NEW INTERACTION MEASURES FOR DECENTRALIZED CONTROL SYSTEMS

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We present new efficient interaction measures which can be used for control operability analysis and control structure selection in decentralized control systems. These measures can indicate not only the stability of decentralized control systems but also the true closed-loop performance of the decentralized control structure. Relationships between published measures and proposed ones are clarified. Some important characteristics of these measures are rigorously analyzed. The significance and the usefulness of the proposed method have been illustrated through examples found in the literature.

INTRODUCTION

Virtually all chemical processes are multivariable in nature.There would be two general approaches to controlling multivariable systems, i.e. centralized control and decentralized control. Although the constraints on the controller structure invariably lead to performance deterioration when compared to the systems with centralized controllers, decentralized controllers have been preferred to more complex multivariable controllers and widely used in chemical industries because of their significant advantages such as ease of design, hardware simplicity, improved safety and failure tolerance, high modularity and flexibility, and decentralized tuning. One of the most important tasks of decentralized controller design is to decide on the control structure or pairing problem. Thus efficient interaction measures are needed which are capable of screening effectively for the best control structure and providing the information of the control-loop operability for guidelining a controller design. The Relative Gain Array (RGA) method (Bristol,1966) has been popular as an interaction measure because it is easy to use and only requires the information of steady state process gains. However, a number of authors pointed out its limitations. Many techniques have been proposed to overcome these limitations. Nevertheless, all these techniques are limited in their use. Desirable characteristics which an ideal interaction measure should have are as follows ; it can handle nonsquare as well as square systems, it can handle open loop unstable as well as open loop stable systems, it can handle partially decentralized as well as fully decentralized control structure, it can handle dynamic as well as static informations, it does not require any quantitative information of controller, it should provide informations on both system stability and performance, it should be scale indepedent and simple to use. But it is not a trivial task to develop the interaction measure satisfying all the conditions described above, and in fact it is still an active area of research .

The objective of this paper is to develop some new interaction measures which satisfy above conditions as many as possible

GENERAL DECENTRALIZED CONTROL SYSTEM

Let us consider the conventional decentralized feedback system structure (Fig.1). The open loop transfer

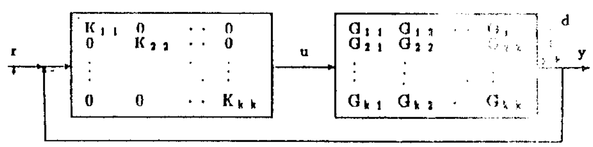

Fig. 1. Conventional decentralized control structure.

function matrix $G(s)\epsilon C^{mxn}$ with kxk block partitioning is

$$G = \begin{matrix} m1 \\ m2 \\ \\ \\ mk \end{matrix} \begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1k} \\ G_{21} & G_{22} & \cdots & G_{2k} \\ \vdots & & \ddots & \vdots \\ G_{k1} & G_{k2} & \cdots & G_{kk} \end{bmatrix} \quad (1)$$
$$\qquad n1 \quad n2 \qquad nk$$

where if $m \geq n$, then $m_i \geq n_i$ for $i = 1,..,k$
and if $m \leq n$, then $m_i \leq n_i$ for $i = 1,..,k$

Then the nominal transfer fucntion $\tilde{G}$ is defined by

$$\tilde{G} = \begin{bmatrix} G_{11} & 0 & \cdots & 0 \\ 0 & G_{22} & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & G_{kk} \end{bmatrix} \quad (2)$$

Let r be the vector of setpoints for the closed loop system, u be the vector of manipulated variables, y be the vector of controlled variables and d be the vector of disturbances. Then r, y, d and u have been partitioned in the manner such as $r=(r_1, r_2,...,r_k)^t$, $y=(y_1, y_2,...,y_k)^t$, $d=(d_1, d_2,...,d_k)^t$, $u=(u_1, u_2,...,u_k)^t$. The conventional decentralized controller K is block diagonal

$$K = \begin{matrix} n1 \\ n2 \\ \\ \\ nk \end{matrix} \begin{bmatrix} K_{11} & 0 & \cdots & 0 \\ 0 & K_{22} & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & K_{kk} \end{bmatrix} \quad (3)$$
$$\qquad m1 \quad m2 \qquad mk$$

If we use PI controller for zero off-set at steady-state, then the K is represented as follows

$$K=K_c+K_i/s \quad (4)$$

where K_c and K_i are constant block diagonal matrices with the same structure as K

