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THE ROBUST CONTROLLER DESIGN FOR UNCERTAIN MULTIVARIABLE SYSTEMS USING SWITCHING DYNAMICS

GWI-TAE PARK CHUN-PING KUO DONG-SIK KIM° SUNG-JUN LIM

Department of Electrical Engineering
Korea University

This paper presents the design of simple robust controller for a class of uncertain multivariable systems. We introduce switching dynamics instead of switching logics unlike variable structure control scheme. Also,we can construct the continuous control law from this switching dynamics and consequently remove the chattering motion. The dynamic equations of the range-space of a switching surface matrix C and uniform ultimate boundedness in the presence of parameter uncertainties are described mathematically.

1. Introduction

Variable structure systems (VSS) are a special class of nonlinear systems characterized by a discontinuous control action which changes structure upon reaching a set of switching surfaces $S(x)=0$. A fundermental property of VSS is the sliding motion of the state point on the intersection of the switching surfaces. During the sliding mode the system has invariance properties,yielding motion which is independent of certain parameters and disturbances. Order reduction is a fundermental property of VSS in the sliding mode. This is due to the motion of the state which is constrained to lie on the intersection of the m switching surfaces.

However, due to switching delays,neglected small time constants etc. the trajectories 'chatter' along the switching surfaces resulting in the generation of an undesirable high frequency component which may excite high frequency unmodeled dynamics of the control systems. The drawback mentioned above can be removed by increasing the switching frequency and by reducing the feedback gains in the vicinity of the switching surface. The gains may be adjusted by applying available adaptive algorithms. Some authors suggested the boundary layer in the vicinity of switching surface.[1][6][10] But these methods have difficulties in the parameter selection for the boundary layer.

In this paper we develop the algorithm for the sliding mode control which is able to remove the chattering motion. To obtain the sliding mode we introduce switching dynamics instead of the switching logics which is commonly used in VSC systems. Also,we can construct the continuous control law from this switching dynamics and consequently remove the chattering motion. The dynamic behaviors of the range- space of a switching surface matrix C will be described mathematically for two possible situations. It will be shown that every response of the uncertain system in the presence of unknown parameter variations is ultimately bounded within an arbitrarily small neighborhood of the state-space origin. It will be also shown that the eigenvalues of closed-loop system is composed of the eigenvalues of subsystems which govern the range-space dynamics and null-space dynamics,Thus, we can design the robust controller of uncertain multivariable systems which is much simpler than that of VSC systems.

2. The variable structure controller with switching logics

2.1 System definition

Let us consider the uncertain multivariable system described by

$$\dot{X}(t)=[A+\Delta A(t)]X(t)+[B+\Delta B(t)]U(t) \qquad (1)$$
$$X(t_0) = X_0$$

where X is the state n-vector, U is the control m-vector. It is assumed that $n > m$ and that the pair (A,B) is completely controllable. The matrix ΔA represents the variations and uncertainties in the plant parameters, ΔB is the plant/control interface uncertainties. The nominal system - that is,the system without uncertainties - is described as follows.

$$\dot{X}(t)=AX(t)+BU(t) \qquad (2)$$
$$X(t_0) = X_0$$

If some conditions are satisfied - primary among which are the so-called 'matching conditions' [4] - then all uncertain elements can be 'lumped' and the system is described by

$$\dot{X}(t)=AX(t)+BU(t)+Be(t) \qquad (3)$$
$$X(t_0) = X_0$$

where e(t) is the 'lumped' element such that the absolute value of $e_i(t)$ is bounded by a non-negative constant valued function $f_i(t)$, i.e., $|e_i(t)| < f_i(t)$.

2.2 The variable structure controller design

The overall aim of a variable structure controller design is to regulate the system state from an initial condition $X(t_0)= X_0$ asymptotically to the state space origin as $t \longrightarrow \infty$. There are two basic steps in the design of a variable structure controller [3]:

(1) The design of the switching surface so that the behavior of the system has certain prescribed properties on the switching surface.

(2) The design of control strategy to steer the system to the switching surface and to maintain it there.

In conventional variable structure controller design the jth switching surface s_j passing through the state-space origin is defined by

$$s_j = \{ X \; ; \; c^j X = 0 \} \quad j=1,2,\ldots,m \tag{4}$$

where c^j is a row n-vector. The sliding mode occurs when the state lies simultaneously in each of the surfaces s_j for $j=1,2,\ldots,m$. Assembling the rows c^j into a full rank $m \times n$ matrix C, the sliding mode is attained when the state reaches and remains in the intersection S of the m switching surfaces :

$$S = \bigcap_{j=1}^{m} s_j = \{ X \; ; \; C X = 0 \} \tag{5}$$

In geometric terms the subspace S is the null space of C , denoted N(C).

