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MODEL PREDICTIVE CONTROL FOR NONLINEAR SYSTEMS

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Abstract - This paper considers the model predictive control (MPC) problems in nonlinear processes or systems. The MPC method determines the control law such that the predicted output based on the identified process model is equal to the reference output which consists of both the process output at current time and the setting value called as the command generator. In this paper, the nonlinear MPC software for a chemical reactor is developed and analized from the point of view of practical applications.

1. INTRODUCTION

Recently much attention has been focused to practical studies on the model predictive (heuristic) control (denoted as MPC) or eqiuvalently model algorithmic control [1]-[3]. The MCP method determines the control law such that the predicted output based on the identified process model is equal to the reference output which consists of both the process output at current time and the setting value called as the command generator.

The main difference between the MPC and the adaptive control is that in order to suppress undesirable variation of the process perturbation or disturbance the reference output adjusts it by including the process output into the reference output in the MPC. On the other hand, the adaptive control adjusts it by modifying the identified process model.

It is reported that the MPC methods for linear processes have been developed and applied for control of industrial processes effectively [1]-[2]. However, it seems that the MPC probrems for nonlinear processes have not yet been discussed from both a theoretical and practical point of view. In this paper, the nonlinear MPC software is developed and analized from a point of view of practical applications. The paper provides us with the methods or tools

(1) how to determine a ratio of the weight between the process output and the command generator in the reference output in order to assure both the stability and the desired transient response of the process,

(2) how to increase the robustness of the process applying the MPC,

(3) how to construct the database based on the control simulation of the nonlinear processes.

It is assumed in this paper that the process model has been identified by a simple indetification method such as the step response technique etc.. The results obtained from the simulation studies are illustrated. In this paper our attention is focused on the general aspects of the MPC for nonlinear processes and the preliminary discussions for the practical application in the next step are presented. The MPC algorithm which forces the concentration of hydrogen in a chemical reactor to the descied concentration is presented in this study.

2. MODEL PREDICTIVE CONTROL (MPC)

For easy understanding, a brief conceptual blockdiagram for the nonlinear process, which describes the relation between the input flow of hydrogen and the output melt flow rate of the mixed polymer generated, in shown in Fig.1.

The MPC control law [1] is given by

$$x^p(i+1)=x^r(i+1) \qquad (1)$$

where $x^p(i+1)$ and $x^r(i+1)$ are the predicted value of the concentration of

hydrogen and the refencence trajectory of it at the sampling time $(i+1)T_0$, respectively. T_0 is the sampling interval. In the MPC , $x^p(i+1)$ is expressed by

$$x^p(i+1)=x_M(i+1)+d(i+1) \tag{2}$$

$$d(i+1)=x(i)-x_M(i) \tag{3}$$

where $x_M(i+1)$ is the predicted value of the concentration of hydrogen using the model, $d(i+1)$ is the predicted value of disturbance, $x(i)$ is the observed value of the concentration at the current sampling time.

In the meanwhile, $x^r(i+1)$ is given as

$$x^r(i+1)=\beta\, x(i)+(1-\beta)x_D(i+1), \tag{4}$$

where β is the parameter which takes into account the robustness of the process and $x_D(i+1)$ is the desired setting value of the concentration of hydrogen at the sampling time $(i+1)T_0$. Rewriting (1) with the aid of (2)-(4), we have

$$x_M(i+1)+x(i)-x_M(i)=\beta\, x(i)+(1-\beta)X_D(i+1). \tag{5}$$

It is assumed that the continuous value of the concentration of hydrogen $x(t)$ in the reactor process obeys the nonlinear differential equation,

$$c_1\frac{dx(t)}{dt}=-c_3(x(t))^{C_4}+c_2x(t)+u(t), \tag{6}$$

where $c_i\,(i=1\sim4)$ are parameters related to the process and $u(t)$ is the input flow of hydrogen at the continuous time t.