Input and output transfer functions are

$$u=K(I+GK)^{-1}(r-d) \quad (5)$$

$$y=GK(I+GK)^{-1}(r-d)+d \quad (6)$$

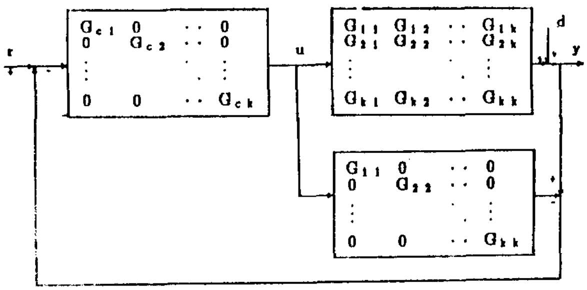

Fig. 2. **Decentralized internal model control structure.**

For the Decentralized Internal Model Control (DIMC) structure (Fig.2), input and output transfer functions are

$$u=G_c(I+(G-\tilde{G})G_c)^{-1}(r-d) \quad (7)$$

$$y=GG_c(I+(G-\tilde{G})G_c)^{-1}(r-d)+d \quad (8)$$

By the IMC controller design strategy, $\tilde{G}$ can be factorized into two factors for a square system such as $\tilde{G}=\tilde{G}_+\tilde{G}_-$ where $\tilde{G}_+$ represents the inherent limitations arising from RHP zeros or time delays. Then we can use $\tilde{G}_-^{-1}$ as the DIMC controller detuned by the adjustable filter F such as $G_c=\tilde{G}_-^{-1}F$

THE CONCEPT OF RSDM AND RDDM

Let us consider square systems. If we define the relative sensitivity as an interaction measure to evaluate the effect of a set point change in one subsystem on the other subsystems, and the relative disturbance as an interaction measure to evaluate the effect of a disturbance in one subsystem on the manipulated variables of the other subsystems. There would be two types of each Relative Sensitivity Matrix (RSM) and Relative Disturbance Matrix (RDM). The first types are defined by

$$Rs_{ji}=(\partial y_j/\partial r_i)(\partial y_i/\partial r_i)^{-1} \text{ for } i,j=1,..,k \quad (9)$$

$$Rd_{ji}=(\partial u_j/\partial d_i)(\partial u_i/\partial d_i)^{-1} \text{ for } i,j=1,..,k \quad (10)$$

The second types are defined by

$$Rs'_{ij}=(\partial y_i/\partial r_i)^{-1}(\partial y_i/\partial r_j) \text{ for } i,j=1,..,k \quad (11)$$

$$Rd'_{ij}=(\partial u_i/\partial d_i)^{-1}(\partial u_i/\partial d_j) \text{ for } i,j=1,..,k \quad (12)$$

Consider asymptotic characteristics of each RSM at low and high limiting frequency ranges in order to modify these RSM and RDM to the forms which does not require the controller information.

At low frequency ranges $\|K_{ii}\|>>1$ and $\tilde{G}_+F\text{->}I$

Therefore The asymptotic Rd and Rd' at low frequencies (denoted by $\hat{Rd}$ and $\hat{Rd}'$) are

$$\hat{Rd}=\begin{bmatrix} I & G^{-1}_{12}(G^{-1}_{22})^{-1}\cdots & G^{-1}_{1k}(G^{-1}_{kk})^{-1} \\ G^{-1}_{21}(G^{-1}_{11})^{-1} & I & \cdots G^{-1}_{2k}(G^{-1}_{kk})^{-1} \\ \vdots & \ddots & \vdots \\ G^{-1}_{k1}(G^{-1}_{11})^{-1} & G^{-1}_{k2}(G^{-1}_{22})^{-1}\cdots & I \end{bmatrix} \quad (13)$$

$$\hat{Rd}'=\begin{bmatrix} I & (G^{-1}_{11})^{-1}G^{-1}_{12}\cdots & (G^{-1}_{11})^{-1}G^{-1}_{1k} \\ (G^{-1}_{22})^{-1}G^{-1}_{21} & I & \cdots (G^{-1}_{22})^{-1}G^{-1}_{2k} \\ \vdots & \ddots & \vdots \\ (G^{-1}_{kk})^{-1}G^{-1}_{k1} & (G^{-1}_{kk})^{-1}G^{-1}_{k2}\cdots & I \end{bmatrix} \quad (14)$$

At high frequency ranges $K \cong K_c$ and $\tilde{G}_+F\text{->}0$

Therefore the asymptotic Rs and Rs' at high frequencies (denoted by $\hat{Rs}$, $\hat{Rs}'$) are

$$\hat{Rs}=\begin{bmatrix} I & G_{12}(G_{22})^{-1}\cdots & G_{1k}(G_{kk})^{-1} \\ G_{21}(G_{11})^{-1} & I & \cdots G_{2k}(G_{kk})^{-1} \\ \vdots & \ddots & \vdots \\ G_{k1}(G_{11})^{-1} & G_{k2}(G_{22})^{-1}\cdots & I \end{bmatrix} \quad (15)$$

$$\hat{Rs}'=\begin{bmatrix} I & (G_{11})^{-1}G_{12}\cdots & (G_{11})^{-1}G_{1k} \\ (G_{22})^{-1}G_{21} & I & \cdots (G_{22})^{-1}G_{2k} \\ \vdots & \ddots & \vdots \\ (G_{kk})^{-1}G_{k1} & (G_{kk})^{-1}G_{k2}\cdots & I \end{bmatrix} \quad (16)$$

Fig.3 illustrates that $\hat{Rs}$, $\hat{Rs}'$, $\hat{Rd}$ and $\hat{Rd}'$ imply the true closed-loop performance for specific frequency ranges.

