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An Application of Manual Control to Swinging-up of a One-link Pendulum

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It is difficult to obtain a swinging-up control sequence of a one-link pendulum analytically or numerically. In this paper, we obtain a proper control sequence through manual control experiments. However, no proper control sequence will be obtained if the rotational velocity of the pendulum is fast for the human operator. To overcome such a disadvantage, we propose a method for training the operator by using a pendulum simulator.

1. INTRODUCTION

After enough training, a human operator can control various complicated systems, such as a car, even if the operator does not know previously the dynamics or the corresponding mathematical model of them. The objective of our research is to obtain proper control sequences for complicated systems by applying a manual control method.

A one-link pendulum is one of the most popular systems, and it is often used to validate new control theories. The pendulum is also used as a model for a biped system. There are many researches concerning about stabilizing control of a pendulum at the upright position[1-3], but few researches about swinging-up control are reported[4]. Swinging-up motion of a one-link pendulum is governed by a nonlinear equation. Therefore, it is difficult to obtain the proper control law of the pendulum with analytical methods or numerical methods.

In this paper, we discuss a manual control method to obtain the control law of swinging-up motion of a one-link pendulum driven by small torque applied at its rolling axis. Since it is difficult for a human operator to stabilize the pendulum at the upright position, we regard that the goal is achieved provided the rotational velocity of the pendulum around the upright position is sufficiently small.

First, we show that a proper control sequence can be obtained through manual control experiments. A human operator can control the swinging-up motion of the one-link pendulum after enough training. By using the obtained sequence, we can achieve the swinging-up control of the pendulum. An example of the experimental results is shown. However, this method mentioned above has a problem. If the movement of the system so fast that the operator cannot follow its motion, then no proper control sequence can be obtained. To overcome such a disadvantage, we propose a method for training the operator by using a pendulum simulator.

2. MATHEMATICAL MODEL OF A PENDULUM

The model of a one-link pendulum

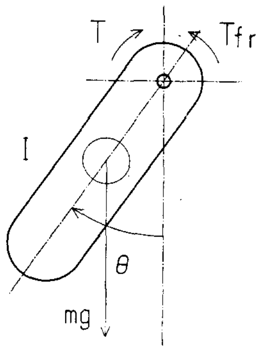

Fig.1 Model of a one-link pendulum

is shown in Fig.1. The governing equation is obtained as follows:

$$I\ddot{\theta}(t) + c\dot{\theta}(t) + mgl\,\sin\theta(t) = T + T_{fr} \qquad (1)$$
$$+ : \dot{\theta}(t) < 0$$
$$- : \dot{\theta}(t) > 0$$

where the notations are

I = inertia of the pendulum, kgm^2,
T = driving torque, Nm,
T_{fr} = friction torque, Nm,
c = coefficient of the viscous damping, Nm/s,
l = distance between the rolling axis and the center of gravity, m.

Equation(1) can be rewritten into the following nondimensional form:

$$\ddot{\theta}(\tau) + 2\zeta\dot{\theta}(\tau) + \sin\theta(\tau) = \alpha + \alpha_{fr} \qquad (2)$$
$$+ : \dot{\theta}(\tau) < 0$$
$$- : \dot{\theta}(\tau) > 0$$

where

$$\dot{\theta}(\tau) = \frac{d\theta(\tau)}{d\tau}, \quad \tau = \omega t,$$

$$2\zeta = \frac{c}{\sqrt{mglI}}, \quad \omega^2 = \frac{mgl}{I},$$

$$\alpha = \frac{T}{\omega^2 I}, \quad \alpha_{fr} = \frac{T_{fr}}{\omega^2 I}$$

We assume that the driving torque is limited as

$$|\alpha| = \alpha_{max.}, \qquad (3)$$
$$\alpha_{max.} = \text{constant}$$

If the maximum driving torque is small, it cannot make the pendulum swing up to the upright position at once. In such a case, it requires to switch the torque direction to swing up the pendulum. The number of switching times are obtained according to the magnitude of the driving torque, the friction torque $_{fr}$ and the viscous damping factor[5].