A sufficient condition for the existence of the sliding mode on the intersection S is that the following inequalities are satisfied.[8]

$$S^T \dot{S} < 0 \tag{6}$$

In general the controller in VSS varies its structure depending on the position relative to the switching surface and has the form:

$$u_i = u_{ieq} + u_{iN} \qquad i=1,2,\ldots,m \tag{7}$$

$$u_{iN} = \begin{cases} u_{iN}^{+}(X) & \text{if } s_i(X) > 0 \\ u_{iN}^{-}(X) & \text{if } s_i(X) < 0 \end{cases}$$

where u_{ieq} is the ith component of the equivalent control -which is continuous- and where u_{iN} is the discontinuous or switched part. Note that there exist several possible discontinuous control structure for u_{iN} .

2.3 The global reachability of variable structure control system

In order to insure the existence of a sliding mode and attractiveness to the surface in the presence of parameter variations, we choose the most simple generalized Lyapunov function as follows.

$$V = ½ \; S^T S \tag{8}$$

Differentiating (8) and using (3) yields

$$\begin{aligned} \dot{V} = S^T \dot{S} &= S^T C(AX+BU+Be) \\ &= S^T C(AX+BUeq+BUn+Be) \\ &= S^T C(BUn+Be) \\ &= S^T (Un+e) \end{aligned} \tag{9}$$

Note that CB is assumed identity matrix for simplicity.

We select the discontinuous control part as follows

$$Un = -\Omega \; SGN(S) \tag{10}$$

where $\Omega = \text{diag}\,(\alpha_1, \alpha_2, \ldots, \alpha_m)$ and $SGN(S)^T = [\,Sgn(s_1), Sgn(s_2), \ldots, Sgn(s_m)\,]$.

Substituting (10) into (9),

$$\begin{aligned} \dot{V} &= S^T[-\Omega\, SGN(S)+e] = S^T e - S^T \Omega\, SGN(S) \\ &= \sum_{i=1}^{m} (s_i e_i - \alpha_i \mid s_i \mid) \end{aligned} \tag{11}$$

If we select $\alpha_i > f_i$, the negative definiteness of $\dot{V}$ can be verified. From the above result we can know that the sliding mode is globally reachable in spite of the existence of parameter variations.

2.4 The range-space dynamics of variable structure control system (VSCS)

In examining the behavior of VSCS the primary interest is in the behavior of the range-space dynamics of the linear operator C which defines the desired performance of the closed-loop system in the sliding mode.

To derive the range-space dynamic equation we consider the derivative of S :

$$\begin{aligned} \dot{S} = C\dot{X} &= C[AX+B(Ueq+Un)] \\ &= C[AX+BUeq]+CBUn \\ &= CBUn \; . \end{aligned} \tag{12}$$

For simplicity let us assume that CB = I, the identity. Then $\dot{S}(X) = Un$. This conditions allows an easy verification of the sufficient conditions for the existence and reachability of a sliding mode, i.e., the conditions that $s_i \dot{s}_i < 0$ when $s_i(x) \neq 0$.

Therefore, we can obtain the range-space dynamic equations

$$\begin{aligned} &\dot{s}_i = u_{iN} = -\alpha_i \, Sgn(s_i) \quad i=1,2,\ldots,m \quad \text{if } s_i \neq 0 \\ &\dot{s}_i = 0 \qquad \text{if } s_i = 0 \end{aligned} \tag{13}$$

These equations are valid only in the case of no parameter variations.

If there exist uncertainties in the system (2), the range-space dynamic equations are more complex. Provided that the matching conditions are satisfied we can obtain the following range-space dynamic equations in the presence of uncertainties

$$\begin{aligned} \dot{S}(X) = C\dot{X} &= C[AX+BU+Be] \\ &= C[AX+BUeq+BUn+Be] \\ &= CB[Un+e] \\ &= Un+e \end{aligned} \tag{14}$$

where CB is assumed identity matrix for simplicity.

Hence the range-space dynamic equations can be written as follows

$$\begin{aligned} &\dot{s}_i = -\alpha_i \, Sgn(s_i) + e_i \quad \text{if } s_i \neq 0 \\ &\dot{s}_i = e_i \quad \text{if } s_i = 0 \qquad i=1,2,\ldots,m \end{aligned} \tag{15}$$

Fig.1a and Fig.1b show the dynamic behavior of range-space in the ith switching surface for two possible situations. From these figures we can know that the variable structure controller with switching logics has chattering motion.

