OPTIMAL SCHEDULING OF IDEALIZED MULTI-PRODUCT BATCH OPERATION

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A heuristic model which determines the scheduling of serial flowshops with minimization of the makespan is proposed for an idealized batch chemical plant. It generates an initial sequence by heuristic reasoning and improves it recursively until no improvement is possible. The heuristic reasoning is based on Johnson's Rule which gives the sequence with the minimum makespan for a two-unit flowshop. The evolutionary step searches the neighborhood of the current sequence for sequences with lower makespan. The robustness of this model is also examined by comparing the minimum makespan of literature examples with the theoretical one.

1. Introduction

Although continuous operation has been adopted in many chemical processes over the last few decades, batch operation is still commonly used in many processes, specifically fine chemical production, biotechnoloy, food processing, etc. The most appealing feature of batch processes is their flexibility in producing multiple products in a single plant through sharing of process equipment. However, for a chemical plant consisting of many batch processes with various operating times to produce multiple products, optimum operating policies have to be found for the improvement of productivity.

Three major recent research topics for batch processes are plant design, intermediate storage tank sizing, and scheduling. Of these production scheduling is the most important topic since the overall productivity and the economic effectiveness depend critically on it. In fact, Parakram (1) identified batch plant production for computer aided production.

Batch chemical plants are classified into two categories: multiproduct and multipurpose. In multiproduct plants, all products follow essentially the same sequence of processing steps. In multipurpose plants, the products may follow different paths through the plant (3).

In general, most batch chemical plants produce multiple products using a series of processing stage, where the order in which products are produced affects the total operating time (called makespan). The criterion of maximizing the productivity or equivalently minimizing makespan required to produce final products has been the subject of many studies in the literature.

Even though mathematical modeling based on combinatorial method has developed to solve minimum makespan, it requires lots of computation and leads to many difficulties in using optimization techniques. Heuristics and evolutionary methods have thus been preferred. Moreover, transfer times and set-up times have been considered to accommodate real problems (2).

In this paper, we present a heuristic algorithm which minimizes the makespan for an idealized simple multiproduct batch chemical plant with infinite number of intermediate storage tanks. Compared with the theoretical minimum solution solved by an implemented computer program, the optimality of the solution is guaranteed for standard literature problems.

2. Mathematical Representation

For a multiproduct batch plant, we assume that the plant usually produces similar products, and the order in which the operation must be carried out is the same for all products. To simplify the problem, unlimited intermediate storage tanks and negligible transfer time of the job from one unit to another are assumed. For this idealized flowshop, the completion time of each job in the sequence on each equipment can be computed from the following equation (N products and M processing units).

$$C_{ij} = \max [C_{(i-1)j}, C_{(j-1)}] + t_{ij} \qquad (1)$$

$$\text{for } i = 1,2,\ldots,N \text{ and } j = 1,2,\ldots,M$$

where C_{ij} is the completion time of the ith job over the jth unit and t_{ij} is the processing time of product k on unit j. Here, the completion time refers to the time at which a product finishes processing on a unit.

This recurrence relation indicates that the completion time of a job on a unit is its processing time plus the time at which processing can start. The latter time is the larger of the two times: the time at which the previous job in the sequence finishes processing on the current unit and the time at which the current job finishes on the previous unit.

With the initial conditions

$$C_{oi} = 0 \qquad \text{for } i = 1,2,\ldots,N \qquad (2)$$

$$C_{jo} = 0 \qquad \text{for } j = 1,2,\ldots,M \qquad (3)$$

and from the recurrence relationship equation of (1), the makespan of each sequence is calculated as C_{NM}. Applying these equations recursively, the completion times for the entire sequence of jobs on all processing units are calculated to find the minimum value. With this combinatorial approach, the minimum makespan is mathematically guaranteed.

3. Heuristic Approach

As the complexity of the flowshop increases, the mathematical representation becomes more complex to formulate. In view of this fact, a considerable amount of effort has been spent on developing suboptimal algorithms that are based on heuristic reasoning (3). In this case, we first try to find an initial sequence and then improve it repeatedly until no improvement is possible.

Most of the heuristic approaches utilize Johnson's algorithm which gives the optimal sequence with the minimum makespan for a simple two-unit flowshops. That is, process first the product whose processing time on the second unit is higher than the first unit. If this condition is satisfied by many products, arrange them in the ascending order of their processing times on the first unit.

For multiple-unit systems, we can make use of this heuristic algorithm. Then the following rules can be proposed.

Rule 1:
Give the priority to the product whose processing time on each unit is the longest comparing with the second one.

Rule 2:
Select the product which has the processing time as close as the expected available time on the unit whose usefulness is expected lower during the processing times of the product, which is selected from Rule 1, on each unit.