In order to obtain the model equation of the process the linearization method is used for (6). Namely, letting

$$x_M(t)=\Delta x_M(t)+x_0 \tag{7a}$$
$$u(t)=\Delta u(t)+u_0 \tag{7b}$$

and substituting (7) into (6), the value u_0 and the linearized process equation are obtained as follows, respectively,

$$u_0=-c_2x_0+10^{-3}(x_0)^{C_4}c_3, \tag{8}$$

$$\Delta x_M(s)=\frac{K}{1+Ts}\Delta u(s), \tag{9}$$

where T is the time constant and K is the process gain given by, respectively,

$$T=\frac{c_1}{c_2+c_3c_4\dfrac{(x_0)^{C_4}}{x_0}}, \tag{10a}$$

$$K=\frac{1}{c_2+c_3c_4\dfrac{(x_0)^4}{x_0}}. \tag{10b}$$

Here, $\Delta x_M(s)$ and $\Delta u(s)$ are the Laplace transform of $\Delta x_M(t)$ and $\Delta u(t)$, respectively, x_0 and u_0 are the nominal values. Discretizing (9) yields

$$\Delta x_M(i+1)=\alpha\,\Delta x_M(i)+K(1-\alpha)\Delta u(i), \tag{11}$$

where $\alpha=\exp(-T_0/T)$.

We substitute the equation of the process model (11) into (5) to obtain the MPC law given by

$$\Delta u(i)=\frac{(1-\beta)}{K(1-\alpha)}(x_D(i+1)-x(i))+\frac{1}{K}\Delta x_M(i), \tag{12}$$

where the increment of the input flow of hydrogen $\Delta u(i)$ is assumed to be constrained by the inequality given by

$$u_{min}\leqq\Delta u(i)\leqq u_{max}. \tag{13}$$

If the above inequality is violated, the increment of the input flow is set to either the lower limit or upper limit such that

$$\Delta u(i)=\begin{cases} u_{min}, & \Delta u(i)\leqq u_{min} \\ u_{max}, & \Delta u(i)\geqq u_{max} \end{cases} \tag{14}$$

3. GENERATION OF SETTING VALUE $x_D(i+1)$

There are several methods how to generate the desired setting value of the concentration of hydrogen $x_D(i+1)$, In this study, we empoly the same MPC algorithm in order to determine the desired setting value.

The melt flow rate of the mixed polymer generated $y(t)$ is related to the melt flow rate of the polymer generated instantanuously $y_p(t)$ such that

$$\ln y(s)=\frac{1}{1+\tau s}\ln y_p(s), \tag{15}$$

where τ is the unknown parameter. The discrete form of (15) becomes

$$\ln y(i+1)=\gamma\ln y(i)+(1-\gamma)\ln y_p(i), \tag{16}$$

where $\gamma=\exp(-T_0/\tau)$. Applying the same MPC method as before for (16) we have

$$y(i+1)+y(i)-y_M(i)=\beta y(i)+(1-\beta)y_D, \quad (17)$$

where y_M is the output of the model, y_D is the desired melt flow rate, and β is the coefficient which takes into consideration of the robustness of the process stated as before. The model is specified by

$$\ln y_M(i+1)=\hat{\gamma}\ln y_M(i)+(1-\hat{\gamma})\ln y_D(i), \quad (18)$$

where the unknown parameter $\hat{\gamma}=\exp(-T_s/\hat{\tau})$ is assumed to be identified by an appropriate identification method.

It should be noted that the coefficient β in (17) is equal to the coefficient appead in (4). This means that the property on the robustness of the concentration of hydrogen is the same as that of the melt flow rate of the mixed polymer. We can choose the different value from β as the coefficient of the melt flow rate process. We see from (17) that

$$\ln y(i+1)=\ln[(1-\beta)(y_D-y(i))+y_M(i)]. \quad (19)$$

Hence, the desired instantenuos melt flow rate $y_{P,D}(i)$ can be calculated from (16) as follows:

$$\ln y_{P,D}(i)=\frac{1}{1-\gamma}\{\ln[(1-\beta)(y_D-y(i))+y_M(i)]-\gamma\ln y_M(i)\}. \quad (20)$$

we assumed that the instantenuous melt flow rate is related to the concentration of hydrogen as follows.

$$\ln y_P(t)=a_1+a_2\ln(x(t)) \quad (21)$$

Hence, the desired concentration $x_D(i+1)$ is written as

$$x_D(i+1)=\exp\left[\frac{\ln y_{P,D}(i+1)-a_1}{a_2}\right] \quad (22)$$

At this stage, the MPC algorithm is sumarized in Table 1.