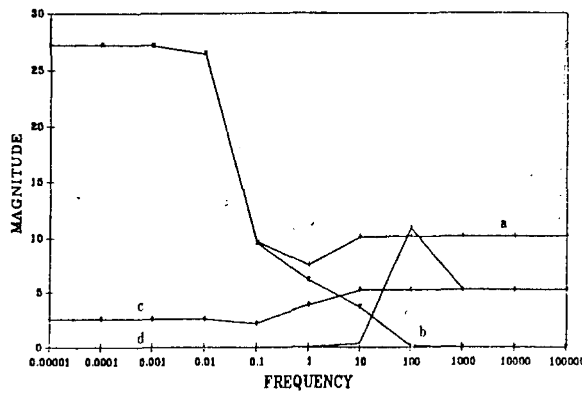

Fig. 3. Comparison Rsd (Rdd) with $\hat{\text{Rsd}}$ ($\hat{\text{Rdd}}$) of pairing structure (y_1:x_1) (y_2:x_2) (y_3:x_3) in example 3. a:Rsd, b:$\hat{\text{Rsd}}$, c:$\hat{\text{Rdd}}$, d:Rdd

Relative Sensitivity Difference Matrix (RSDM) and Relative Disturbance Difference Matrix (RDDM) are defined by

$$Rsd=Rs_{real}-Rs_{nominal}=Rs-I \quad (17)$$

$$Rsd'=Rs'_{real}-Rs'_{nominal}=Rs'-I \quad (18)$$

$$Rdd=Rd_{real}-Rd_{nominal}=Rd-I \quad (19)$$

$$Rdd'=Rd'_{real}-Rd'_{nominal}=Rd'-I \quad (20)$$

Therefore it is clear that RSDM(RDDM) describes the difference between relative sensitivity (relative disturbance) of real system and that of nominal(i.e. interaction free) system. Furthermore the following relationships are established

$$\hat{Rsd}=(G-\tilde{G})\tilde{G}^{-1} \quad (21)$$

$$\hat{Rsd}'=\tilde{G}^{-1}(G-\tilde{G}) \quad (22)$$

$$\hat{Rdd}=G^{-1}(\overline{G^{-1}})^{-1}-I \quad (23)$$

$$\hat{Rdd}'=(\overline{G^{-1}})^{-1}G^{-1}-I \quad (24)$$

where $(\overline{G^{-1}})$ means $\text{diag}\{(G^{-1})_{11},..,(G^{-1})_{kk}\}$

For a system without interactions all RSDM and RDDM are null matrices. Norms of RSDM and RDDM increase as the system deviates from a non-interacting system.

Relationships between RSDM and perturbation concepts

If we interpret a real plant G as the perturbed system from a nominal system by off-diagonal block terms (Fig.4), then the real plant G would be represented with common perturbations as follows

$$G=\tilde{G}+\Delta_a \quad (25)$$

$$G=(I+\Delta_o)\tilde{G} \quad (26)$$

$$G=\tilde{G}(I+\Delta_o') \quad (27)$$

From Eq.26–27 one can observe the relationships such as $\Delta_o = \hat{R}sd$ and $\Delta_o'=\hat{R}sd'$. These perturbations have clear physical meanings and useful properties for stability.

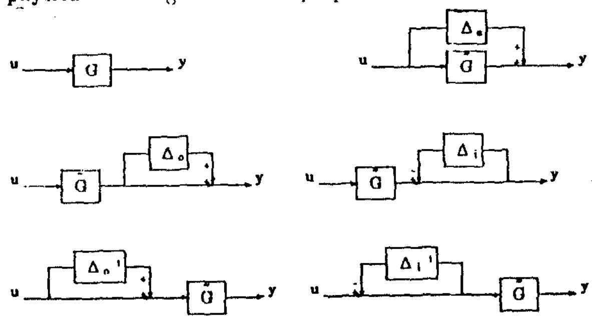

Fig. 4. Rearranged transfer functions with various perturbations.

Properties of RSDM and RDDM.

Property 1. If the transfer matrix is a (block) diagonal matrix, then

$\|\hat{R}sd\|=\|\hat{R}sd'\|=\|\hat{R}dd\|=\|\hat{R}add\|=\|\hat{R}dd'\|=\|\hat{R}add'\|=0.$

Property 2. $\hat{R}sd$, $\hat{R}dd'$ and $\hat{R}add'$ are input scale independent. $\hat{R}sd'$,$\hat{R}dd$ and $\hat{R}add$ are output scale independent.

Property 3. If the transfer matrix is 2x2 block partitioned matrix, then $\hat{R}sd=-\hat{R}dd'$ and $\hat{R}sd'=-\hat{R}dd$. therefore $\|\hat{R}sd\|=\|\hat{R}dd'\|$ and $\|\hat{R}sd'\|=\|\hat{R}dd\|$

Property 4. If the transfer matrix is a triangular matrix, then $\|\hat{R}dd\|=\|\hat{R}add\|$ and $\|\hat{R}dd'\|=\|\hat{R}add'\|$.

