'89 KACC 1989. 10. 27～28

ROBUSTNESS IMPROVEMENT OF DIRECT DECENTRALIZED MODEL REFERENCE ADAPTIVE CONTROL

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The control of a class of large scale systems formed by an arbitrary linear interconnections of linear time-invariant subsystems with unknown parameters is investigated. An approach is developed for improving the robustness of such a large scale system. In doing so, the new parameter adaptation algorithm(PAA) is used and a sufficient condition of stability is discussed by using the sector theory.

1. Intorduction

Decentralized adaptive control(DAC) of a large scale system with unknown parameters has the advantages in computational aspect, but it can be unstable due to unmodeled interconnections. Therefore, in a decentralized adaptive control system, the problem of improving the robustness becomes very important. And this leads to the design problem of robust adaptive controller for a plant with unknown parameters.

Recent attempt to design the robust adaptive controller has invariably proceeded. And the modification of the parameter adaptation algorithm (PAA) takes a large part in the studies of the robutness improvment. Among many modification of the adaptive law suggested, three of them are widely accepted. They are the introduction of the dead-zone in the PAA[1], the restriction of the parameter search region[2] and σ-modification which introduces an additional term of the form $\sigma\theta$ in the PAA for adjusting the parameter vector[3]. And σ-modification PAA is the more adequate algorithm for the DAC because it has the advantages over the two earlier methods in that it assures the boundedness of solutions without any additional informations regarding the system.

Ioannou and Kokotovic gave a general solution for the design of a continuous DAC by using the σ-modification PAA. Most earlier studies for the design of a DAC are restricted to a special class of interconnected subsystems[4][5]. Discrete version of DAC was presented by Wiemer and Unbehauen[6]. They also used the σ-modification.

In this paper, we proposed the new PAA to make the discrete decentralized adaptive control systems more robust than the others with existing PAAs. Newly proposed PAA is the modification of recursive leastsquare (RLS) algorithm. It differs from RLS algorithm in that it introduces the signal normalization and the additional term which is proportional exponentially to the nomalized local tracking errors.

2. Problem formulation

Consider a linear time-invariant system which is described as an interconnection of N subsystems and is represented by

$$x_i(k+1) = A_i x_i(k) + b_i u_i(k) + f_i(x) \qquad (1.a)$$

$$f_i(x) = \sum_{\substack{j=1 \\ j \neq i}}^{N} A_{ij} x_j \qquad (1.b)$$

$$y_i = h_i^T x_i, \qquad i = 1,2, \ldots, N \qquad (1.c)$$

which for the i-th subsystem $x_i \in R^{n_i}$ is the state vector, $u_i \in R^1$ is the control variables, $y_i \in R^1$ is the output, and $f_i(x) \in R^{n_i}$ is the interaction vector from the other subsystems. The parameters A_i, A_{ij}, b_i, and h_i are unknown constant matrices.

In this representation the composite system is described as

$$x(k+1) = Ax(k) + Bu(k) + Hx(k) \qquad (2.a)$$

$$y(k) = C^T x(k) \qquad (2.b)$$

where $x = [\, x_1^T, x_2^T, \ldots\ldots, x_N^T]^T$ is the composite state vector, $y = [\, y_1 \,, y_2 \,, \ldots \; y_N]^T$ is the composite output vector, $u = [\, u_1, \; u_2 \,, \; \ldots u_N \,]^T$ is the composite control vector and

$$H = \begin{bmatrix} 0 & A_{12} & A_{13} & \cdots & A_{1N} \\ A_{21} & 0 & \ddots & & \vdots \\ \vdots & & & \ddots & \vdots \\ A_{N1} & \cdots & \cdots & \cdots & 0 \end{bmatrix} \qquad (2.c)$$

is the interconnection matrix. Furthermore

$$A = \mathrm{diag}(A_i), \; B = \mathrm{diag}(b_i) \text{ and } C = \mathrm{diag}(h_i^T)$$

For the above plant, the input-output representation of the i-th local subsystem is described as follows

$$A_i(q^{-1})Y_i(k) = q^{-d} B_i(q^{-1}) u_i(k) + \delta_i(k) \qquad (3)$$

where $\delta_i(k)$ is the disturbance sequence considering the effect of the interconnections.