3. EQUIPMENTS FOR MANUAL CONTROL EXPERIMENTS

Figure 2 shows the experimental equipments of manual control. These are composed of a pendulum unit, a computer and a joy-stick controller. A human operator manipulates the joy-stick controller so as to swing up the pendulum to the upright position. The computer samples and records the manipulation of the human operator and the angular trajectory of the pendulum.

The joy-stick controller generates 3 level signal, namely, zero, and plus and minus unity voltage. The voltage is converted to the driving torque by the bipolar constant current power amplifier. The pendulum is driven by the D.C. servo motor through two gears.

If the rotational velocity of the pendulum is so fast that the human operator cannot follow the motion, then no proper manipulation will be achieved. Therefore, it is important to select the parameters of the equipment. The parameter which determines the rotational velocity is the natural frequency omega. We determined its value as 4.342 through several experiments. The required switching times of the the driving torque direction depends on the magnitude of the nondimensional torque alpha. We selected its value as 0.488 so that the minimal required switching times are 2[5]. All of parameters of the pendulum are shown in table 1.

4. AN EXAMPLE OF MANUAL CONTROL EXPERIMENT

In this section, we show an experimental result of the manual

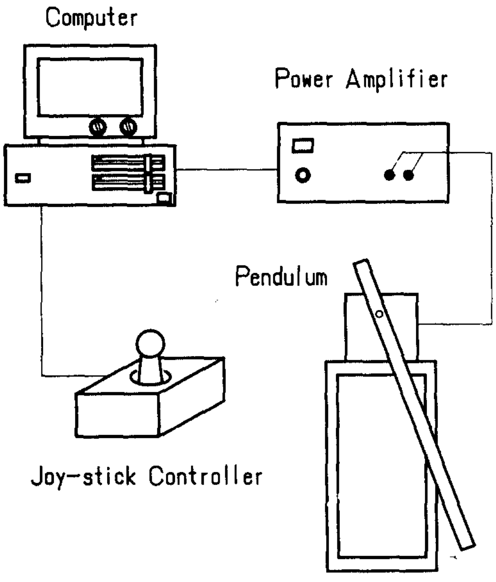

Fig.2 Experimental equipment

Table 1 Parameters of the pendulum system

Parameter	Value	Unit
Inertia I	3.76×10^{-3}	kgm^2
Mass m	50.5×10^{-3}	kg
Length l	143.0×10^{-3}	m
Natural Freq. ω	4.342	sec^{-1}
Damping Factor ζ	0.0536	-
Friction Trq. α_{fr}	0.010	-
Torque α	0.488	-

control. The block diagram of the experimental equipment is shown in Fig.3. In the training phase, the human operator observes the motion of the pendulum and manipulates the joystick controller in order to swing up the pendulum. The manipulation and the trajectory of the pendulum is sampled by the computer. In the playback phase, the sampled manipulation data is used instead of the operator's manipulation.

We show an experimental result. After enough training, the human operator has achieved swinging-up the pendulum as shown in Fig.4. The obtained manipulation data can effectively be used to control the pendulum by using the playback method[8,9].

5. TRAINING WITH THE PENDULUM SIMULATOR

In the previous section, we showed that the human operator was able to generate a proper control sequence. However, if the rotational velocity of the pendulum is fast, then no proper control sequence is obtained because the human operator can not follow the motion of the pendulum. An experimental result indicating this situation is shown in Fig.5. No proper control sequence was obtained after all. In order to overcome the disadvantage, we propose another method for training the human operator by using a pendulum simulator.