RELATIONSHIPS WITH STABILITY

Let us consider the conventional decentralized control system. Postlethwaite and Foo (1985) derived useful Theorems for robust stability. An application of their work to the conventional decentralized control structure yields the following theorem.

Theorem 1. Assume that i) G and $\tilde{G}$ have the same number of poles in D_d and ii) nominal closed loop system $\tilde{H}$ is stable, then the perturbed closed loop system H is stable if $\rho(\Delta_o\tilde{H})<1$. $\forall s\in D_d$

where D_d denotes direct Nyquist D–contour and $\rho(.)$ denotes the spectral radius.

If a real plant can be expressed as a block diagonal matrix (i.e. $G=\tilde{G}$), then $\Delta_o=0$. Therefore the perturbed closed loop system H is always stable provided that the decentralized controller is tuned so that each nominal closed loop system is stable. The more the real plant resembles a block diagonal matrix, the more Δ_o resembles a null matrix. Therefore, for given $\tilde{H}$ the range in which $\rho(\Delta_o\tilde{H})$ be kept as small as possible should be large. On the other hand, the more the real plant deviates from the block diagonal matrix, the more it is difficult for $\rho(\Delta_o\tilde{H})$ to be kept as small as sufficient to maintain stability of the overall plant. Nwokah (1986) derived a tight criterion for robust stability by using M matrix theory. The following Corollaries are developed based on his work.

Corollary 1–1. Let's define matrices $R\in R^{kxk}$, $B\in R^{kxk}$ and $C\in R^{kxk}$ such as

$R=\{r_{ij}\}$ where $r_{ij}=\|(\Delta_o\tilde{H})_{ij}\|$

$B=\{b_{ij}\}$ where $b_{ij}=\|\Delta_{oij}\|$

$C=\{c_{ij}\}$ where $c_{ij}=\|\tilde{H}_{ij}\|$

Then under the same assumptions in Theorem 1, the perturbed closed loop system is stable if $\rho(BC)<1$. $\forall s\in D_d$

Corollary 1–2. Under the same assumptions in Theorem 1, the perturbed closed loop system is stable if $\|\tilde{H}_i\|<\rho^{-1}(B)$ $\forall i$, $\forall s\in D_d$

From Corollary 1–2 it can be known that the increase of $\rho(B)$ causes decrease of stability boundary $\|\tilde{H}_i\|$. Small stability boundary $\|\tilde{H}_i\|$ means poor control performance. So the deterioration of control performance can be known from suggested interaction measures.

According to Morari (1989), a plant is Decentralized Integral Controllable (DIC) if there exists a diagonal controller with integral action such that closed loop system is stable and the gains of any subset of loops can be reduced to zero without affecting the closed loop stability. The tight sufficient condition for DIC is

Corollary 1–3. For the square system under assumptions in Theorem 1, the sufficient condition for DIC is as $\rho(B(0))<1$ $\forall s\in D_d$

Corollary 1–4. For the square system under assumptions in Theorem 1 when we realize perfect control for each one of the nominal loops, the sufficient condition for overall stability is as

$\rho(\Delta_o)<1$ $\forall s\in D_d$

According to the Small Gain Theorem, the stability of the DIMC structure is guaranteed

$$\text{if } \rho(\Delta_o\tilde{G},F)<1 \quad \forall s\in D_d \quad (28)$$

By the application of M matrix theorem, the following stability criterion of explicite type is obtained.

$$\|F_i\|<\rho^{-1}(B) \quad \forall i, \forall s\in D_d \quad (29)$$

Extension to nonsquare systems.

The DIMC controller for a nonsquare system with more inputs than outputs is designed as $G_c=\tilde{G}^+F$ where $\tilde{G}^+$ denotes the pseudoinverse of $\tilde{G}$.

Asymptotic RDDM at low frequencies are

$$Rdd'_{ij}=((G_{ii})^+(G\tilde{G}^+)_{ii}^{-1})^+(G_{ii})^+(G\tilde{G}^+)_{ij}^{-1}-\delta_{ij}I \quad (30)$$

Asymptotic RSDM at high frequencies are

$$Rsd_{ij}=G_{ij}(G_{jj})^+-\delta_{ij}I \quad (31)$$

SUGGESTED PAIRING AND DESIGN RULE.

The suggested pairing and design rule is as follows :

i) Make a frequency plot of $\|\hat{R}sd\|$ and $\|\hat{R}dd'\|$ ($\|\hat{R}sd'\|$ and $\|\hat{R}dd\|$) for every pairing cases.

ii) Select the pairing case with the smallest value of $\|\hat{R}sd\|$ at high frequencies and $\|\hat{R}dd'\|$ at low frequencies ($\|\hat{R}sd'\|$ at high frequencies and $\|\hat{R}dd\|$ at low frequencies) Ideally the value less than one is desirable.

iii) Check the stability characteristics and performance deterioration by $\rho(B)$ over the whole frequency range.

iv) If there exists no desirable case in fully decentralized structures, then consider the block partitioning and repeat step i) – iii).

v) If there exists no desirable case in partially decentralized structures either, then consider a decoupler or a fully centralized multivariable controller.