It is assumed that $B_i(q^{-1})$ is a monic Hurwitz polynomial and $A_i(q^{-1})$ is a monic polynomial of degree m_i. The coefficients of $A_i(q^{-1})$, $B_i(q^{-1})$ are unknown constants.

The problem is to design LACRs (Local Adaptive Controllers) such that the output of the composite system (2.1) (2.2) are regulated to zero or track the output of a given reference model formed of the local reference models. Each subsystem is controlled independently on the basis of its own performance criterion and locally provided information: that is, there is no sharing of information among the indivisual adaptive control agents.

3. Adaptive Control Using the local output of the plant

The adaptive decentralized control scheme is shown in Fig.1

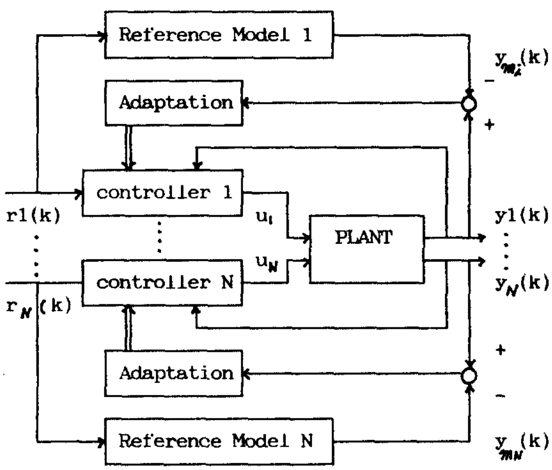

Fig.1 Adaptive decentralized control scheme

We now consider the general problem of DAC of a large scale system which is described by (3). In this representation only local outputs are available for measurement and output of each subsystem is required to track the output y_{m_i} of an n_i-th order reference model.

$$A_{m_i}(q^{-1})y_{m_i}(k) = q^{-d'}gB_{m_i}(q^{-1})r_i(k) \quad (4)$$

where $r_i(k)$ is a uniformly bounded reference input signal.

We assume that $A_{m_i}(q^{-1})$ is stable and $d = d'$

The controller structure for the i-th subsystem is designed as follows

System model (3) can be expressed in predictor form for $A_{m_i}(q^{-1})y_i(t+d)$

$$A_{m_i}(q^{-1})y_i(k+d) = \alpha_i(q^{-1})y_i(k) + \beta_i(q^{-1})u_i(k) \quad (5)$$

where $\alpha_i(q^{-1}) = G_i(q^{-1})$, $\beta_i(q^{-1}) = F_i(q^{-1})B_i(q^{-1})$

$F_i(q^{-1})$ and $G_i(q^{-1})$ are unique polynomials of order $m+d-1$, n_i-1, respectively satisfying

$$A_{m_i}(q^{-1}) = F_i(q^{-1})A_i(q^{-1}) + q^{-d}G_i(q^{-1}) \quad (6)$$

For the reference model shown in (4), the corresponding model reference control law is as follows

$$\alpha_i(q^{-1})y_i(k) + \beta_i(q^{-1})u_i(k) = g\,B_{m_i}(q^{-1})r_i(k) \quad (7)$$

when Θ_{i0} is known, the control law (7) can be written as

$$u_i(k) = \phi_i^T(k)\Theta_{i0} \quad (8.a)$$

where

$$\Theta_{i0}(k) = [\ \alpha_{i0},\dots,\alpha_{i\,n-1},\beta_{i1},\dots,\beta_{i\,m+d-1},1/\beta_{i0}\ ] \quad (8.b)$$

$$\phi_i(k) = [-y_i(k),\dots,-y_i(k-n+1),-u_i(k-1),\dots$$
$$-u_i(k-m-d+1),ra_i(k)]^T \quad (8.c)$$

$$\text{with } ra_i(k) = gB_{m_i}(q^{-1})r_i(k) \quad (8.d)$$

When we know the plant parameters, we can design the controller in the form of (7) by using the plant parameters. But in the case of unknown parameters, i.e. in the adaptive control system, we must determine the controller parameters using the input,output information through a PAA. Therefore in the adaptive control system, PAA is the most important factor in all respect.