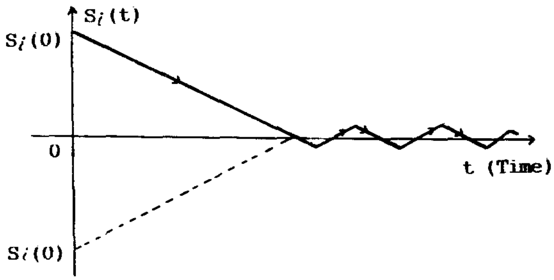

Fig.1a The range-space dynamics in the ith switching surface in the case of no parameter uncertainties

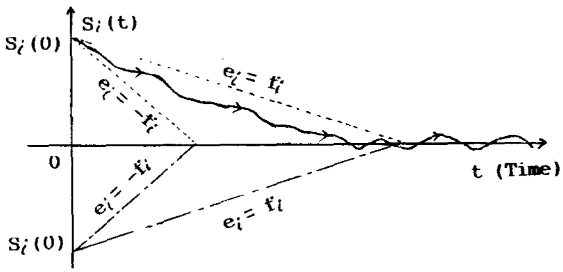

Fig.1b The range-space dynamics in the ith switching surface in the presence of parameter uncertainties

3. The robust controller with switching dynamics

3.1 The switching dynamics in the range-space and the design of robust controller with this switching dynamics

In order to occur the sliding mode on the intersection S the condition (6) must be satisfied.In variable structure controller the trajectories essentially chatter due to switching logics. Therefore, we introduce the switching dynamics instead of switching logics to remove chattering motion:

$$\dot{S} = -\Gamma S \tag{16}$$

where $\Gamma = diag(r_1, r_2, \ldots, r_m)$, $r_i > 0$.

The condition for the existence of a sliding mode is easily checked as follows

$$S^T \dot{S} = -S^T \Gamma S < 0 \quad \text{if} \quad S \neq 0 \tag{17}$$

We can construct the controller using the above switching dynamics.

Differentiating (5) and using (16) yields

$$\dot{S}=C\dot{X}=C(AX+BU)=-\Gamma S \tag{18}$$

Solving for U

$$U=-(CB)^{-1}[CAX+\Gamma S]=-(CB)^{-1}CAX-(CB)^{-1}\Gamma S \tag{19}$$
$$= U_{eq} + U_{sd}$$

where Ueq is equivalent control and Usd is continuous control with above switching dynamics. Note that the control input (19) is bounded due to the sliding mode condition and the reachability condition.

Here let us examine the closed-loop eigenvalues. Substituting (19) into the nominal system without uncertainties (2), we obtain the following closed-loop system.

$$\dot{X}=[A_{eq}-B(CB)^{-1}\Gamma C]X \tag{20}$$

where $A_{eq}=[I_n-B(CB)^{-1}C]A$.

We introduce a similarity transformation M which decouples the system (20) into the fast and slow subsystems.

Let $M \in R^{n\times n}$ be defined by

$$M=[W^g : C]^T \tag{21}$$

where $W \in R^{n\times(n-m)}$ is the eigenvector matrix and W^g is a generalized inverse of W. Note that M is invertible with $M^{-1}=[W : B]$.

We introduce the new coordinates

$$\tilde{X}=MX \tag{22}$$

Then in the new coordinates the closed-loop system becomes

$$\dot{\tilde{X}}=M[A_{eq}-B(CB)^{-1}\Gamma C]M^{-1}\tilde{X} \tag{23}$$

$$= \left[\begin{array}{c|c} W^g[A_{eq}-B(CB)^{-1}\Gamma C]W & W^g[A_{eq}-B(CB)^{-1}\Gamma C]B \\ \hline C\,[A_{eq}-B(CB)^{-1}\Gamma C]W & C\,[A_{eq}-B(CB)^{-1}\Gamma C]B \end{array}\right]\tilde{X}$$

From $M M^{-1}=I_n$ we obtain the following relations

$$\begin{aligned} W^g W &= I_{n-m} \\ W^g B &= 0 \\ C W &= 0 \\ C B &= I_m \end{aligned} \tag{24}$$

From the definition of a generalized inverse and the above relations (24) we can obtain the following simplified state-space model in the new coordinates.

$$\dot{\tilde{X}} = \left[\begin{array}{c|c} W^g A W & W^g A B \\ \hline 0 & -\Gamma \end{array}\right]\tilde{X} \tag{25}$$

Therefore the closed-loop eigenvalues can be determined by the following characteristic equation

$$\det(\lambda I_{n-m} - W^g A W)\det(\lambda I_m + \Gamma)=0 \tag{26}$$

Since $W^g A W = J$ we obtain the following result.

$$\det(\lambda I_{n-m} - J)\det(\lambda I_m + \Gamma)=0 \tag{27}$$

where J is $(n-m)\times(n-m)$ Jordan matrix with n-m prescribed negative eigenvalues.