4. SIMULATION STUDY

In the simulation study, the following numerical values are used:

$$c_1=1.43\times 523.2\times(1-0.4713)\times\frac{2.016}{42.08}\times 10^{-6},$$

$$c_2=-(569.34-247.43)\times\frac{2.016}{42.08}\times 10^{-6},$$

$$c_3=247.43\times 0.60372\times 10^{-3},$$

$$c_4=0.5037,$$

$$a_1=-4.798 \quad a_2=1.079.$$

The objective of the MPC is to bring the MFR y(0) of the mixed polymer into the desired MFR y_D. We use the following numerical values as the inetial values.

$$y(0)=4.3\ ,\ y_D=13\ ,\ x_s=330,$$
$$y_P(0)=y(0)=y_M(0)=4.3.$$

In the simulation study, the parameter β and the maximun value of the input flow of hydrogen u_{max} are changed. Some of the simulated results are shown in Figs. 2 - 4. Fig. 2 shows the output runs of the related variables when $\beta=0.2$ and $u_{max}=0.1$. Fig. 3 shows the output runs when $\beta=0.5$ and $u_{max}=0.1$. Fig. 4 shows the corresponding output runs when $\beta=0.8$ and $u_{max}=0.1$.

For the reason of space, the other simulated results were ommitted in this paper. A NEC PC-9801 microcomputer was used to obtain the numerical results in this study and the 4th order Runge-Kutta -Gill method was employed in order to solve the nonlinear differential equation given by (6).

5. CONDUSIOUS

This paper considered the design problem of MPC for the nonlinear processes of chemical reactors. The nonlinear MPC algorithm was newly presented and simulated in order to see how it works. In this study we forcused mainly our attention to the theoretical aspects of the MPC for the nonlinear process. The next step is to consider an application of the algorithm derived to a real process.

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Table 1 Model Predictive Control Algolism

Variables	Equations
$x(t)$ $x(i)$	$C_1\frac{dx(t)}{dt}=-C_3(x(t))^{C_4}+C_2x(t)+u(t)$ $x(i)=x(iT_0)$
$\Delta x_M(i)$	$\Delta x_M(i+1)=\alpha\,\Delta x_M(i)+K(1-\alpha)\Delta u(i)$ $\alpha=EXP(-T_0/T)$ $T=C_1/(C_2+C_3C_4\frac{(x_0)^{C_4}}{x_0})$ $K=1/(C_2+C_3C_4\frac{(x_0)^{C_4}}{x_0})$
$\Delta u(i)$	$\Delta u(i)=\frac{(1-\beta)}{K(1-\alpha)}(x_D(i+1)-x(i))+\frac{1}{K}\Delta x_M(i))$ $\Delta u(i)=\begin{cases}u_{min}, & \Delta u(i)<u_{min}\\ u_{max}, & \Delta u(i)>u_{max}\end{cases}$
$u(i)$	$u(i)=\Delta u(i)+u_0$
$x_M(i)$	$x_M(i)=\Delta x_M(i)+x_0$
u_0	$u_0=-C_2x_0+10^{-3}(i)(x_0)^{C_4}C_3$
$lny(i+1)$	$lny(i+1)=\gamma\,lny(i)+(1+\gamma)lny_P(i)$ $\gamma=EXP(-T_0/\tau)$
$lny_M(i+1)$	$lny_M(i+1)=\hat{\gamma}\,lny_M(i)+(1-\hat{\gamma})lny_D(i)$ $\hat{\gamma}=exp(-T_0/\hat{\tau})$
$lny_P(i)$	$lny_P(i)=a_1+a_2\ln(x(i))$
$lny_{P,D}(i)$	$lny_{P,D}(i)$ $=\frac{1}{1-\gamma}\ln[(1-\beta)(y_D-y(i))+y_M(i)]$ $-\gamma\,lny_M(i)\}$
$x_D(i+1)$	$x_D(i+1)=exp\{[\,lny_{P,D}(i+1)-a_1]/a_2\}$

c_i ($i=1\sim4$): Known parameters
x : Concentration of hydrogen
T : Time constant of nonlinear process considered
K : Gain
t_0: Sampling interval
x_M, Δx_M: Concentration of hydrogen of process model and its increment
u, Δu: Input flow of hydrogen and its increment
β : Parameter to be adjusted in command generator
τ, $\hat{\tau}$: Parameter and its estimate

