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A note on an adaptive control to certain discrete-time linear system with 2 ordered performance function

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Abstract

To discrete-time linear control system the dynamic characteristics of which include unknown parameters, that is, system equation:

$$A(z^{-1})y_k = B(z^{-1})u_{k-1} \quad k=1,2,3,\ldots$$

and 2 ordered performance function:

$$J(u_k) = \lim_{T\to\infty} \frac{1}{T}\cdot\sum_{s=0}^{T}\left\{ (y_{k+s} - g\cdot\gamma_s\cdot y^*_{k+s})^2 + \lambda u^2_{k+s} \right\} \quad ;\min$$

in this paper, the authors will investigate a control method for this adaptive control system, by the comparison of outputs y_k, in order that output y_k pursues given reference output y^*_k : and based on the calculations by personal computer, the authors will comment on perfect tracking and a large indifference at an early steps with respect to scalar g of this control system.

These results are a little extensional ones of C. Samson.

1. Introduction

It is necessary that when the systems whose dynamic characteristics are not known or are changable little by little with time will be controlled, cotrol inputs must be determined estimating the characteristics of systems. The way of control in these systems is called adaptive control[1),2)]

In this paper, to discrete-time linear single input single output (SISO) control system included unknown parameters ${}_i$, ${}_i$, $i=1,2,\ldots,n$;
that is, system equation:

$$A(z^{-1})y_k = B(z^{-1})u_{k-1} \tag{1}$$

$k= 1,2,3,\ldots$

where

$$A(z^{-1}) = 1-\alpha_1 z^{-1} - \ldots - \alpha_n z^{-n}$$

$$B(z^{-1}) = \beta_1 + \beta_2 z^{-1} + \ldots + \beta_n z^{-n+1}$$

and 2 ordered performance function:

$$J(u_k) = \lim_{T\to\infty} \frac{1}{T}\cdot\sum_{s=0}^{T}\left\{ (y_{k+s} - g\cdot\gamma_s\cdot y^*_{k+s})^2 + \lambda u^2_{k+s} \right\} \quad : \min \tag{2}$$

here

$$\lambda > 0, \quad \gamma_s = \begin{cases} 1 & s=0,1,2,\ldots,N \\ 0 & \text{otherwise} \end{cases},$$

an adaptive control will be considered in order that outputs y_k tracks given reference output y^*_k. Now, scalar g in (2) is unknown constant so that $y_k = y^*_k$ may hold if y^*_k is constant.

An interesting similar research is done by C. Samson[3)], but there are two weak points in it (in case of g=1),

[1] perfect tracking can't be done

[2] a large error between y_k and y^*_k at early steps.

To improve these weak points, the research is done by R. Kaji[4)]. But the way to decide g is more difficult and complex than that research.

Refering to that research, in this paper, the note on scalar g is presented, along with a numerical example.

2. Adaptive control to this discrete-time SISO system

2.1 Adaptive control method

If the parameters α_i, β_i, i=1,2, ...,n are estimated by, for example, least square method etc., that is, a_i, b_i, i=1,2,...,n, respectively, the value of (2) is minimized by the following control u_k in Reference 5):

$$x_{k+1} = Ax_k + bu_k \quad (3)$$

$$u_k = -lx_k + s_k \quad (4)$$

$$y_k = cx_k \quad (5)$$

here

$$\left.\begin{array}{l} A=\begin{bmatrix} a_1 & \vdots & \\ a_2 & \vdots & I_{n-1} \\ \cdot & \vdots & \\ \cdot & \vdots & \\ a_n & \vdots & 0 \quad 0 \quad \ldots 0 \end{bmatrix}, \\ b=\begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ b_n \end{bmatrix}, \quad c=\begin{bmatrix} 1 & 0 & .. & 0 \end{bmatrix} \end{array}\right\} (6)$$

where

$$\left.\begin{array}{l} R = c^T c + A^T RA - l^T ul \\ l = b^T RA / u \\ u = b^T Rb + \lambda \end{array}\right\} (7)$$

$$\left.\begin{array}{l} V_{k,s+1} = (A - bl)V_{k,s} + g \cdot c^T y^*_{k+N-s} \\ \quad s= 0,1,\ldots,N \\ V_{k,0} = 0 \\ s_k = b^T V_{k,N} / u \end{array}\right\} (8)$$

Now, A^T is the transpose of A.

In actual, the values of state vectors x_k can't be measured, so x_k needs to be estimated by outputs y_k. Samson[3] proposed the next expression:

$$\hat{x}_k = A\hat{x}_{k-1} + bu_{k-1} + k(y_{k-1} - c\hat{x}_{k-1}) \quad (9)$$

$$u_k = -l\hat{x}_k + s_k \quad (10)$$

here

$k = \begin{bmatrix} a_1 & a_2 & \ldots & a_n \end{bmatrix}^T$.

In this paper, the expression:

$$\hat{x}_k = A\hat{x}_{k-1} + bu_{k-1} + k(y_k - c\hat{x}_k) \quad (11)$$

will be used.

From (11) and (10), canceling $\hat{x}_k$, the expression:

$$F(z^{-1})u_k + G(z^{-1})y_k = Q(z^{-1})s_k \quad (12)$$

will be got.

Where, $F(z^{-1})$, $G(z^{-1})$, $Q(z^{-1})$ are polynomials with z^{-1}.