Consequently, the closed-loop eigenvalues are determined by both the Jordan matrix J and the positive definite diagonal matrix Γ . Note that J determines the null-space dynamics and Γ the range-space dynamics. Therefore, the stability of closed-loop system can always be guaranteed. From the above result we can know that the sliding mode is globally reachable when the parameter variations do not exist.

3.2 The uniform ultimate boundedness of uncertain systems with switching dynamics

Let us consider the following system with uncertainties

$$\dot{X} = AX(t) + BU(t) + Be(t) \tag{28}$$
$$X(t_0) = X_0$$

Concerning the uncertain system (28) we introduce the following assumptions

(1) The norm of the uncertain element is bounded by a known function, i.e., $\| e(t) \| < h(t)$ where $h(t)$ is a non-negative scalar valued function.

(2) The control input is bounded by nonnegative scalar valued function $\tau(t)$ i.e., $\| U(t) \| < \tau(t)$.

(3) The origin, X=0, is uniformly asymptotically stable for the uncontrolled nominal system $\dot{X}(t) = AX(t)$. In particular, there are a Lyapunov function $V(\cdot,\cdot)$ and continuous, strictly increasing functions $\beta_i(\cdot) : R^+ \longrightarrow R^+$, i=1,2,3 which satisfy

$$\beta_i(0) = 0 \quad i=1,2,3$$
$$\lim_{r \to \infty} \beta_i(r) = \infty \quad i=1,2 \tag{29}$$

such that for all $X \in R^n$

$$\beta_1(\| X \|) \leq V(t,X) \leq \beta_2(\| X \|) \tag{30}$$

$$\frac{\partial V(t,X)}{\partial t} + \text{grad}V(t,X)AX \leq -\beta_3(\| X \|) \tag{31}$$

Note that grad $V(t,X) = (\partial S(X)/\partial X)^T S = C^T S$. In other words, there is a Lyapunov function $V(\cdot,\cdot)$ for the uncontrolled nominal system.[9]

Theorem : Let us consider the system (28) with a state feedback control (19). Then the following results hold.

[1] Uniform boundedness : If $X(\cdot) : [t_0, t_1] \longrightarrow R^n$, $X(t_0) = X_0$ is a solution of (28), then

$$\| X_0 \| \leq r \Longrightarrow \| X(t) \| \leq d(r) \;\forall t \in [t_0, t_1]$$

$$\text{where } d(r) = \begin{cases} (\beta_1^{-1} \circ \beta_2)(R) & \text{if } r \leq R \\ (\beta_1^{-1} \circ \beta_2)(r) & \text{if } r > R \end{cases} \tag{32}$$

and $R = \beta_3^{-1}(2\epsilon)$ (33)

Futhermore, the solution has a continuation over $[t_0, \infty)$.

[2] Uniform ultimate boundedness : If $X(\cdot) : [t_0, \infty) \longrightarrow R^n$, $X(t_0) = X_0$ is a solution of (28) with $\| X_0 \| < r$, then for given $\bar{d} > (\beta_1^{-1} \circ \beta_2)(R)$

$$\| X(t) \| \leq \bar{d} \quad \forall t \geq t_0 + T(\bar{d}, r)$$

$$\text{where } T(\bar{d},r) = \begin{cases} 0 & \text{if } r \leq \bar{R} \\ \dfrac{\beta_2(r) - \beta_1(\bar{R})}{\beta_3(\bar{R}) - 2\epsilon} & \text{if } r > \bar{R} \end{cases} \tag{34}$$

and $\bar{R} = (\beta_2^{-1} \circ \beta_1)(\bar{d})$ (35)

Proof : [1] Let us consider a solution $X(\cdot) : [t_0, t_1] \longrightarrow R^n$, $X(t_0) = X_0$, with $\| X_0 \| \leq r$ and $R \leq r$. Let $\tilde{r} = \max(r, R)$ so that $\| X_0 \| \leq \tilde{r}$ and $R \leq \tilde{r}$.