Note that if only the steady–state information is available, then we can use $\|\hat{R}dd'(0)\|$ and $\|\hat{R}dd(0)\|$ and check DIC from $\rho(B(0))$.

EXAMPLES

We present some examples to illustrate the proposed method. Dynamic simulation is performed to confirm our predictions. The DIMC controller with the second order filter is used for an equal base comparison among candidate pairing structures.

Example 1. This example consists of a distillation column with side streams presented by Ray(1981). The only feasible pairing structure is the diagonal (block diagonal) pairing. For the case with only steady state information availabe, according to our method, a severe interaction effect at steady–state is expected due to high $\|\hat{R}dd(0)\|$ and $\|\hat{R}dd'(0)\|$ although the system is DIC since $\rho(B(0))<1$. So our method suggests that there exist no proper pairing case both in the fully decentralized system and the partially decentralized system. For the case with dynamic information available, Our method suggests that all pairing cases are not proper due to their high $\|\hat{R}sd\|$, $\|\hat{R}sd'\|$, $\|\hat{R}dd\|$ and $\|\hat{R}dd'\|$. Fig. 5c–e show dynamic simulation with filter constant $\alpha=0.5$.

Example 2. This example was presented originally by Friedly (1984). For the case with only steady state information available, according to our method, a severe interaction effect at steady–state is expected due to high $\|\hat{R}dd'(0)\|$ although the system is DIC since $\rho(B(0))<1$. For the case with dynamic information available, the relative sensitivities seem very good due to low $\|\hat{R}sd\|$ and $\|\hat{R}sd'\|$, but relative disturbances are bad due to high $\|\hat{R}dd'\|$. Thus our method suggests that there exist no proper pairing. One can observe the large difference between $\|\hat{R}dd\|$ and $\|\hat{R}dd'\|$. This means the large different dynamic characteristics between each loops in that pairing structure. Fig. 6b–c show dynamic simulation with filter constant $\alpha=0.5$.

Example 3. This is presented to illustrate the dynamic interaction effect(Gagnepain and Seborg,1982).
Some plausible pairing structures after a coarse screening are as follows.

a: $(y_1{:}x_3)(y_2{:}x_2)(y_3{:}x_1)$
b: $(y_1,y_2{:}x_2,x_3)(y_3{:}x_1)$
c: $(y_1{:}x_1)(y_2{:}x_3)(y_3{:}x_2)$
d: $(y_2,y_3{:}x_2,x_3)(y_1{:}x_1)$

For the case with only steady state information availabe, our method recommand the pairing c. For the case with dynamic information available, we can know that the closed loop performances of both a and c are not bad in low frequency since each $\|\hat{R}dd\|$ and $\|\hat{R}dd'\|$ has similar low values. But for high frequency the structure c and d show large dynamic interaction effect due to their high $\|\hat{R}sd\|$ and $\|\hat{R}sd'\|$. Therefore the proposed method recommends the structure a and b because of their good dynamic interaction. Fig. 7c–d show dynamic simulations for the pairing a and b with filter constant $\alpha=0.05$.

Example 4. This is an example for a non–square system presented by Reeves and Arkun (1989).
From Fig. 8a it is clear that the pairing $(y_1{:}x_1,x_2)$ $(y_2{:}x_3)$ is the best pairing structure for both high and low frequencies because $\|\hat{R}sd\|$ and $\|\hat{R}sd'\|$ show the most desirable feature among all other structures. Fig. 8b–c show dynamic simulations for that pairing case with filter constant $\alpha=0.05$.

CONCLUSIONS

New interaction measures have been proposed by introducing the concept of Relative Sensitivity Difference Matrix (RSDM) and Relative Disturbance Difference Matrix (RDDM). It has been shown that without any quantitative information of the controller it is possible to predict the true closed–loop performance by using the asymptotic characteristics of these matrices at specific frequency ranges. Several useful properties for control loop operability have been obtained from two types of RSDM and RDDM. It has been shown that there exist close relationships between the proposed interaction measure matrices and multiplicative perturbations. Using this fact, tight stability conditions of explicit type have been driven from robust stability theorems. The suggested method can be used to predict not only the stability of decentralized systems but also the performance degradation due to decentralized control. A guideline for pairing and design of decentralized control system has been suggested. Through some illustrative examples we confirmed the reliability of the proposed method.

Literature Cited

1. Arkun, Y."Relative Sensitivity: A Dynamic Closed Loop Interaction Measure and Design Tool," *AIChE J.*, 34(4), 672 (1988).
2. Bristol, E. H.,"ON a New Measure of Interaction for Multivariable Process Control," *IEEE Trans. Auto. Control*, 11, 133 (1966).
3. Doyle, J. C.,"Analysis of Feedback Systems with Structured Uncertainties," *IEE Proc. D, Control Theory & Appl.*, 129, 242 (1982).
4. Economou, C. and M. Morari,"Internal Model Control. – Multiloop Design," *Ind. Eng. Chem. Process Des. Dev.*, 25, 411 (1986).
5. Friedly, J. C.,"Use of the Bristol Array in Designing Noninteracting Control Loops. A Limitation and Extension," *Ind. Eng. Chem. Process Des. Dev.*, 23, 469 (1984).
6. Gagnepain, J. P. and Seborg, D. E.,"An Analysis of Process Interactions with Applications to Multiloop Control System Design," *Ind. Eng. Chem. Process Des. Dev.*, 21, 5 (1982).
7. Nwokah, O. D. I.,"On nonsingular value based design of controllers for robust stability," *IEE Proc. D, Control Theory & Appl.*, 133(2), 57 (1986).
8. Postlethwaite, J. and V. K. Foo,"Robustness with Simultaneous Pole and Zero Movement across the jw–Axis," *Automatica.*, 23(4), 433 (1985).
9. Reeves, D. E. and Arkun, Y. "Interaction Measures of Nonsquare Decentralized Control Structures," *AIChE J.*, 35(4), 603 (1989).

