# A Simple Method for Treating Nonlinear Control Systems through State Feedback

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**Abstract**

If the nonlinear term in a nonlinear control system equation can be deleted by state feedback control, the original system becomes a linear system. For this linear control system, many well known methods may be used to handle it, and then reverse it back to nonlinear form. Many problems of nonlinear control systems can be solved in this way. In this paper, this method will be used to transfer the identification problem of non-linear systems into a linear control problem. The nonlinear observer is established by constructing linear observer. Then the state control of nonlinear systems is realized. Finally, the technique of the PID controller obtained by using bang-bang tracker as a differentiator provides a stronger robust controller.

Even though the method in this paper may not theoretically perfect, many numerical simulations show that it is applicable.

1. In the last decade, the differential geometric method is flourishing in the nonlinear system control theory. But there are still many obstacles to be overcome when it is used in practical engineering problems as a general method.

If the nonlinear terms in a nonlinear control system may be deleted by state feedback control, problems about nonlinear control systems may be solved as those of linear control systems

## 2. Identification of Nonlinear Systems.

Given a nonlinear system

$$(1)\quad \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x_1,x_2) \\ y = x_1 \end{cases}$$

where $f(x_1,x_2)$ is unknown. Our purpose is to construct $f(x_1,x_2)$ through the measurement $y(t)$.

We may consider system (1) as the result system obtained from linear system

$$(2)\quad \begin{cases} \dot{\bar{x}}_1 = \bar{x}_2 \\ \dot{\bar{x}}_2 = -a_1\bar{x}_1 - a_2\bar{x}_2 + u \\ \bar{y} = \bar{x}_1 \end{cases}$$

by adding state feedback $u=a_1\bar{x}_1+a_2\bar{x}_2+f(x_1,x_2)$, where $a_1$ and $a_2$ are given positive constants.

Now, we may choose $u(t)$ in such a way that, the output $y(t)$ of the system (2) approaches $y(t)$. Then

$u(t)$, $\bar{x}_1(t)$, $\bar{x}_2(t)$

may be obtained from system (2). Next, we may eliminate t by using the above three variables to get function

$u=u(\bar{x}_1,\bar{x}_2)$

It follows that

$f(x_1,x_2) = u(x_1,x_2) - a_1x_1 - a_2x_2$

The process of producing $u(\bar{x}_1,\bar{x}_2)$ from $u(t)$, $\bar{x}_1(t)$, $\bar{x}_2(t)$ is a typical function approximation problem. Various methods can be used for it. The information required to produce $u(\bar{x}_1,\bar{x}_2)$ are obtained from linear system (2) and measurement $y(t)$. Practically if the purpose is to control the system, it is not necessary to construct $f(x_1,x_2)$. In this circumstance it is enough to estimate $a(t) = f(x_1(t),x_2(t))$.

## 3. Nonlinear Observer

Assume system

$$(3)\quad \begin{cases} \dot{x}_1 = x_1 + f_1(x_1,\ x_2) \\ \dot{x}_2 = f_2(x_1,\ x_2) \\ y\ = x_1 \end{cases}$$

is observable. We may consider $f_1(x_1,x_2)$ and $f_2(x_1,x_2)$ as the state feedback controls

$u_1 = f_1(x_1,x_2),\quad u_2=f_2(x_1,x_2)$

of the linear system

$$(4)\quad \begin{cases} \dot{x}_1 = x_2 + u_1 \\ \dot{x}_2 = u_2 \\ y\ = x_1 \end{cases}$$

For the above linear system (4) the linear observer may be established as

$$\begin{cases} \dot{\hat{x}}_1 = -l_1\hat{x}_1 + \hat{x}_2 + l_1 y + u_1 \\ \dot{\hat{x}}_2 = -l_2\hat{x}_1 \qquad + l_2 y + u_2 \end{cases}$$

Then we revers $u_1$ and $u_2$ back as

$$u_1 = f_1(y,\hat{x}_2),\quad u_2 = f_2(y,\hat{x}_2)$$

That is

$$(5)\quad \begin{cases} \dot{\hat{x}}_1 = -l_1\hat{x}_1 + \hat{x}_2 + l_1 y + f_1(y,\hat{x}_2) \\ \dot{\hat{x}}_2 = -l_2\hat{x}_1 \qquad + l_2 y + f_2(y,\hat{x}_2) \end{cases}$$

The error equation fo the systems (3) and (5) is

$$(6)\quad \begin{cases} \delta\dot{x}_1 = -l_1\delta x_1 + \delta x_2 \\ \qquad\qquad + (f_1(y,x_2)-f_1(y,x_2-\delta x_2)) \\ \delta\dot{x}_2 = -l_2\delta x_1 + (f_2(y,x_2)-f_2(y,x_2-\delta x_2)) \end{cases}$$

It is seen from equation (6) that when $\delta x_2$ varies inside a given region the system (5) acts as a state observer of the system (3).

## 4. State Feedback Realization of Nonlinear Control Systems Consider the Control System

Consider the control system

$$(7)\quad \begin{cases} \dot{x}_1 = x_2 + f_1(x_1,\ x_2) \\ \dot{x}_2 = f_2(x_1,x_2) + u \\ y\ = x_1 \end{cases}$$

The goal of this section is to design a state feedback $u(x_1,x_2)$ which makes closed loop system to be stable. Even though x is not measurable directly, we may use the state observer to realize the feedback $u(\hat{x}_1,\hat{x}_2)$ or $u(x_1,\hat{x}_2)$, where $\hat{x}_1$, $\hat{x}_2$ are outputs of the observer.