5. A New Parameter Adaptation Algorithm

The presence of interconnection H can change the stability properties of the decoupled subsystems, and it is necessary to obtain the sufficient conditions to guarantee the stability of the overall system. In this section, to relax the stability condition for the interconnections, we propose the new PAA as follows.

$$\hat{\Theta}_i(k) = E_i(k)\hat{\Theta}_i(k-d) + P_i(k)\bar{\phi}_i(k)\bar{e}_i(k) \quad (9.a)$$

$$E_i(k) = \exp(-\lambda_i|\ \bar{e}_i(k)\ |\ ) \quad (9.b)$$

$$\text{where } e_i(k) = A_{m_i}(q^{-1})(y_i(k) - y_{m_i}(k)) \quad (9.c)$$

λ_i is a positive constant

$$P_i(k) = P_i(k-d) - \frac{P_i(k-d)\bar{\phi}_i^T(k-d)\bar{\phi}_i(k-d)P_i(k-d)}{\beta_i + \bar{\phi}_i^T(k-d)P_i(k-d)\bar{\phi}_i(k-d)} \quad (9.d)$$

The control is then generated from

$$u_i(k) = \phi_i^T(k)\hat{\Theta}(k) \quad (10)$$

In the above $(\bar{\ })$ is used to denote normalized variables and corresponding operators are defined as follows

$$\bar{\phi}_i(k-d) = \rho_i(k)^{-\frac{1}{2}}\phi_i(k-d),\ \bar{e}_i(k) = \rho_i(k)^{-\frac{1}{2}}e_i(k) \quad (11)$$

Normalization factor $\rho_i(k)$ is defined as follows

$$\rho_i(k) = \tau\rho_i(k-1) + \max(|\phi_i(k-d)|, \rho_i), \rho_i > 0, \tau \in (0,1) \quad (12)$$

In the case of standard RLS, $E_i(k) = 1$.

Note that exponentially propotional to the normalized tracking error, $E(k)$ retards the parameter adaptation.

5. The convergence properties of the new PAA

Theorem 1. For the algorithm (9.a) (9.b) the signal $\tilde{\theta}_i(k) = \hat{\theta}_i(k) - \theta_i^*$ is bounded as

$$\{ \tilde{\theta}_i \mid \frac{\bar{e}_i(k-d)^2}{1 + \phi_i^T(k-d)\, P_i(k-d)\phi_i(k-d)} + S2_i \| \tilde{\theta}_i(k) + (I - E_i(k)I + Sm_i(k))^T\theta_i^*/2 \|^2 < S2_i \| \theta_i^*(I - E(k)I + Sm_i(k)) \|^2 /4 \} \quad (13)$$

If $(E_i(k)I - \dfrac{P_i(k-d)\phi_i^T(k-d)\phi_i(k-d)}{1 + \phi_i^T(k-d)\, P_i(k-d)\phi_i(k-d)})$ (14)

is positive definite

where $S2_i, Sm_i$ are defined in the proof

proof:

For convenience, we set the forgetting factor equal to unity.

Substracting θ_i^* from the both sides of (9.b) we obtain

$$\tilde{\theta}_i(k) = \tilde{\theta}_i(k-d) - \frac{P_i(k-d)\phi_i^T(k-d)\phi_i(k-d)\ \tilde{\theta}_i(k-d)}{1 + \phi_i^T(k-d)\, P_i(k-d)\phi_i(k-d)} + (E_i(k)-1)\hat{\theta}_i(k-d) \quad (15)$$

Then, using (9.b) and the matrix inversion lemma, we have

$$\tilde{\theta}_i(k) = P_i(k)P_i(k-d)^{-1}\,\tilde{\theta}_i(k-d) + (E_i(k) -1)\hat{\theta}_i(k-d) \quad (16)$$