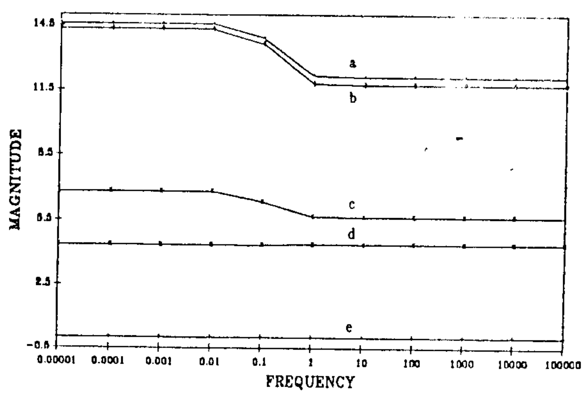

Fig. 5a. $\hat{R}sd$, $\hat{R}dd'$ and $\rho(B)$ of various pairing structures in example 1. a: $\hat{R}dd'$ of $(y_1{:}x_1)$ $(y_2{:}x_2)$ $(y_3{:}x_3)$, b: $\hat{R}sd=\hat{R}dd'$ of $(y_1,y_2{:}x_1,x_2)$ $(y_3{:}x_3)$, c: $\hat{R}sd$ of $(y_1{:}x_1)$ $(y_2{:}x_2)$ $(y_3{:}x_3)$, d: $\hat{R}sd=\hat{R}dd'$ of $(y_1{:}x_1)$ $(y_2,y_3{:}x_2,x_3)$, e: $\rho(B)$ of every pairing case

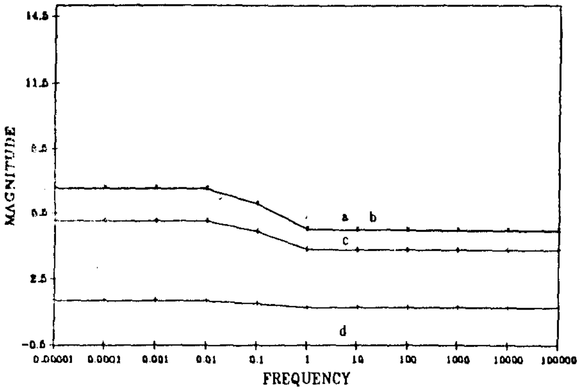

Fig. 5b. $\hat{R}sd'$, $\hat{R}dd$ of various pairing structures in example 1. a: $\hat{R}dd$ of $(y_1{:}x_1)$ $(y_2{:}x_2)$ $(y_3{:}x_3)$, b: $\hat{R}sd'=\hat{R}dd$ of $(y_1{:}x_1)$ $(y_2,y_3{:}x_2,x_3)$, c: $\hat{R}sd'$ of $(y_1{:}x_1)$ $(y_2{:}x_2)$ $(y_3{:}x_3)$, d: $\hat{R}sd'=\hat{R}dd$ of $(y_1,y_2{:}x_1,x_2)$ $(y_3{:}x_3)$.

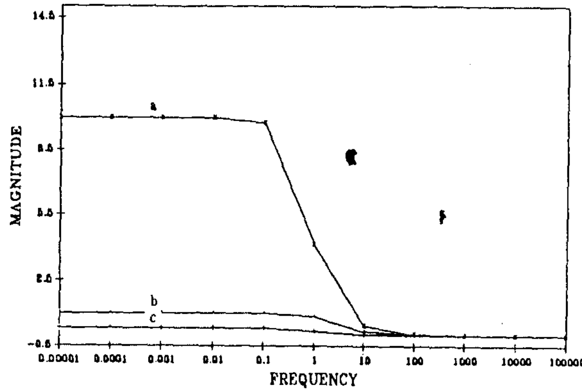

Fig. 6a. $\hat{R}sd$, $\hat{R}dd'$, $\hat{R}sd'$, $\hat{R}dd$ and $\rho(B)$ of the pairing structure $(y_1{:}x_1)$ $(y_2{:}x_2)$ in example 2. a: $\hat{R}sd=\hat{R}dd'$, b: $\hat{R}sd'=\hat{R}dd$, c: $\rho(B)$.