Also, by (32) $d(r) = (\beta_1^{-1} \circ \beta_2)(\tilde{r})$. Futhermore, in view of (30), $\beta_1(\tilde{r}) \leq \beta_2(\tilde{r})$ so that $\tilde{r} \leq (\beta_1^{-1} \circ \beta_2)(\tilde{r}) = d(r)$. Thus, $\| X(t_0) \| = \| X_0 \| \leq \tilde{r} \leq d(r)$. Now suppose there is a $t_3 > t_0$ such that

$$\| X(t_3) \| > d(r) \tag{36}$$

Since $X(\cdot)$ is continuous and $\| X(t_0) \| \leq \tilde{r} \leq d(r) \leq \| X(t_3) \|$ there is $t_2 \in [t_0, t_3]$ such that $\| X(t_2) \| = \tilde{r}$ and $\| X(t) \| \geq \tilde{r}, \forall t \in [t_2, t_3]$

Here, we consider the Lyapunov derivative :

$$\begin{aligned} L(t,X) &\triangleq \frac{\partial V(t,X)}{\partial t} + \text{grad}V(t,X)[AX+BU+Be] \\ &\leq -\beta_3(\| X \|) + C^T SBU + C^T SBe \\ &\leq -\beta_3(\| X \|) + \| \phi(t) \| + \| \varphi(t) \| \\ &< -\beta_3(\| X \|) + 2\epsilon \end{aligned} \tag{37}$$

where $\phi(t) = C^T SB\tau(t)$, $\varphi(t) = C^T SBh(t)$ and $\epsilon = \sup_t \{ \| \phi(t) \|, \| \varphi(t) \| \}$.

Now, in view of (30) and (37)

$$\begin{aligned} \beta_1(\| X(t_3) \|) &\leq V[t_3, X(t_3)] \\ &= V[t_2, X(t_2)] + \int_{t_2}^{t_3} L[\tau, X(\tau)] d\tau \\ &\leq \beta_2(\| X(t_2) \|) + \int_{t_2}^{t_3} [-\beta_3[X(\tau)] + 2\epsilon] d\tau \\ &\leq \beta_2(\tilde{r}) + \int_{t_2}^{t_3} [-\beta \; (R) + 2\epsilon] d\tau \\ &= \beta_2(\tilde{r}) \end{aligned}$$

Hence, $\| X(t_3) \| \leq (\beta_1^{-1} \circ \beta_2)(\tilde{r}) = d(r)$

However, that contradicts supposition (36) ; hence, $\| X(t) \| \leq d(r)$, $\forall t \in [t_0, t_1]$. Consequently, there is a set $\delta(X)$, say, $\delta(X) = \{ X \in R ; \| X \| \leq \tilde{d} > d(r) \}$ from which no solution (with $\| X_0 \| < r$) can escape. Hence every such solution can be extended over any interval and hence over $[t_0, \infty)$.

[2] Let $X(\cdot) : [t_0, \infty) \longrightarrow R^n$, $X(t_0) = X_0$ with $\| X_0 \| \leq r$, denote a solution of (28). We consider a $\bar{d} > (\beta_1^{-1} \circ \beta_2)(R)$. By definition, $\bar{R} = (\beta_2^{-1} \circ \beta_1)(\bar{d})$ so that $\bar{R} > R$ and $d(\bar{R}) = (\beta_1^{-1} \circ \beta_2)(\bar{R}) = \bar{d}$.

If $r \leq \bar{R}$, then $\| X_0 \| \leq \bar{R}$; hence, in view of the uniform boundedness result [1], $\| X(t) \| \leq d(\bar{R}) = \bar{d}$, $\forall t \in [t_0, \infty)$ so that $T(\bar{d}, r) = 0$.

Next we consider $r > \bar{R}$, and suppose that

$$\| X(t) \| > \bar{R}, \quad \forall t \in [t_0, t_1] \tag{38}$$

where

$$t_1 = t_0 + T(\bar{d}, r), \quad T(\bar{d}, r) \triangleq \frac{\beta_2(r) - \beta_1(\bar{R})}{\beta_3(\bar{R}) - 2\epsilon}$$

Then, in view of (30) and (37),

$$\begin{aligned} \beta_1(\| X(t_1) \|) &\leq V[t_1, X(t_1)] \\ &= V[t_0, X(t_0)] + \int_{t_0}^{t_1} L[\tau, X(\tau)] d\tau \\ &\leq \beta_2(\| X_0 \|) + \int_{t_0}^{t_1} [-\beta_3(\| X(\tau) \|) + 2\epsilon] d\tau \\ &\leq \beta_2(r) + T(\bar{d}, r)[-\beta_3(\bar{R}) + 2\epsilon] \\ &= \beta_2(r) + \frac{\beta_2(r) - \beta_1(\bar{R})}{\beta_3(\bar{R}) + 2\epsilon} \times [-\beta_3(\bar{R}) - 2\epsilon] \\ &= \beta(\bar{R}). \end{aligned}$$

The range-space dynamics for the robust controller can be derived from section 3.3.

$$\dot{S} = -CBUn = \begin{bmatrix} 10s_1 \\ 24s_2 \end{bmatrix} \qquad (46)$$

Note that above range-space dynamics (44),(46) are derived under assumption that no parameter uncertainties exist.