Hence introducing $V_i(k) = \tilde{\theta}_i(k)\ P_i(k)^{-1}\,\tilde{\theta}_i(k)$, we have

$$\Delta V_i(k) = V_i(k) - V_i(k-d)$$
$$= \tilde{\theta}_i(k)\ P_i(k)^{-1}\ \tilde{\theta}_i(k) - \tilde{\theta}_i(k-d)\ P_i(k-d)^{-1}\ \tilde{\theta}_i(k-d) \quad (17)$$

Substituting the Eq.(16) into (17)

$$\Delta V_i(k) = (\tilde{\theta}_i(k) - \tilde{\theta}_i(k-d))^T P_i(k-d)^{-1}\,\tilde{\theta}_i(k-d) + \tilde{\theta}_i^T(k)P_i(k)^{-1}\ (E_i(k)-1)\hat{\theta}_i(k-d) \quad (18)$$

Eq.(15) can be written as follows

$$P_i(k-d)^{-1}(\tilde{\theta}_i(k) - \tilde{\theta}_i(k-d)) = -\frac{\phi_i(k-d)\phi_i^T(k-d)\ \tilde{\theta}_i(K-d)}{1 + \phi_i^T(k-d)\, P_i(k-d)\phi_i(k-d)} + P_i(k-d)^{-1}(E_i(k)-1)\hat{\theta}_i(k-d) \quad (19)$$

Substituting Eq(19) into (18), we obtain

$$\Delta V_i(k) = -\frac{\tilde{\theta}_i^T(k-d)\phi_i(k-d)\phi_i^T(k-d)\tilde{\theta}_i(k-d)}{1 + \phi_i^T(k-d)\, P_i(k-d)\phi_i(k-d)} + 2\tilde{\theta}_i^T(k)P_i^{-1}(E_i(k) -1)\hat{\theta}_i(k-d) \quad (20)$$

where $P_i^{-1} = P_i(k-d)^{-1} + P_i(k)^{-1}$

To replace $\hat{\theta}_i(k-d)$ with $\tilde{\theta}_i(k)$ relation we obtain the following

$$\tilde{\theta}_i(k) = Sm_i(k)\tilde{\theta}_i(k-d) + (E_i(k)-1)\theta_i^* \quad (21)$$

where

$$Sm_i(k) = (E_i(k)I - \frac{P_i(K-d)\phi_i^T(k-d)\phi_i(k-d)}{1 + \phi_i^T(k-d)\, P_i(k-d)\phi_i(k-d)}) \quad (22)$$

trerefore

$$\tilde{\theta}_i(k-d) = Sm_i^{-1}(k)(\tilde{\theta}_i(k) - (E_i(k) -1)\theta_i^*)$$
$$\hat{\theta}_i(k-d) = Sm_i^{-1}(k)(\tilde{\theta}_i(k) - (E_i(k) -1)\theta_i^*) + \theta_i^* \quad (23)$$

Substituting EQ.(23) into (20)

$$\Delta V_i = -\frac{\bar{e}_i(k-d)^2}{1 + \phi_i^T(k-d)\, P_i(k-d)\phi_i(k-d)} + 2(E_i(k) -1)(\ \tilde{\theta}_i^T(k)\ P_i^{-1}Sm_i^{-1}(k)\tilde{\theta}_i(k) + \tilde{\theta}_i^T(k)P_i^{-1}(Sm_i^{-1}(k) - E_i(k)Sm_i^{-1}(k)) + I)\theta_i^*) \quad (24)$$

Hence ,if $Sm_i(k) > 0$, we obtain the relation $\Delta V < 0$ in D_i^c

The set D_i is defined as follows

$$D_i = \{ \tilde{\theta}_i \mid \frac{\bar{e}_i(k-d)^2}{1 + \phi_i^T(K-d)\, P_i(k-2d)\phi_i(k-d)} + S2_i \| \tilde{\theta}_i(k) + (Sm_i^{-1}(k) - E_i(k)Sm_i^{-1}(k) + I)\theta_i^*/2 \|^2_{P^{-1}} < S2_i \| (Sm_i^{-1}(k) - E_i(k)Sm_i^{-1}(k) + I)\theta_i^* \|^2_{P^{-1}} /4 \} \quad (25)$$

where $S2_i = 2(1-E_i(k))$ (26)