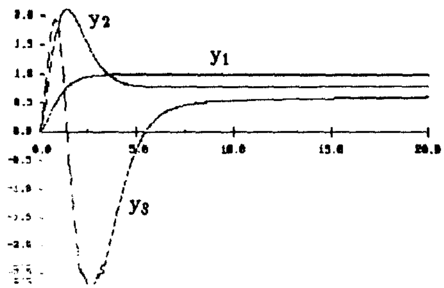

Fig. 5c. Output responses for the pairing $(y_1{:}x_1)$ $(y_2{:}x_2)$ $(y_3{:}x_3)$ in example 1. by the simultaneous setpoint change $r_1{:}1.$, $r_2{:}0.8$, $r_3{:}0.6$

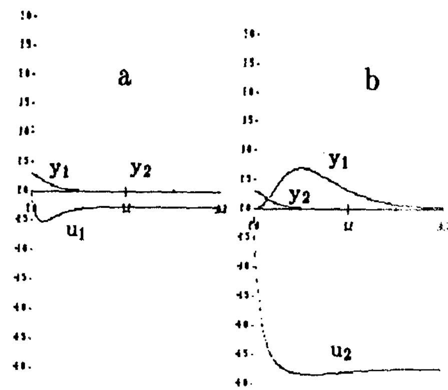

Fig. 6c. Manipulated variable and output responses for the pairing $(y_1{:}x_1)$ $(y_2{:}x_2)$ by the step disturbance in example 2. a: $d_1{:}0.3$, b: $d_2{:}0.3$

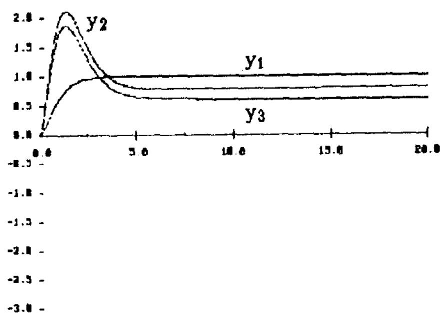

Fig. 5d. Output responses for the pairing $(y_1{:}x_1)$ $(y_2,y_3{:}x_2,x_3)$ by the simultaneous setpoint change $r_1{:}1.$, $r_2{:}0.8$, $r_3{:}0.6$ in example 1.

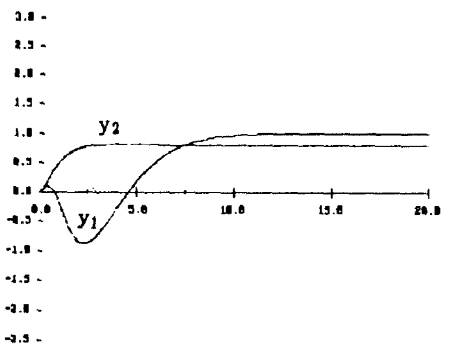

Fig. 6b. Output responses for the pairing $(y_1{:}x_1)$ $(y_2{:}x_2)$ by the simultaneous setpoint change $r_1{:}1.$, $r_2{:}0.8$ in example 2.

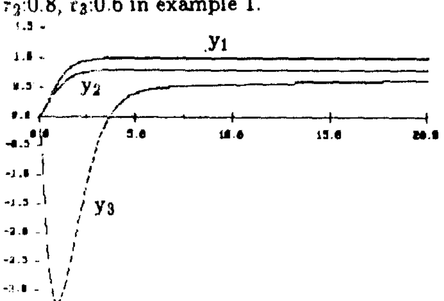

Fig. 5e. Output responses for the pairing $(y_1,y_2{:}x_1,x_2)$ $(y_3{:}x_3)$ by the simultaneous setpoint change $r_1{:}1.$, $r_2{:}0.8$, $r_3{:}0.6$ in example 1.

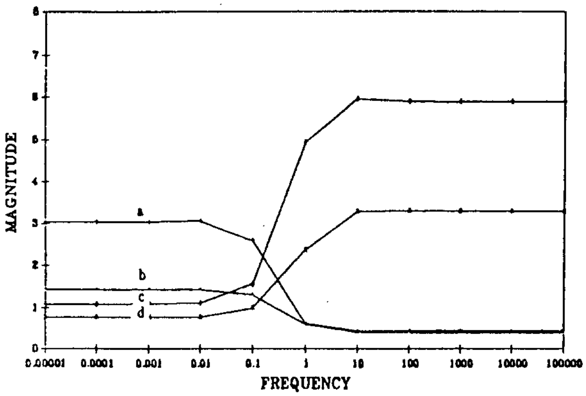

Fig. 7a. $\hat{R}sd$, $\hat{R}dd'$, $\hat{R}sd'$ and $\hat{R}dd$ of various pairing structures in example 3. a: $\hat{R}sd=\hat{R}sd'$ of $(y_1{:}x_3)$ $(y_2{:}x_2)$ $(y_3{:}x_1)$, b: $\hat{R}dd=\hat{R}dd'$ of $(y_1{:}x_3)$ $(y_2{:}x_2)$ $(y_3{:}x_1)$, c: $\hat{R}sd=\hat{R}sd'$ of $(y_1{:}x_1)$ $(y_2{:}x_3)$ $(y_3{:}x_2)$, d: $\hat{R}dd=\hat{R}dd'$ of $(y_1{:}x_1)$ $(y_2{:}x_3)$ $(y_3{:}x_2)$.