In Fig.3 ,Fig.4, Fig.5 and Fig.6 the respective state trajectories for the variable structure controller and robust controller are shown. Also, each control input is shown in Fig.7 and Fig.8. Note that the control inputs for the robust controller have no chattering. The solid line denotes the variable structure controller and the solid line with circles the robust controller.

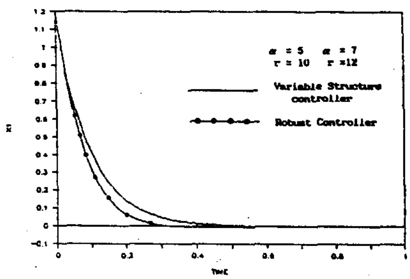

Fig.3 The state trajectories for x_1 in the case of no parameter uncertainties

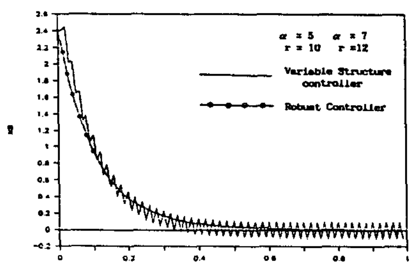

Fig.4 The state trajectroies for x_2 in the case of no parameter uncertainties

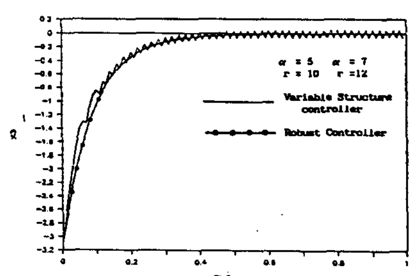

Fig.5 The state trajectroies for x_3 in the case of no parameter uncertainties

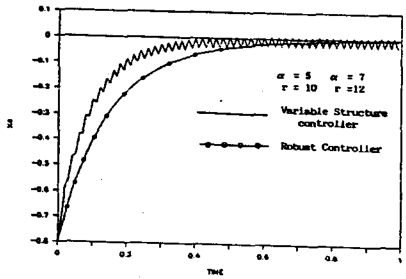

Fig.6 The state trajectories for x_4 in the case of no parameter uncertainties

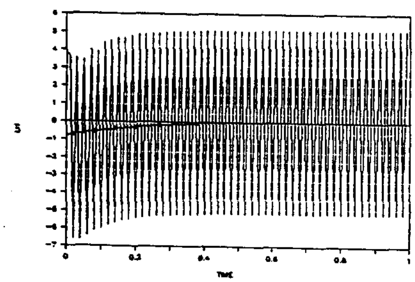

Fig.7 The control inputs for u_1 in the case of no parameter uncertainties

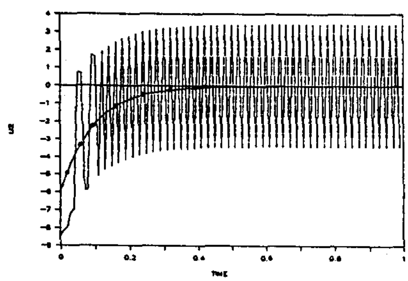

Fig.8 The control inputs for u_2 in the case of no parameter uncertainties

Assuming that the system has the following parameter uncertainties, i.e., $e^T = [e_1\ e_2] = [0.9\sin(t)\ 0.75\cos(t)]$, the range-space dynamics for two controllers in the presence of parameter uncertainties can be derived repectively as follows

$$\dot{S} = \begin{bmatrix} -5\,\mathrm{Sgn}(s_1) \\ -14\mathrm{Sgn}(s_2) \end{bmatrix} + \begin{bmatrix} 0.9\sin(t) \\ 0.75\cos(t) \end{bmatrix} \qquad (47)$$

$$\dot{S} = \begin{bmatrix} 10s_1 \\ 24s_2 \end{bmatrix} + \begin{bmatrix} 0.9\sin(t) \\ 0.75\cos(t) \end{bmatrix} \qquad (48)$$

The state trajectories for two controllers in the presence of parameter uncertainties are shown in Fig.9, Fig.10, Fig.11 and Fig.12, respectively. The control inputs for two controllers in the presence of parameter uncertainties are shown in Fig.13 and Fig.14.