Q.E.D

Note that even when $Sm_i < 0$, there is a possibility that $\Delta V_i < 0$ if

$$\left| \frac{\bar{e}_i(k-d)^2}{1 + \phi_i^T(K-d)\, P_i(k-2d)\phi_i(k-d)} \right| > S2_i \left| \tilde{\theta}_i(k)P_i^{-1}\tilde{\theta}_i(k) + \tilde{\theta}_i(k)P_i^{-1}(Sm_i(k) - E_i(k)I + I)\theta_i^* \right|$$

In the next section, we will obtain the sufficient condition of the stability of decentralized adaptive control system with the sector theory when the new PAA is used.

6. Reformulation into a decentralized error model

In order to analyze the adaptive control system using the input-output stability theory, we reformulate the adaptive system into an error feedback system.

Define a set of tuned parameters

$$\theta_i^* = [\ \alpha_i^* \quad \beta_i^*\ ] \tag{27}$$

Define the polynomial C_i in terms of the tuned parameters

$$C_i = A_i\alpha_i^* + q^{d}\beta_i^* B_i \tag{28}$$

Multiplying Eq.(3) by polyomial C_i of Eq.(28) gives

$$C_i\bar{y}_i(k) = B_i[\alpha_i^*\bar{u}_i(k-d) + \beta_i^*\bar{y}_i(k-d)] + S_i^*\bar{\delta}_i(k) \tag{29}$$

Thus, from Eq.(29)

$$y_i(k) = C_i^{-1}B_i\,\theta_i^{*T}\bar{\Phi}_i(k-d) + C_i^{-1}\alpha_i^*\bar{\delta}_i(k) \tag{30}$$

We can calculate the normalized filtered tracking error from Eq.(19.c) and Eq.(30)

$$\begin{aligned}\bar{e}_i(k) &= A_{m_i}\bar{y}_i(k) - r_i(k)\\ &= -H2(\hat{\theta}_i(k-d) - \theta_i^*)\bar{\Phi}_i(k-d)\\ &\quad + A_{m_i}C_i\,\alpha_i^*\bar{\delta}_i(k) + (H2_i-1)r_i(k)\\ &= -\bar{H}2_i\,\bar{\psi}_i(k) + \bar{e}_i(k)\end{aligned} \tag{31.a}$$

where $\bar{H}2_i = A_{m_i}C_i^{-1}B_i$, $\bar{e}_i(k) = A_{m_i}C_i^{-1}\alpha_i^*\bar{\delta}_i(k) + (\bar{H}2_i-1)\bar{r}_i(k)$

$$\psi_i(k) = \tilde{\theta}_i^{T}(K-d)\ \phi_i(k-d) \tag{31.b}$$

$$\bar{\psi}_i(k) = \rho(k)^{-\frac{1}{2}}\psi_i(k) \tag{31.c}$$

$$\bar{H}i_i = \rho_i(k)^{-\frac{1}{2}}Hi_i[\ \rho_i(k)^{\frac{1}{2}}\ ]\ ;\ i=1,2 \tag{31.d}$$

This error feedback system can be described by Fig.2

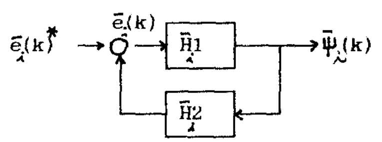

Fig.2 Error model

In this error model, $\bar{H}1_i : \bar{e}_i(k) \rightarrow \bar{\psi}_i(k)$ denotes an operator defined by the new PAA. The adaptive control problem may now be analyzed as a feedback system using input-output stability theory.

Note that normalization permits unmodeled interconnections δ_i to be treated as bounded disturbance.

7. Sufficient condition for stability

To analyze the stability of the decentralized adaptive control systems, the sector theory will be applied to the error model in Fig.2 The theory gives conditions for the global stability of a feedback configuration consisting of two conic bounded operators. Now, a sector condition for H1 is cosidered and the corresponding H2's sector condition stabilizing the closed loop shown in Fig.1 is also considered. The sector condition for H1 is given in the Theorem 2.