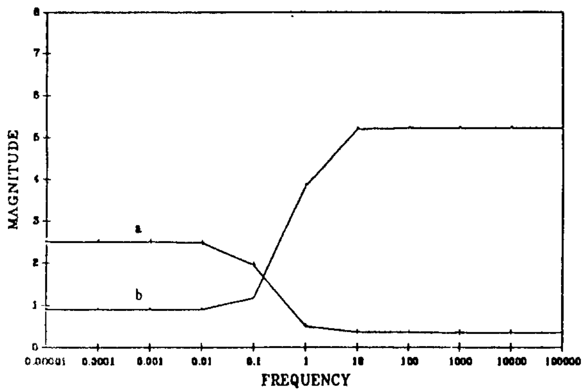

Fig. 7b. $\hat{R}sd=\hat{R}dd'=\hat{R}sd'=\hat{R}dd$ of various pairing structures in example 3. a: $(y_1,y_2{:}x_2,x_3)$ $(y_3{:}x_1)$ or $(y_1{:}x_3)$ $(y_2,y_3{:}x_1,x_2)$, b: $(y_1{:}x_1)$ $(y_2,y_3{:}x_2,x_3)$ or $(y_1,y_2{:}x_1,x_3)$ $(y_3{:}x_2)$

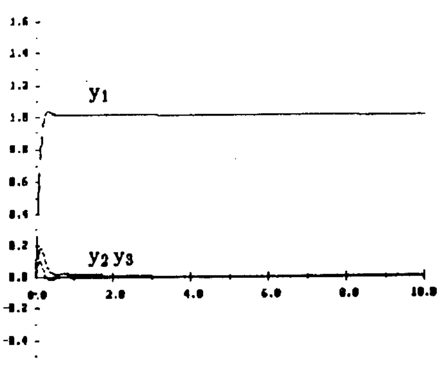

Fig. 7c. Output responses for the pairing $(y_1{:}x_3)$ $(y_2{:}x_2)$ $(y_3{:}x_1)$ by the setpoint change $r_1{:}1.$ in example 3.

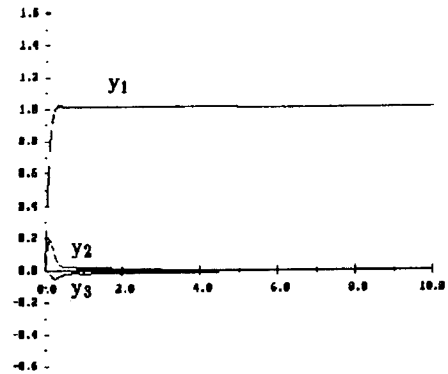

Fig. 7d. Output responses for the pairing $(y_1,y_2{:}x_2,x_3)$ $(y_3{:}x_1)$ by the setpoint change $r_1{:}1.$ in example 3.

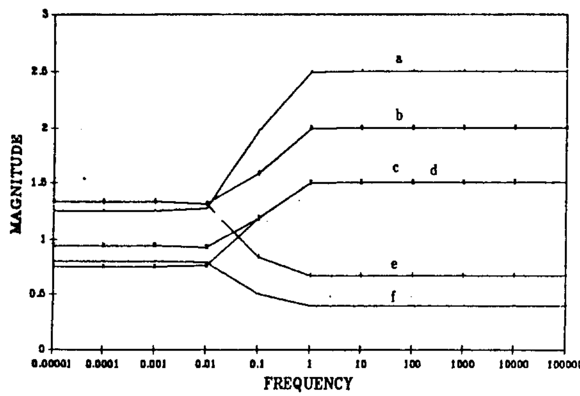

Fig. 8a. $\hat{R}sd=\hat{R}dd'$ of various pairing structures in example 4. a: $(y_2{:}x_1,x_2)$ $(y_1{:}x_3)$, b: $(y_1{:}x_1,x_3)$ $(y_2{:}x_2)$, c: $(y_1{:}x_2)$ $(y_2{:}x_1,x_3)$, d: $(y_1{:}x_1)$ $(y_2{:}x_2,x_3)$, e: $(y_1{:}x_2,x_3)$ $(y_2{:}x_1)$, f: $(y_1{:}x_1,x_2)$ $(y_2{:}x_3)$.

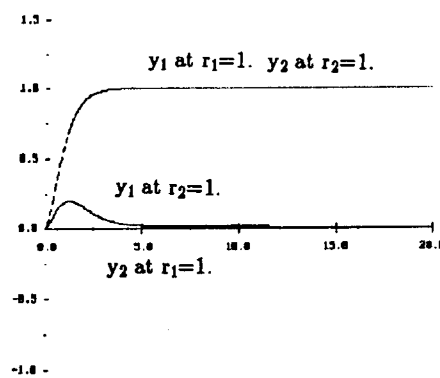

Fig. 8b. Output responses for the pairing $(y_1{:}x_1,x_2)$ $(y_2{:}x_3)$ by the setpoint change in example 4.