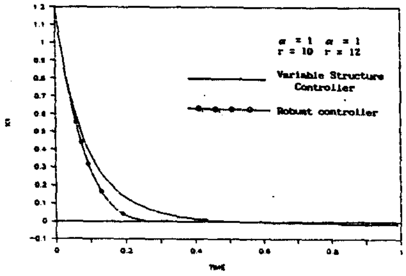

Fig.9 The state trajectories for x_1 in the presence of parameter uncertainties

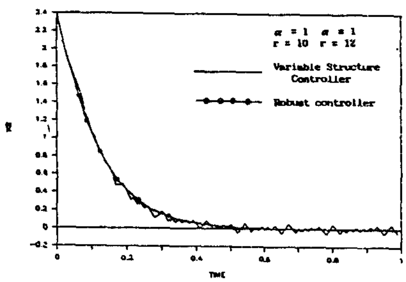

Fig.10 The state trajectories for x_2 in the presence of parameter uncertainties

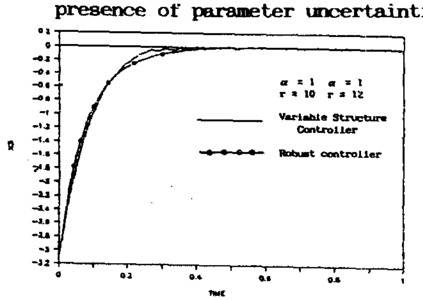

Fig.11 The state trajectories for x_3 in the presence of parameter uncertainties

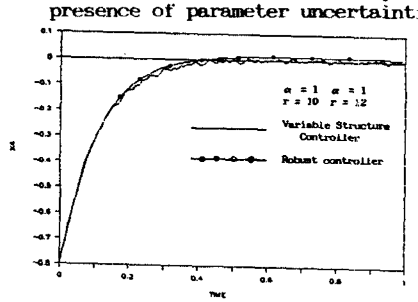

Fig.12 The state trajectories for x_4 in the presence of parameter uncertainties

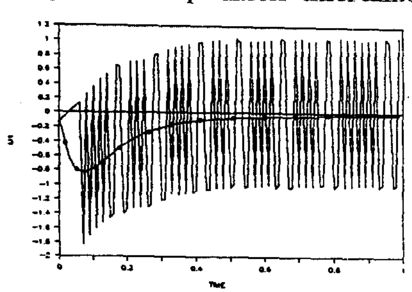

Fig.13 The control inputs for u_1 in the presence of parameter uncertainties

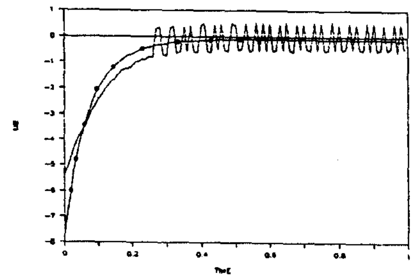

Fig.14 The control inputs for u_2 in the presence of parameter uncertainties

5. Conclusion

In this paper, we developed the algorithm for the sliding mode control which is able to remove the chattering motion. To obtain a sliding mode we introduce the switching dynamics instead of switching logics. The proposed algorithm does not require the computations for the switching gains of control inputs unlike the conventional variable structure control (VSC) scheme and requires only the selection of γ in the range-space in order to occur a sliding mode. Hence the real-time control is realizable. By the adjustment of γ the almost ideal sliding mode which can not be realized by variable structure controller can be obtained. Also, the dynamic behaviors of the range-space of a switching surface matrix C are analysed mathematically for two possible situations. The uniform ultimate boundedness for the proposed robust controller proved in the presence of unknown parameter variations. Finally we can construct the robust controller of uncertain multivariable systems which is much simpler than that of VSC systems.

6. References

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That is $\| X(t_1) \| \le \bar{R}$. But that contradicts supposition (38). Hence there must be a $t_2 \in [t_0, t_1]$ such that $\| X(t_2) \| \le \bar{R}$. Then, as a consequence of the uniform boundedness result [2], $\| X(t) \| \le d(\bar{R}) = \bar{d}, \forall t \ge t_2$. Hence, $\| X(t) \| \le \bar{d}, \forall t \ge t_1 + T(\bar{d}, r)$. Q.E.D.

From above theorem it is guaranteed that every response of the uncertain system in the presence of unknown parameter variations is uniformly ultimately bounded within a set E containing the state-space origin.

3.3 The range-space dynamics of the uncertain system with switching dynamics.

In this subsection, we examine the dynamic behavior of the uncertain system with switching dynamics.