First of all,we may find a function $g(x_1,x_2)$, which stabilizes the system

$$(8)\quad \begin{cases} \dot{x}_1 = x_2 + f_1(x_1,x_2) \\ \dot{x}_2 = g(x_1,x_2) + y \\ y\ = x_1 \end{cases}$$

Then we may build a state observer for system (8) as

$$(9)\quad \begin{cases} \dot{\hat{x}}_1 = -l_1(\hat{x}_1-y) + \hat{x}_2 + f_1(y,\hat{x}_2) \\ \dot{\hat{x}}_2 = -l_2(\hat{x}_1-y) \qquad + g(y,\hat{x}_2) + v \end{cases}$$

To transfer the system (7) to system (8),we need the following feedback

$$u = -f_2(x_1,x_2) + g(x_1,x_2) + v$$

What we obtained from observer (9) is $\hat{x}_1$ and $\hat{x}_2$. Thus, the real feedback acting on system (7) is

$$u = -f_2(\hat{x}_1,\hat{x}_2) + g(\hat{x}_1,\hat{x}_2) + v$$

Finally, the closed loop system obtained from the state feedback is

$$(10)\quad \begin{cases} \dot{x}_1 = x_2 + f_1(x_1,x_2) \\ \dot{x}_2 = f_2(x_1,x_2)-f_2(\hat{x}_1,\hat{x}_2)+g(\hat{x}_1,\hat{x}_2)+v \\ \dot{\hat{x}}_1 = -l_1(\hat{x}_1-x_1) + \hat{x}_2 + f_1(x_1,\hat{x}_2) \\ \dot{\hat{x}}_2 = -l_2(\hat{x}_2-x_1) \qquad + g(x_1,\hat{x}_2) + v \end{cases}$$

The error equation between the system states and the outputs of the observer is

$$(11)\quad \begin{cases} \delta\dot{x}_1 = -l_1\delta x_1 + \delta x_2 \\ \qquad + (f_1(x_1,x_2)-f_1(x_1,x_2-\delta x_2)) \\ \delta\dot{x}_2 = -l_2\delta x_1 \\ \qquad + (f_2(x_1,x_2)-f_2(x_1-\delta x_1,x_2-\delta x_2)) \\ \qquad + (g(x_1-\delta x_1,x_2-\delta x_2)-g(x_1,x_2-\delta x_2)) \end{cases}$$

It is clear from equation (11) that when $\delta x_1$ and $\delta x_2$ vary inside a certain region system (11) is stable. That is, $\hat{x}_1 \longrightarrow x_1$ and $\hat{x}_2 \longrightarrow x_2$, which ensure the stability of system (10).

## 5. Robust Controller

The PID controller is widely used in the practical control engineering. The reason for that is it has a nice adaptability and robustness. Moreover it is easy to be realized. It is an interesting problem to improve the robustness and the adaptability the PID controller.

Hence we suggest a technique to improve the PID regulator.

Observe the following bang-bang fast tracker.

$$(12) \quad \begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = -M \,\mathrm{sign}(z_1 - y(t) + z_2|z_2|/2M) \end{cases}$$

When $y(t)$ is input into this system, $z_1(t)$ tracks it pretty fast. When $y(t)$ varies rather slowly, $z_2(t)$ approaches $\dot{y}(t)$, and that the system (12) acts as a differentiator. System (12) has a property as follows: When $y(t)$ jumps, $z_1(t)$ tracks $y(t)$ with the acceleration at most as a quadratic form of $M$, no jump happens. This property is important for PID controller.

According to the above property of the system (12), we suggest the following control scheme: Where the PID control law is

$$(13) \qquad u = K_1(z_{21} - y) + K_2(z_{22} - z_{12}) + K_3\int_0^t (z_{21} - y)\,dt$$

For the following systems

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x_1, x_2) + u_0(t) + u \\ y = x_1 \end{cases}$$

the computer simulation shows that when

$$M_1 = 50, \; M_2 = 0.5$$

$$K_1 = 130, \; K_2 = 20, \; K_3 = 120$$

over a large scale, $y(t)$ can track the reference input $u_0(t)$ which is almost independent of the particular form of $f(x_1, x_2)$. Moreover if the function $f(x_1, x_2)$ is known, the control law (13) may be rewritten as

$$u = \bar{K}_1(z_{21} - y) + \bar{K}_2(z_{22} - z_{12}) + \bar{K}_3\int_0^t (z_{21} - y)\,dt - a(t)$$

where

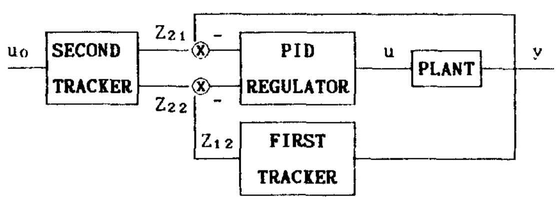
$$a(t) = f(y(t), z_{12}(t))$$

and

$$\bar{K}_i = K_i/3$$

Finally, we would like to point out that the above control scheme still is improvable. There is a bright future for improving it by using some nice properties of the particular nonlinear functions.

CONTROL SCHEME




The First Tracker :

$$\dot{Z}_{11} = Z_{12}$$

$$\dot{Z}_{12} = M_1 \,\mathrm{sign}\,(Z_{11} - Y(t) + Z_{12}|Z_{12}|/2M_1)$$

The Second Tracker :

$$\dot{Z}_{21} = Z_{22}$$

$$\dot{Z}_{22} = -M_2 \,\mathrm{sign}\,(Z_{21} - U_0(t) + Z_{22}|Z_{22}|/2M_2)$$