From these rules, we can generate an initial sequence which is not always the optimum with the minimum makespan. In the evolutionary step, we search the neighborhood of the current sequences with lower makespans. The neighborhood is defined as the N-1 sequences resulting from pairwise interchanges of adjacent products in the current sequence. However, if we find many sequnces with lower makespans, we can apply those interchanges simultaneously. Then select the sequence with the lowest makespan as the new current sequence and repeat this evolutionary step.

4. Algorithm

For idealized multiproduct batch processes, the optimal sequence with the minimum makespan can be obtained via the following procedures.

Phase 1: Invention Procedure
Given M-unit flowshop problem, find an initial sequence by applying Rules 1 and 2.

Phase 2: Evolution Procedure
Improve the current sequence by interchanging the adjacent products simultaneously. Repeat it recursively until no improvement is possible.

5. Application

For a standard literature problem whose processing time matrix is shown in Table 1(3), the makespans of all sequences are calculated by a computer program using the mathematical representation. The results are listed in Fig. 1. Four optimal solutions, 1-2-4-3, 1-3-4-2, 1-4-2-3, and 1-4-3-2 exist. All of them begin with the product 1 and the minimum make span for this problem is 90.

Table 1: Processing times

Products	Units 1	2	3	4
1	10	20	5	30
2	15	8	12	10
3	20	7	9	5
4	13	7	17	10

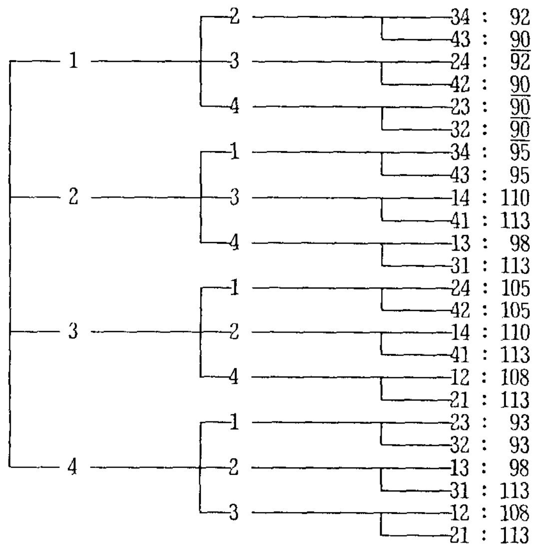

Fig. 1: Makespans of all the operating sequences

However, in the heuristic approach, we first apply Rule 1. Then the processing time ratios are calculated for each unit.

unit 1 : 20 / 15 = 1.33 (product 3)
unit 2 : 20 / 8 = 2.5 (product 1)
unit 3 : 17 / 12 = 1.4 (product 4)
unit 4 : 30 / 10 = 3.0 (product 1)

Since the product 1 takes the longest time on unit 2 and 4 it has to be processed first. Its processing time is 65 because 10 (unit 1), 20 (unit 2), 5 (unit 3), 30 (unit 4) are required. During this processing, since unit 2 and 4 take longer time than other units, the usefulness of unit 1 and 3 is expected to be very low. Then, from Rule 2, the processing times of the product 2 over unit 1 and 3 are 15 and 12, respectively. For the product 4, they are 20 and 9. Since the expected available times of unit 1 and 3 during the production of the product 1 are 20 and 30, respectively, we can give the priority to the product 4 whose processing times are the closest.

For the 1-4 sequences, total operation times are 23 (=10+13) for unit 1, 37 (=10+30+7) for unit 2, 54 (=37+17) for unit 3, and 75 (=65+10) for unit 4. Thus the expected available times are 14 (=37-23) for unit 1, 17 (=54-37) for unit 2, and 21 (=75-54) for unit 3. Since the processing times are 15,8,12 for the product 2 and 20,7,9 for the product 3 the expected available times are closer to the processing time of product 3. Then the final solution is 1-4-3-2 and its makespan is 90, which is the minimum makespan. Since we cannot improve this sequence in the evolutionary step no better solution is obtained. Thus this is the optimum.

5. Conclusion

A heurisic approach for the minimum makespan of idealized batch operation is proposed. It consists of invention and evolution phases. In the invention step, an initial sequence is generated and it is improved to find the optimal solution in the evolutionary step. Two heuristic rules are also proposed and utilized repeatedly for the sequence invention. The approach is successful in finding the optimum sequence for idealized problems. The same procedure can be applied to more general complex real problems.

Reference

1. Parakrama, R., "Improving Batch Chemical Processes," Chem. Eng., Sept., pp. 24 (1985)

2. Karimi, I. and H. Ku, "A Modified Heuristic for an Initial Sequence in Flowshop Scheduling," Ind. Eng. Chem.Res., 27, pp. 1654 (1988)

3. Ku, H. M., et al., "Scheduling in Batch Processes," Chem. Eng. Prog., 83, pp. 35 (1987)

4. Rajagopalan, D. and I. A. Karimi, "Completion Times in Serial Mixed-Storage Multiproduct Processes with Transfer and Set-up Times," Computers Chem. Engng., 13, pp. 175 (1989)