Theorem 2. The conic relation $\bar{H}1_i$ is outside the CONE($C1_i$, $R1_i$)

$$\text{where } C1_i = -1/E_i(k),\qquad R1_i = (1-\alpha_i)/E_i(k) \tag{32}$$

$$\text{where } \alpha_i > \frac{\bar{\Phi}_i^{T}(k-d)\ P_i(k-d)\bar{\Phi}_i(k-d)}{\beta_i + \bar{\Phi}_i^{T}(K-d)\ P_i(k-d)\bar{\Phi}_i(k-d)} \tag{33}$$

Proof: see appendix

A conic relation H2 stabilizing the closed loop system in Fig.2 is given in Theorem 3.

Theorem 3. For the H1's conic sector, an allowable conic sector H2 stabilizing the closed loop in Fig.2 is outside the CONE(C2,R2)

$$\text{where } C2_i = C1_i/(R1_i^2 - C1_i^2),\ R2_i^2 = R1_i^2/(R1_i^2 - C1_i^2)^2 \tag{34}$$

proof : This is a straightforward application of [7 , Theorem.2b p235)

It is known that the value of α in Eq.(33) and R1 are determined by the maximum eigenvalue of P(0) [8]. From the result of Theorem 2, we can see that the value of R1 can be determined by the term E(k) as well as the maximum eigenvalue of P(0) when the new PAA is employed. This provides a possibility of robustness improvement. When the normalized tracking error becomes large, from Eq.(9.b), the term E(k) becomes smaller and this makes R1 larger. Therefore we can see that allowable conic sector of H2 stabilizing the closed loop in Fig.2 becomes larger than that of the standard RLS($E_i(k)=1$). This explains the improvement of robustness because the larger $\bar{H}2_i$'s conic sector guarantees the more robust adaptive control system (the connection between the size of H2, its allowable cone, and robustness is made clear in [9]).
Up to now, we considered the stability of normalized signal but that of unnormalized signal. The stability of normalized signal doesn't always guarantee the stability of unnormalized signal. The relation between Hi's and Hi's conic sector and the relation between the stability of normalized signal and that of unnormalized signal were shown in [10].

8. Numerical Examples and computer simulation

Plant is given as follows

$$x(k+1) = \begin{bmatrix} 1. & -1. & 1.9 & 0.1053 \\ 0.005 & 0.85 & -1.9 & -0.1053 \\ 0.724 & 0.276 & 1. & 0.01 \\ -0.11 & -0.18 & 0.11 & 1. \end{bmatrix} x(k) + \begin{bmatrix} 0.6277 \\ -0.127706 \\ 1.88383 \\ 0.106166 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 1. & 1. & 0. & 0. \\ 0. & 0. & 1. & 1. \end{bmatrix} x(k)$$

Reference model is given as follows

Model1 : $(1-1.65q^{-1}-0.68q^{-2})y_{m1}(k)$

$= q^{-1}(0.5-0.5q^{-1})r_1(k)$

Model2 : $(1-1.7q^{-1}-0.72q^{-2})y_{m2}(k)$

$= q^{-1}(1-0.5q^{-1})r_2(k)$

$r_i(k) = 1.$

$\rho_i(k) = 0.5\rho_i(k-1) + \max(|\phi(k-1)|, 1)$

$P_i(0) = 100 \times I_{4\times4}$, $I_{4\times4}$ is 4×4 identity matrix

$\phi_i(0) = [\,0.\ 0.\ 1.\ 0.25\,]^T$

$\theta_i(0) = [\,1.\ 1.\ 1.\ 1.\,]^T$

Computer simulation results are shown in Fig.s 3,4,5,6 . From the simulation results, the DAC with new PAA is more robust than that with normalized σ-modification PAA.