Let us consider the following relationship

$$\dot{S}=C\dot{X}=C[AX+BUeq-B(CB)^{-1}\Gamma S] \\ =-(CB)\,(CB)^{-1}\Gamma S=-\Gamma S \qquad (39)$$

Therefore, we obtain the following range-space dynamic equations :

$$\dot{s}_i = -r_i s_i \quad i=1,2,\ldots,m \quad r_i > 0 \qquad (40)$$

From (39) we can know that the states approach to the switching surface exponentially from the initial states.

If there exist uncertainties in the system (2), the range-space dynamic equations are more complex. Provided that the matching conditions are satisfied, we can derive the following range-space dynamic equations in the presence of uncertainties.

$$\dot{S}=C\dot{X}=C[AX+BU+Be] \\ =C[AX+BUeq+BUn+Be] \\ =CB[Un+e] \\ =-\Gamma S+e \qquad (41)$$

Hence, the range-space dynamic equations can be written as follows

$$\dot{s}_i=-r_i s_i + e_i \qquad (42)$$

Fig.2a and Fig.2b show the range-space dynamics in the ith switching surface for two possible situations.

From these figure we can know that the robust controller with switching dynamics has no chattering motion.

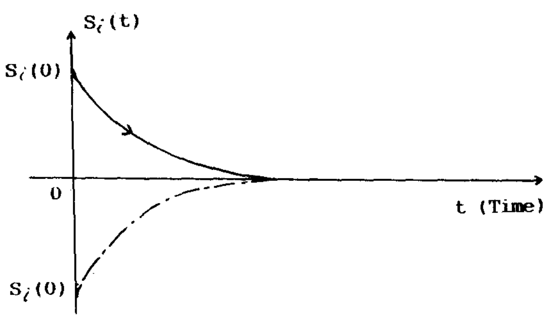

Fig.2a The range-space dynamics in the ith switching surface in the case of no parameter uncertainties

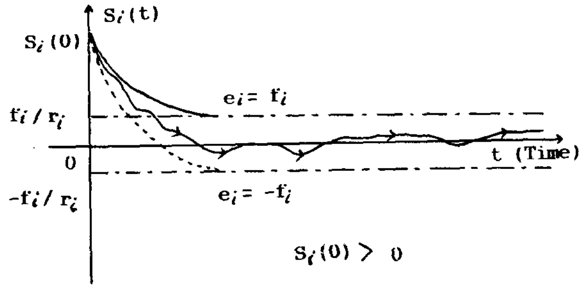

Fig.2b The range-space dynamics in the ith switching surface in the presence of parameter uncertainties

4. Numerical examples

4.1 The variable structure controller for the 4th order system with two inputs.

Let us consider the following 4th order system with two inputs.

$$\dot{X}=\begin{bmatrix} 1.4 & -0.2 & 6.7 & -5.6 \\ 10.1 & -4.3 & 9.9 & -13.8 \\ 1.0 & 0.0 & -5.0 & 2.0 \\ 2.2 & 4.3 & 3.3 & -5.0 \end{bmatrix} X + \begin{bmatrix} 0.0 & 0.0 \\ 5.6 & 0.0 \\ 1.1 & -3.2 \\ 1.1 & 0.0 \end{bmatrix} U$$

$$X(0) = [\ 1.2 \quad 2.4 \quad -3.1 \quad -0.8\ ]^T$$

To design the switching surface described in section 2.2 we assign two repeated eigenvalues at - 10. Provided the matrix CB = H is selected the diagonally dominant matrix,i.e.,CB=diag(-1,-2), we can construct the switching surface matrix C.

$$C=\begin{bmatrix} 0.0751 & -0.1128 & 0.0000 & -0.3163 \\ 1.0773 & -0.0196 & 0.6357 & -0.5373 \end{bmatrix}$$

We can determine the control inputs as follows.

$$U=Ueq+Un \\ =-(CB)^{-1}[CAX-\Omega\, SGN(S)] \qquad (43)$$

where $SGN(S)=[Sgn(s_1),Sgn(s_2)]$, Ω=diag(5,7).

The range-space dynamics for the variable structure controller can be drived from section 2.4.

$$\dot{S} = [\ \dot{s}_1 \ \ \dot{s}_2]^T = CBUn \\ = \begin{bmatrix} -5\ Sgn(s_1) \\ -14 Sgn(s_2) \end{bmatrix} \qquad (44)$$

4.2. The robust controller for the 4th order system with two inputs.

In order to occur a sliding mode on a switching surface selected in section 4.1 we determine the continuous control inputs from our switching dynamics

$$U=[u_1\ u_2]^T=Ueq+Un \\ =-(CB)^{-1}CAX-(CB)^{-1}\Gamma S \qquad (45)$$

where Γ is positive definite,i.e.,Γ=diag(10,12).