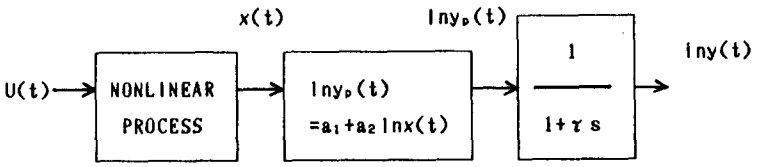

Fig. 1 Hydrogen flow and melt flow rate process (Nonlinear Process)

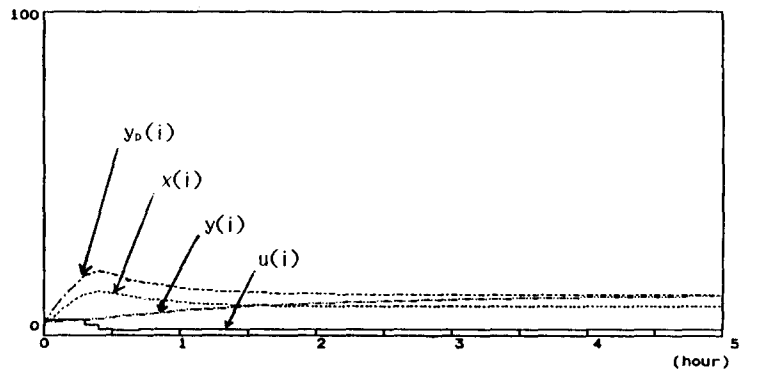

Fig. 2 Output runs of u(i), x(i), y(i), and $y_p(i)$ when β=0.2 and u_{max}=0.1

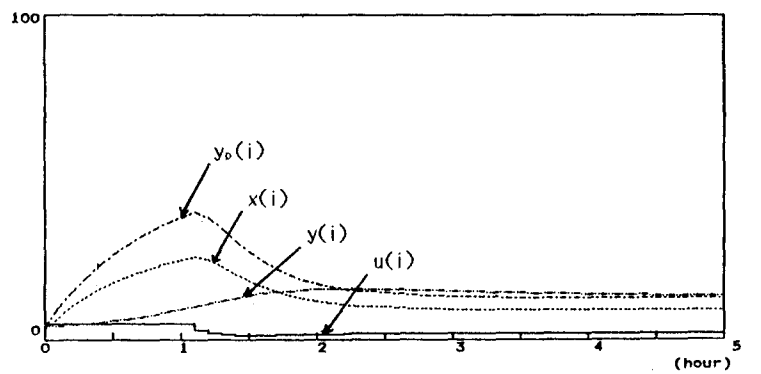

Fig. 3 Output runs of u(i), x(i), y(i), and $y_p(i)$ when β=0.5 and u_{max}=0.1

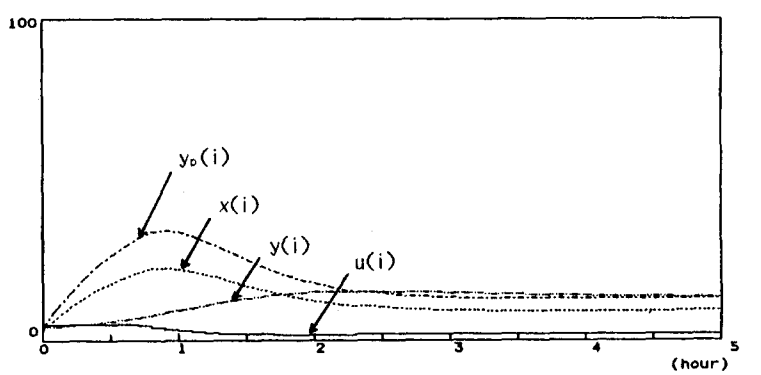

Fig. 4 Output runs of u(i), x(i), y(i), and $y_p(i)$ when β=0.8 and u_{max}=0.1