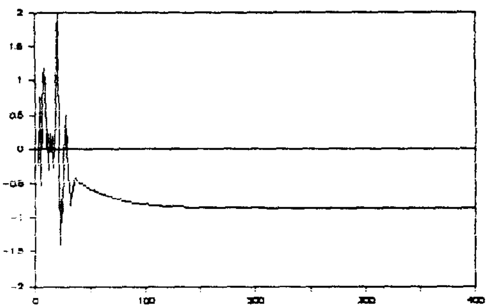

Fig.3 Filtered tracking error of the first subsystem with normalized σ-modification RLS algorithm in the case of $\sigma = 0.97$

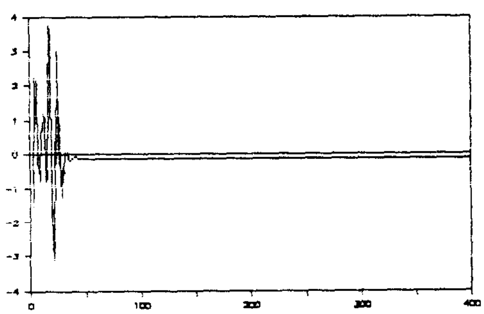

Fig.4 Filtered tracking error of the second subsystem with normalized σ-modification RLS algorithm in the case of $\sigma = 0.97$

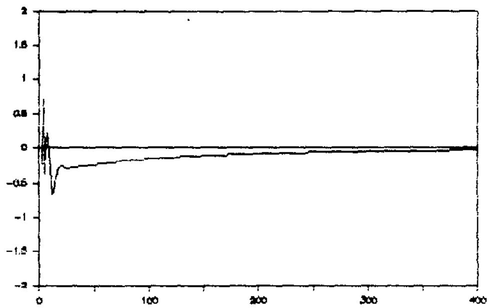

Fig.5 Filtered tracking error of the first subsystem with new PAA in the case of $\lambda = 0.8$

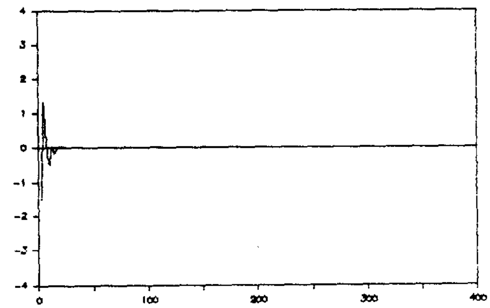

Fig.6 Filtered tracking error of the second subsystem with new PAA in the case of $\lambda = 0.8$

9. Conclusion

The problem of decentralized adaptive control was considered. To prove the robustness improvement of the DAC scheme, the parameter adaptation algorithm and the effect of interconnections of the different control loops were separated into different blocks, i.e. a time varying feedforward and a linear time invariant block. Gloval stability of the associate error model has been proven by applying the powerful sector stability theorem [10]. It was shown that conventional adaptation algorithms have to be modified. The allowable class of unmodeled interconnections is extended when the new PAA is applied. Therefore robustness improvement is achived. References

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Appendix

Proof of the Theorem 1.

Consider the quadratic function

$$V_i(k) = \hat{\tilde{\theta}}_i(k)\ P_i(k)^{-1}\ \hat{\tilde{\theta}}_i(k) \qquad (A.1)$$

where $\hat{\tilde{\theta}}_i = \hat{\theta}_i - \theta_i^*$

Substracting θ_i^* from both sides of Eq.(9.a), we obtain the relation as follows

$$\hat{\tilde{\theta}}_i(k) = E_i(k)\hat{\tilde{\theta}}_i(k-d) + P_i(k)\bar{\phi}_i(k-d)\bar{e}_i(k) + D_i(k) \qquad (A.2)$$

where $D_i(k) = (E_i(k) - 1.)\theta_i^*$

$V_i(k)$ is rearranged using the Eq.(A.2)

