informations of pressure and flowrate at the other remote location from a desired location to measure instantaneous flowrate, and hydraulic pipeline dynamics relating to the pressures and flowrates between two cross sections of the pipeline. The estimated flowrate waveforms are compared with directly measured ones by cylindrical choke-type instantaneous flowmeter, which is used for calibration. The excellent agreement between estimated and directly measured flowrate waveforms illustrates the validity of this method.

It is shown that *remote instantaneous flowrate measurement method* proposed here is useful in estimating unsteady flowrate through an arbitrary cross section of hydraulic pipelines and components. It is expected that *remote instantaneous flowrate measurement method* can be conveniently used for measuring unsteady flowrate in hydraulic control systems. We intend to present *quasi-remote instantaneous flowrate measurement method* in the near future.

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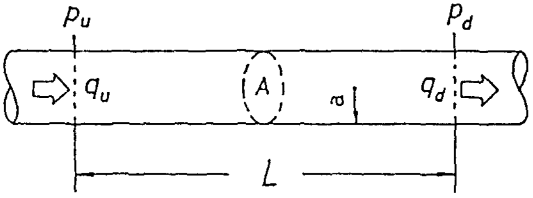

Fig.1 Schematic of hydraulic pipeline

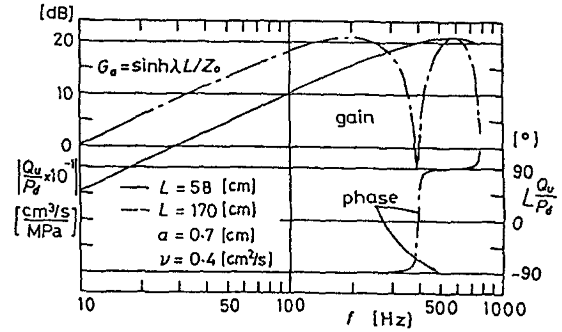

Fig.2 Frequency characteristics of $G_a(j\omega)$

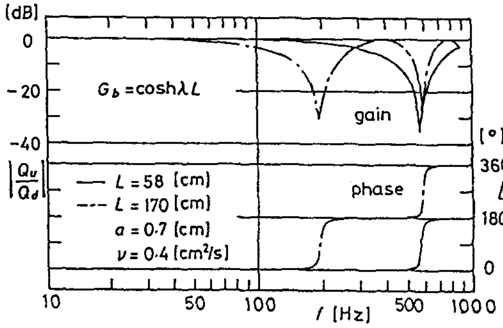

Fig.3 Frequency characteristics of $G_b(j\omega)$

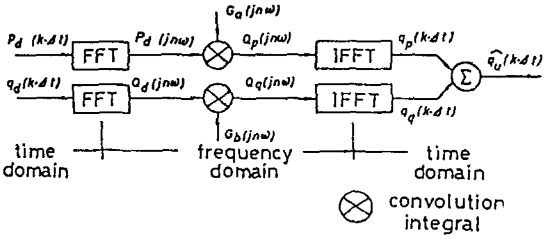

Fig.4 Flowrate estimation procedure by FFT

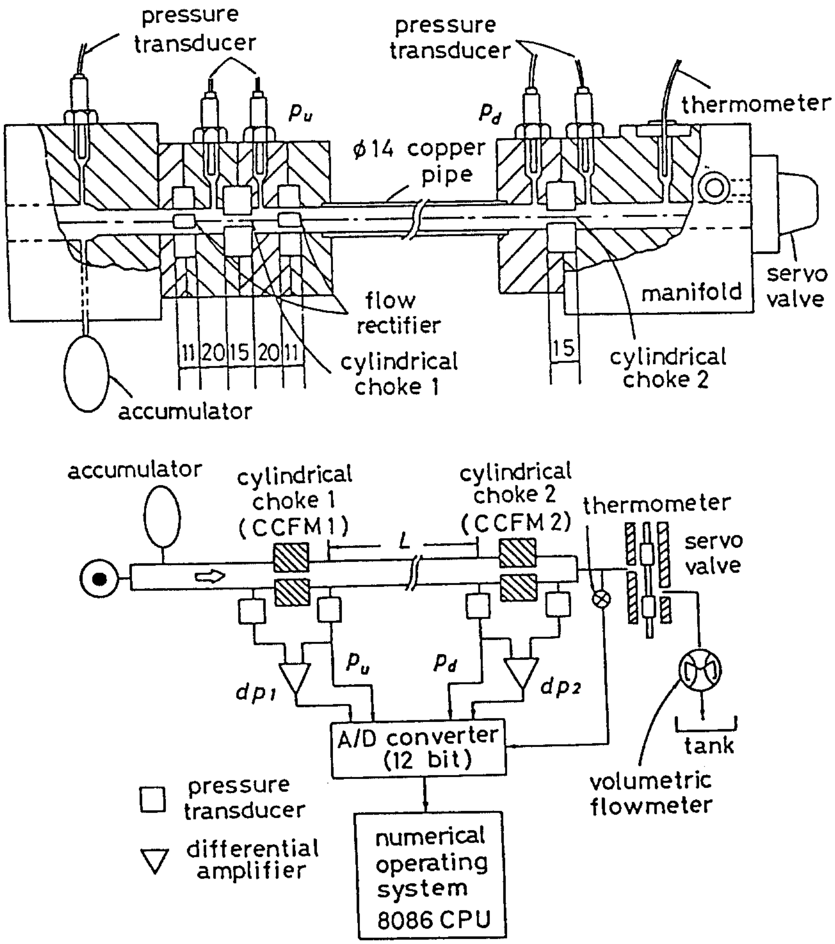

Fig.5 Schematic of experimental setup

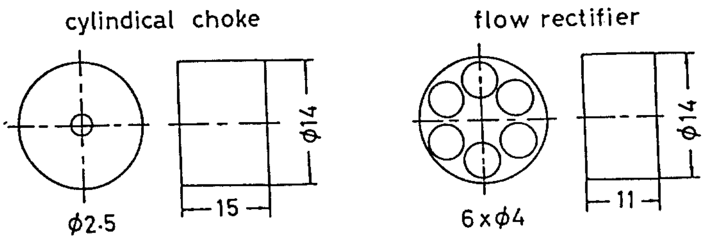

Fig.6 Configuration of cylindrical choke and flow rectifier

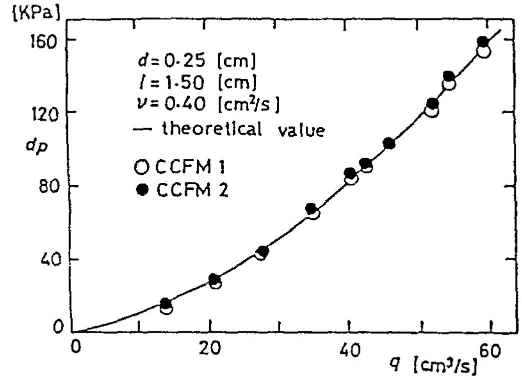

Fig.7 Static characteristics of CCFM

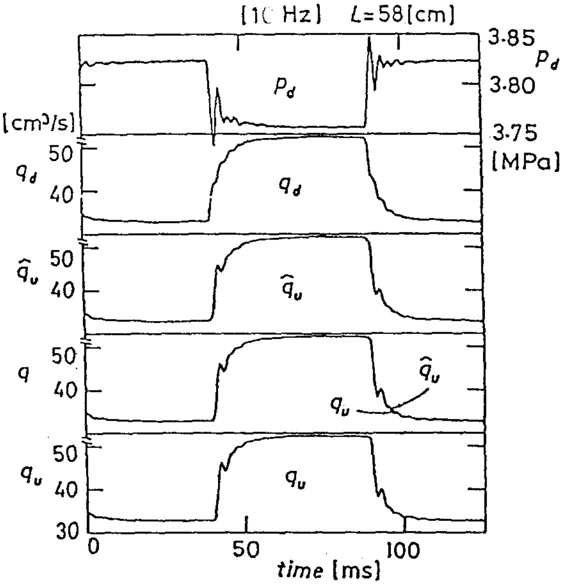

Fig.8 Typical examples of estimated and measured waveforms for rectangular wave input [10Hz]

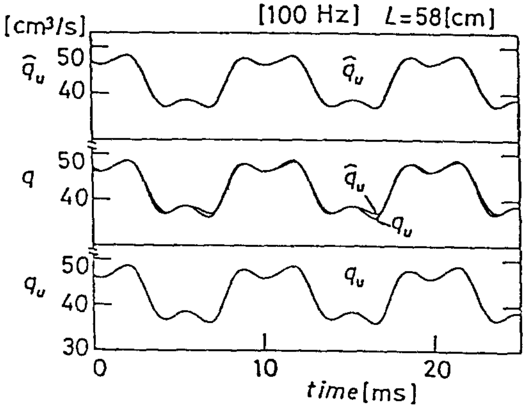

Fig.9 Comparisons of estimated and measured upstream flowrate waveforms for rectangular wave input [100Hz]

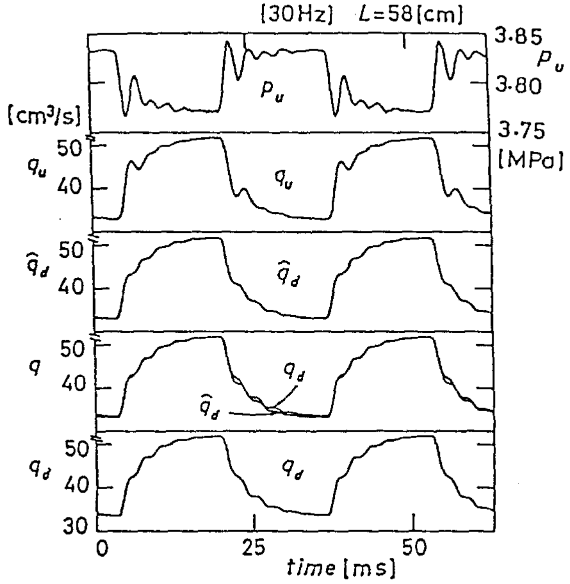

Fig.10 Typical examples of estimated and measured waveforms for rectangular wave input[30Hz]

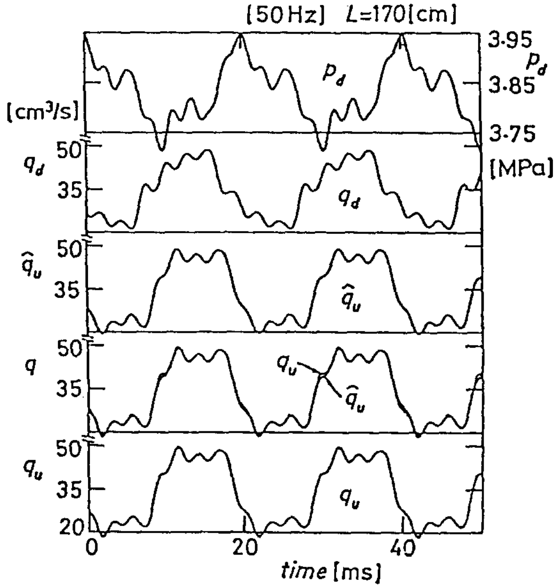

Fig.11 Typical examples of estimated and measured waveforms for rectangular wave input[50Hz]