The structure of the simulator is shown in Fig.6. The pendulum follows the reference angle, and the reference is generated by the computer by calculating the governing equation of the pendulum. Therefore, it is easy to modify the parameters of the

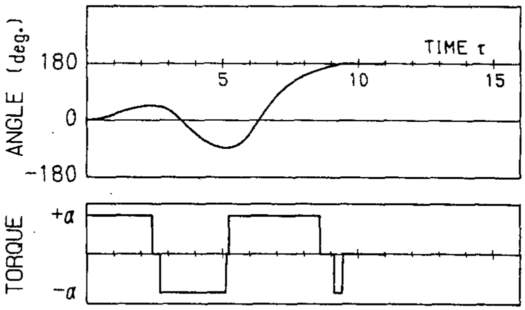

Fig.4 An experimental result of the manual control

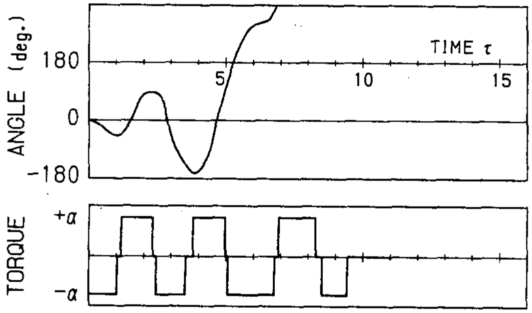

Fig.5 An example of failure case

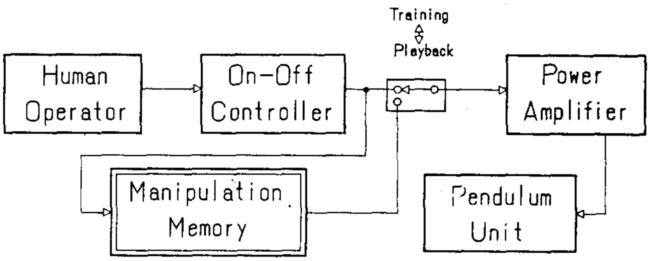

Fig.3 Block diagram of the manual control

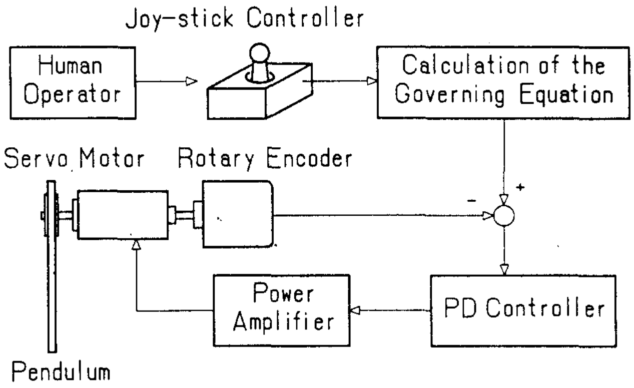

Fig.6 Block diagram of the pendulum simulator

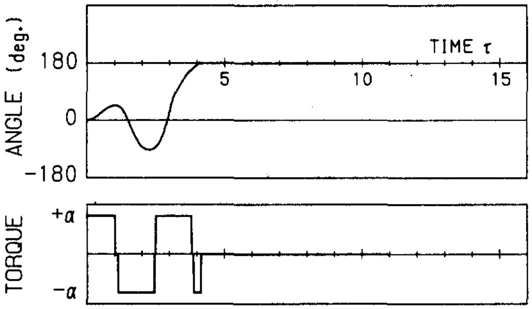

Fig.7 An experimental result with the pendulum simulator

pendulum because changing the parameters is corresponding to modify some constants in the computer program. By using this simulator, we trained the human operator as follows. First, we select the parameters of the simulator so as to swing slowly. The human operator is trained with the pendulum. After then, the parameters are modified a little, and the training is continued. This procedure is repeating until the parameters reach the desired value. A result of the training is shown in Fig.7. By using this method, a proper control sequence is obtained.

6. CONCLUSION

We have considered an application of manual control to swinging-up of a one-link pendulum. The objective of our research is to find a way how we can obtain a proper control law for a complicated system which can hardly be described by mathematical methods. In this paper, we have applied the manual control method to the one-link pendulum system, and have obtained a proper control sequence through some experiments. However, if the movement of the pendulum is so fast that the operator cannot follow the motion, then we cannot directly apply the manual control method to the system. We have proposed a method for training the operator by using a pendulum simulator, and the disadvantage has been overcome successfully.

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