$$V_i(k) = [\ E_i(k)\hat{\tilde{\theta}}_i(k-d) + P_i(k)\bar{\phi}_i(k-d)\bar{e}_i(k) + D_i(k)\ ]^T$$
$$P_i(k)^{-1}[E_i(k)\hat{\tilde{\theta}}_i(k-d) + P_i(k)\bar{\phi}_i(k-d)\bar{e}_i(k) + D_i(k)\ ]$$
$$= E_i(K)^2\ \hat{\tilde{\theta}}_i^T(k-d)\ P_i(k)^{-1}\hat{\tilde{\theta}}_i(k-d)$$
$$+2E_i(k)\hat{\tilde{\theta}}_i^T(k-d)\bar{\phi}_i(k-d)\bar{e}_i(k)+\bar{\phi}_i(k-d)\ P_i(k)\bar{\phi}_i(k-d)\bar{e}_i(k)^2$$
$$+2D_i^T(k)\ P_i(k)^{-1}\ E_i(k)\hat{\tilde{\theta}}_i(K-d) + 2D_i(k)^T\ \bar{\phi}_i(k-d)\bar{e}_i(k)$$
$$+D_i(k)^T\ P_i(k)^{-1}\ D_i(k)$$
$$=E_i(k)^2\ \hat{\tilde{\theta}}_i(k-d)^T(\beta_i P_i(k-d)^{-1} + \bar{\phi}_i(k-d)\bar{\phi}_i^T(k-d)\ )\hat{\tilde{\theta}}_i(k-d)$$
$$+2E_i(k)\hat{\tilde{\theta}}_i^T(k-d)\bar{\phi}_i(k-d)\bar{e}_i(k)+\bar{\phi}_i(k-d)\ P_i(k)\bar{\phi}_i(k-d)\bar{e}_i(k)^2$$
$$+ M_i(k)$$
$$= \beta_i E_i(k)^2\ V_i(k-d) + E_i(k)^2\ \bar{\psi}_i(k)^2 + 2E_i(k)\bar{\psi}_i(k)e_i(k)$$
$$+ \bar{\phi}_i^T(k-d)\ P_i(k)\bar{\phi}_i(k-d)\bar{e}_i(k)^2 + M_i(K) \qquad (A.3)$$

where $M_i(k) = 2D_i^T(k)\ P_i(k)^{-1}\ E_i(k)\hat{\tilde{\theta}}_i(k-d)$

$$+\ 2D_i^T(k)\ \bar{\phi}_i(k-d)\bar{e}_i(k) + D_i^T(k)\ P_i(k)^{-1}\ D_i(k)$$

If we make the term $\beta E(k)^2$ equal to unity, we can obtain the inequality as follows

$$\bar{\psi}_i(k)^2 + 2\bar{\psi}_i(k)e_i(k)/E_i(k)$$
$$+\ \bar{\phi}_i^T(k-d)\ P_i(k)\bar{\phi}_i(k-d)\bar{e}_i(k)^2\ /E_i(k)^2$$
$$\geq (V_i(k) - V_i(k-d))/E_i(k)^2$$
$$- M_i(k)/E_i(k)^2 \qquad (A.4)$$

Summing from 0 to N leads to

$$\| \bar{\psi}_i(k) + \bar{e}_i(k)/E_i(k) \|_N^2 - \| (1 - \bar{\phi}_i(k-d)\ P(k)\bar{\phi}_i(k-d))/E_i(k) \|_N^2\ \| e_i(k) \|_N^2$$
$$\geq -\sum_{k=1}^{d} V_i(k)/E_i(k) - \sum_{k=1}^{N} M_i(k)/E_i(k) \qquad (A.5)$$

We use the equation as follows[10]

$$\bar{\phi}_i^T(k-d)\ P_i(k)\bar{\phi}_i(k-d) = \frac{\bar{\phi}_i^T(k-d)\ P_i(k-d)\bar{\phi}_i(k-d)}{\beta_i + \bar{\phi}_i^T(K-d)\ P_i(k-d)\bar{\phi}_i(k-d)}$$

and whether M(k) is positive or not, inequality (A.5) can be represented as follows

$$\| \bar{\psi}_i(k) + \hat{e}_i(k)/E_i(k) \|_N^2 - \| (1 - \alpha_i)/E_i(k) \|_N^2\ \| e_i(k) \|_N^2 \geq -Q_i \qquad (A.6)$$

where Q_i is a positive constant greater than $\sum V_i(k) + \sum M_i(k)$ in the case of positive $M_i(k)$ or $\sum V_i(K)$ in the case of negative $M_i(k)$

Q.E.D.