By setting (12) into (1), the expression:

$$y_k = \frac{B(z^{-1})Q(z^{-1})}{A(z^{-1})F(z^{-1}) + z^{-1}B(z^{-1})G(z^{-1})} s_{k-1} \quad (13)$$

will be got.

On the other hand, from (3),(4) and (5), the expression:

$$y_k = \left[B(z^{-1}) / P(z^{-1}) \right] s_{k-1} \quad (14)$$

here

$$P(z^{-1}) \equiv \det \left[I - (A - bl)z^{-1} \right] \quad (15)$$

will be taken.

To coincide (13) with (14), the condition:

$$A(z^{-1})F(z^{-1}) + z^{-1}B(z^{-1})G(z^{-1}) = P(z^{-1})Q(z^{-1}) \quad (16)$$

have to hold. From (16), $F(z^{-1})$ and $G(z^{-1})$ are determined. Now, $Q(z^{-1})$ is given.

To decide s_k, relations (7) are denoted the following:

$$\left.\begin{array}{l} d_{k,s} = \dfrac{B(z^{-1})}{P(z^{-1})} (\gamma_s \cdot y^*_{k+N-s}) \\ d_{k,0} = 0 \\ s_k = \left(\dfrac{g}{u}\right) \cdot d_{k,N} \end{array}\right\} (17)$$

Where, g is decided so that $y_k = y^*_k$ may hold if y^* is constant.

In the first expression in (17), when y^*_k is set 1, let the value of $d_{k,N}$ h. Now, g is determined the following:

$$\frac{g}{u} = \frac{P(1)}{B(1)\ h} \quad (18)$$

2.2 Algorithm of adaptive control

By the results of 2.1, in comparison with the case of Samson(g= 1), y_k given in (18) is close to y^*_k certainly. But the calculation in (18) is difficult and complex. So it is meaningful that based on the value of u, the value of scalar g is given $g>1$ or $g<1$ beforehand.

Step 1 Calculate R, l and u with the relations:

$$\begin{cases} R = c^T c + A^T R A - l^T u l \\ l = b^T R A / u \\ u = b^T R b + \lambda \end{cases}$$

Step 2 Calculate $P(z^{-1})$ with the expression:

$$P(z^{-1}) = \det[I - (A - bl) z^{-1}] .$$

Step 3 Set $Q(z^{-1}) = 1$. Derive $F(z^{-1})$ and $G(z^{-1})$ with the expression:

$$A(z^{-1})F(z^{-1}) + z^{-1}B(z^{-1})G(z^{-1}) = P(z^{-1}) .$$

Step 4 Give the value of N and y_k^*. Calculate s_k with the relations:

$$\begin{cases} d_{k,s} = \dfrac{B(z^{-1})}{P(z^{-1})} (\gamma_s \cdot y^*_{k+N-s}) \\ d_{k,0} = 0 \\ \dfrac{g}{u} = \dfrac{P(1)}{B(1)\ h} \end{cases}$$

$$s_k = \frac{g}{u} \cdot d_{k,N} .$$

Step 5 Calculate y_k and u_k with the expressions:

$$y_k = \frac{B(z^{-1})}{P(z^{-1})} s_{k-1}$$

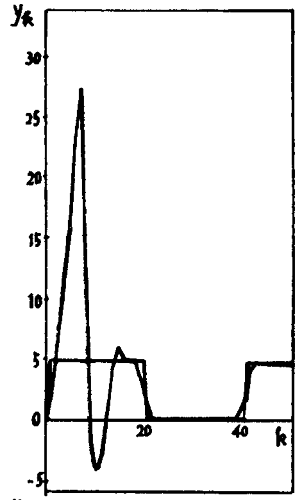
$$u_k = z(1 - f(z^{-1}))u_{k-1} - G(z^{-1}) y_k + s_k .$$

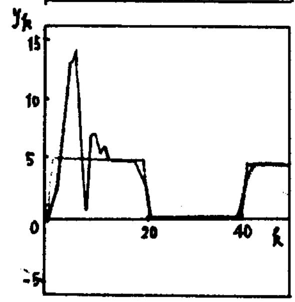
2.3 Numerical example

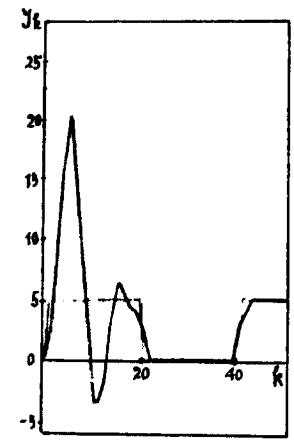
Let $n=3$. Set $a_1 = 1.2$, $a_2 = a_3 = 0$, $b_1 = 0$, $b_2 = 0.5$, $b_3 = 1$ and $\lambda = 1$. Results are the following; now, PC9801 VF is used for the computations.

The first diagram is the case of Samson: g=1 .

The second one is the case of $g = \dfrac{P(1)\ u}{B(1)\ h}$ and the third one is the case of given g= 2 beforehand.







3. Conclusion

The authors, in this paper, investigate the degree of tracking (i.e. the weak points of Samson[3]) to this discrete-time adaptive control system. A matter of course, the results of tracking is improved by using g given in 2.2, compared with the results of Samson. But it is a neck point that the calculation on g is very complex. So by giving the value of g suitably, it is shown that the result superior to one of Samson are taken.

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